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ESTIMATION OF U.S. DESIGN TEMPERATURES USING DAILY MAXIMUM AND MINIMUM TEMPERATURES

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1. INTRODUCTION

Design temperatures are commonly used for the design, sizing, distribution, installation, and marketing of heating and air conditioning equipment (ASHRAE 2001). They are defined as temperatures that are exceeded a specific percentage of time (specific number of hours). The most recent estimates of design temperatures are in the 2001 ASHRAE Handbook (ASHRAE 2001). The calculation of design temperatures requires hourly data and thus values are provided in the ASHRAE Handbook for only about 500 U.S. stations with adequate hourly data.

There are over 6,000 cooperative observer locations that have at least 10 years of daily maximum and minimum temperature data since 1961. Obviously, hourly and daily temperature data are related. Hourly temperatures are constrained to be within the maximum and minimum. Also, since the temporal variation of hourly temperature is usually smooth, some hourly values will usually be close in magnitude to the daily maximum and minimum values. This study, supported by ASHRAE, investigated the development and application of techniques to estimate design temperatures using long-term records of daily maximum and minimum temperatures. If a technique of sufficient accuracy can be developed, the number of locations at which design temperatures can be calculated will be increased significantly.

2. DATA

ASHRAE provided the hourly and daily temperature data that were used to calculate design values for the 2001 Handbook. From the more than 500 locations listed, 14 stations were selected as a test set used in the techniques development. The locations are: Huntsville, AL; Key West, FL; West Palm Beach, FL; Wilmington, NC; Portland, ME; Indianapolis, IN; Amarillo, TX; Grand Island, NE; Minot, ND; Phoenix, AZ; Bakersville, CA; Sacramento, CA; Portland, OR; and Quillayute, WA. They span a wide range of climates, from desert to mid-latitude rain forest, and from subtropical maritime to cold interior continental.

The remaining locations were considered as a validation data set for the results described in Section 4. For validation, daily maximum and minimum temperatures were obtained from the National Climatic Data Center's TD-3200 data set (Phillips 2000). Not all stations listed in the 2001 Handbook have data in TD-3200. For the 48 contiguous United States, there were 379 stations with data in TD-3200.

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3. TECHNIQUES

Two past studies are relevant to the present study. Doesken and McKee (1983) developed a technique to estimate winter design temperatures from daily minimum temperatures for Colorado locations using a power law function. Kunkel (1986) developed a technique to estimate summer design temperatures from daily maximum temperatures for New Mexico locations using an exponential function. The chosen mathematical functions provided a good fit to the extreme portions of the cumulative distribution function (CDFs) of hourly temperatures and of daily maximum and minimum temperatures. The key in both studies is that they found a general way to relate the parameters of the mathematical functions that were fit to the daily CDF to the parameters of the functions that were fit to the hourly CDF. Thus, it was possible to use the daily CDF to estimate the parameters of the function of the hourly CDF and thereby calculate design values. Because the transformations were general (not specific to each hourly station), they could be applied to stations only reporting daily observations. However, since these techniques were developed for a set of stations in a single state, they can be applied with confidence only to locations with similar mountain climates. The challenge addressed in this study was to find techniques that apply across the different climatic regimes that characterize the United States. These two past studies were used as starting points for the present investigation.

A brief description of key aspects of the techniques are described here. Further information about these techniques can be found in Kunkel (2002).

3.1 Winter design temperature functions

The power law function for the hourly distribution can be expressed as follows

$$1 - P_h = a_h (T_h - T_{ow})^{b_h} \tag{1}$$

where P_h= the fractional probability of exceedance, T_h= hourly temperature, and T_{ow}, a_h, and b_h are parameters of the hourly CDF power law. T_{ow} is the value of temperature where P_h equals 1 and thus represents the lower limit of applicability of the function; this will be referred to as the origin temperature value. The parameter a_h is referred to as the scaling factor and the parameter b_h is the power. The corresponding function for daily minimum temperatures can be expressed as

$$1 - P_d = a_d (T_{min} - T_{ow})^{b_d} \tag{2}$$

where P_d = the fractional probability of exceedance for daily minimum temperature, T_{min} = daily minimum temperature, and T_{ow}, a_d, and b_d are the parameters of the daily CDF power law. It should be noted that, of the three power law parameters, T_{ow} was assigned the same value for both the hourly and the daily distributions while the scaling factor and power were varied.

Kunkel (1986) found that an exponential function produced the best results for the high temperature portion of the CDFs for New Mexico locations. In this study, the exponential function was also tested for application to estimation of winter design temperatures. The exponential function for the hourly distribution can be expressed as follows:

$$1 - P_h = e^{-r_h (T_h - T_{ow})^{s_h}} \qquad (3)$$

where T_{ow}, r_h, and s_h are the parameters of the hourly CDF exponential function. The corresponding function for daily minimum temperatures can be expressed as

$$1 - P_d = e^{-r_d (T_{min} - T_{ow})^{s_d}} \qquad (4)$$

where T_{ow}, r_d, and s_d are the parameters of the daily CDF exponential function. As was the case for the power law, T_{ow} was assigned the same value for both hourly and daily distributions. However, this value was not necessarily the same as the value used for the power law.

