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THE IMPACT OF THE ATMOSPHERIC BOUNDARY LAYER ON MOUNTAIN FORCED GRAVITY WAVES

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1. INTRODUCTION

Mountain forced gravity waves have been studied extensively for over 50 years, from theoretical and numerical modelling studies through to laboratory and observational experiments. However, the influence of the atmospheric boundary layer on the dynamics of these waves has received little attention. Recent progress has been made by using surface friction to represent simple boundary layer effects in numerical modelling studies of mountain flow (Richard *et al* 1989, Grubisic *et al* 1995, Olafsson and Bougeault 1997a, and Peng and Thompson, 1998, 2000). These studies show that surface friction acts to reduce the strength and amplitude of lee side phenomena such as gravity wave activity, downslope windstorms and lee vorticies. In the present contribution a simple two layer theory of the atmosphere is proposed and the results of this model are suggestive of the controlling parameters which act in this situation.

2. TWO LAYER MODEL

Consider a simple, two-dimensional ($x - z$), two-layer model of the atmosphere. In full generality the basic state wind profile is considered to be a function of height, $\overline{u} = U(z)$. In an attempt to represent the gross, leading order effect of a well mixed, neutral boundary layer the lower layer is considered to be inviscid with neutral stability, $N = 0$. Above a prescribed boundary layer depth, $z = D$, the upper layer is considered to have constant static stability given by $N = constant$ (where N is the Brunt-Vaisala frequency). Analytical solutions to the problem of atmospheric flow over an isolated mountain are sought for two different basic state wind profiles in the lower boundary layer air: when the windspeed is constant, and when the windspeed is linearly sheared with height. In both situations the wind is taken to be constant in the upper layer, and is continuous with appropriate dynamical conditions across the interfacial boundary.

The steady state, non-rotating governing equation of motion for the considered linear, Boussinesq problem is:

$$\frac{\partial^2 w'}{\partial z^2} + \frac{\partial^2 w'}{\partial x^2} + l^2 w' = 0 \ , \qquad (1)$$

where l^2 is the classical Scorer parameter ($l^2 = \frac{N^2}{U^2} - \frac{1}{U}\frac{d^2U}{dz^2}$). In the lower layer eq. (1) is solved with the lower boundary condition, $w' = U\frac{dh}{dx}$, where $h(x)$ is the mountain profile. The flow in the upper layer is then found from solving (1) with the specified basic state atmospheric conditions and matching interfacial conditions at the layer boundary $z = D$. The classical Witch of Agnesi mountain profile is chosen for its simple Fourier transform, so that

$$h(x) = \frac{Ha^2}{a^2 + x^2}$$

with 'a' denoting the half-width of the mountain.

3. CONSTANT LOW LEVEL WIND

The first situation considered is that of a uniformly constant windspeed throughout the atmosphere. The basic state wind profile is taken to be $\overline{u} = U_o$ in both layers, and the static stability is given by $