3.2 Summer design temperature functions

An analogous set of equations was tested for the estimation of summer design temperatures. In this case, mathematical functions were fit to the high temperature portion of the CDF of hourly temperatures and also to the high temperature portion of the CDF for daily maximum temperatures.

3.3 Empirical technique

An empirical approach was also tested. This approach consisted of the synthetic creation of an hourly CDF. This was based on the observation that the daily cycle of temperature is rather regular and predictable on most days. The maximum and minimum temperatures provide the upper and lower bounds, respectively, between which this cycle occurs. This technique was as follows. For each day, 24 hourly temperatures were estimated using the following equation:

$$T_h(i) = T_{min} + F(i)(T_{max} - T_{min}) \qquad (5)$$

where i = rank-ordered hour index (1-24) and F(i) = empirically-determined function relating the distribution of hourly temperatures to T_{max} and T_{min}. An hourly CDF was calculated from the synthetic hourly temperatures and design temperatures were then calculated directly from the synthetic CDF.

The values of F(i) were determined using the 14 test stations. For each day, the hourly temperatures were rank-ordered from lowest to highest. For each hour, the factor F was calculated as

$$F(i) = \frac{T_h(i) - T_{min}}{T_{max} - T_{min}} \qquad (6)$$

The values of F(i) were then averaged for all days to obtain individual station values of F(i). Finally, the values for the 14 stations were averaged to yield one relationship.

3.4 Origin temperature specification

Both Doesken and McKee (1983) and Kunkel (1986) used constant values of the origin temperature parameters T_{ow} (-37.2°C) and T_{os} (60.0°C). Initial comparisons indicated that this approach was not adequate for the wide range of climatic conditions across the U.S. Instead, much better results were obtained by relating the origin temperatures to each station's daily temperature climatology. Two different methods were tested. In the first method, the origin temperatures were related to the record high (T_{high}) and low (T_{low}) temperatures as follows

$$T_{ow1} = T_{low} - \Delta T_{w1} \qquad (7)$$

$$T_{os1} = T_{high} + \Delta T_{s1} \qquad (8)$$

where ΔT_{w1} and ΔT_{s1} are the origin offset temperatures for winter and summer design values, respectively. This is referred to hereafter as "Method 1." In the second method, the origin temperatures were related to the 0.998 ($T_{0.998}$) and 0.002 ($T_{0.002}$) exceedance values as follows:

$$T_{ow2} = T_{0.998} - \Delta T_{w2} \qquad (9)$$

$$T_{0s2} = T_{0.002} + \Delta T_{s2} \qquad (10)$$

This will be referred to as "Method 2." Method 2 was developed because it was discovered that there was considerable variation across the country in the relationship between record highs and lows and the temperatures in the range of interest (i.e., exceedance values of 0.996-0.980 and 0.02-0.004). From a meteorological viewpoint, some locations experienced single extreme events with record temperatures considerably lower than the second lowest event.

Extensive testing was performed to determine optimum values for origin temperatures, across a range of offsets from 0-20°C.

4. Results

Design values were estimated for the 379 stations listed in ASHRAE (2001) that had data in TD-3200 and that were not used in techniques development. The period of record used to calculate design values in ASHRAE (2001) varied from station to station, generally being either 1961-1993 or 1982-1993.

The period of record for TD-3200 data was matched to the ASHRAE (2001) period and thus also varied from station to station. It was found that Method 2 performed better than Method 1 and the following results are for Method 2. Figures 1, 2, and 3 show scatter plots of the results for the 0.990 probability of exceedance estimates using the empirical technique, the exponential function, and the power law, respectively. All three techniques produce good estimates of the design values. Table 1 provides a statistical summary of the results. All techniques produce estimates with mean absolute errors of less than 1.0°C and mean square errors of around 1.0°C or less. Mean biases are generally very low at less than 0.2°C, except for the empirical technique which underestimates summer design values by 0.5°C and the exponential function which underestimates the 0.990 winter design value by 0.3°C.

5. CONCLUSIONS

This study investigated three techniques to estimate design temperatures from daily minimum and maximum temperatures. Two techniques using mathematical functions (exponential and power law) were adopted from previous geographically-limited studies and further developed so that they could be applied to the entire U.S. A third technique used an empirical approach to develop a synthetic CDF. The major conclusions follow:

- All three techniques produce rather good estimates of design temperatures.
- The two mathematical functions require specification of an origin temperature. Better results were obtained when the origin temperatures were related to the 0.998 probability of exceedance value for daily minimum temperature (winter) and to the 0.002 probability of exceedance value for daily maximum temperature (summer) than relating them to the record low and high temperatures.
- The empirical technique produces best results for winter design values with mean absolute errors about half those produced by the exponential function and power law techniques.
- The power law technique relating the origin temperature to the 0.002 probability of exceedance value produces slightly smaller errors than the other techniques for summer design values. However, all three techniques produce rather small errors, although the mean bias for the empirical technique is considerably larger.

Based on the above results, the empirical technique is recommended for estimation of winter design values, while either the exponential function or power law technique will produce reliable estimates for summer design values.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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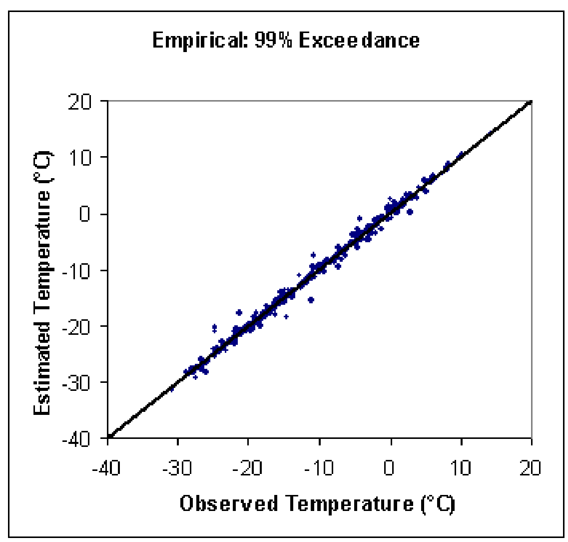

Figure 1. Estimated design values using the empirical technique vs design values published in ASHRAE (2001) for the 0.990 probability of exceedance. Units are °C. The solid line shows a 1:1 relationship.

Table 1. Errors in estimated design temperatures for stations in ASHRAE (2001)

	Probability of Exceedance				
	0.996	***0.990***	***0.020***	***0.010***	***0.004***
Experimental Function - Method 2					
Mean Absolute Error	0.9°C	0.8°C	0.6°C	0.5°C	0.5°C
Mean Square Error	1.2°C	1.2°C	0.8°C	0.7°C	0.7°C
Bias	0.0°C	-0.3°C	0.0°C	0.1°C	0.1°C
Power Law - Method 2					
Mean Absolute Error	0.7°C	0.7°C	0.4°C	0.4°C	0.5°C
Mean Square Error	1.1°C	1.1°C	0.5°C	0.5°C	0.5°C
Bias	-0.2°C	-0.2°C	-0.1°C	-0.1°C	-0.1°C
Empirical					
Mean Absolute Error	0.4°C	0.4°C	0.6°C	0.6°C	0.6°C
Mean Square Error	0.6°C	0.6°C	0.6°C	0.6°C	0.7°C
Bias	0.0°C	-0.1°C	-0.5°C	-0.5°C	-0.5°C

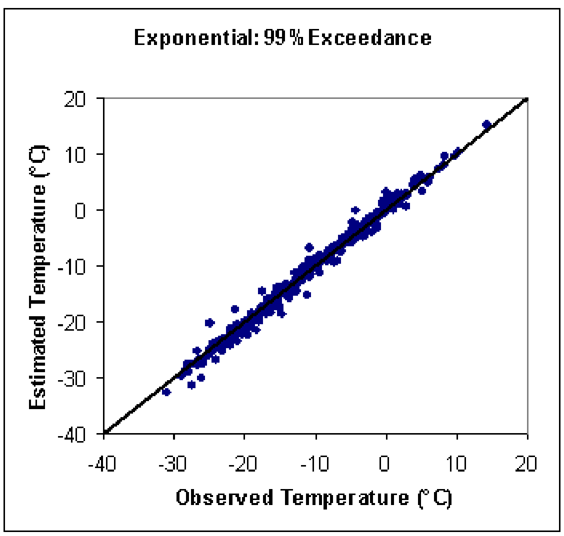

Figure 2. Estimated design values using exponential function - Method 2 vs. design values published in ASHRAE (2001) for the 0.990 probability of exceedance. Units are °C. The solid line shows a 1:1 relationship.

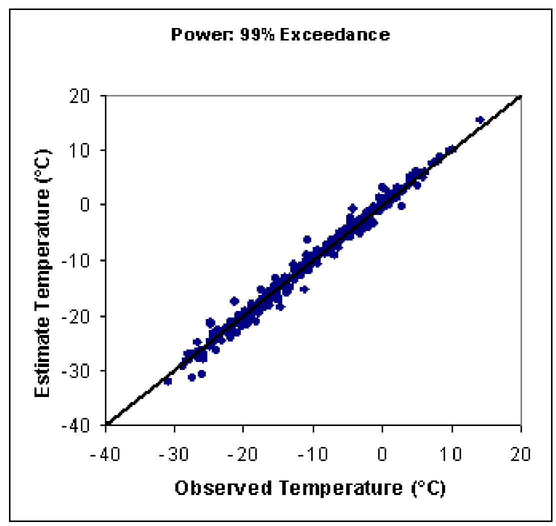

Figure 3. Estimated design values using the power law - Method 2 vs design values published in ASHRAE (2001) for the 0.990 probability of exceedance. Units are °C. The solid line shows a 1:1 relationship.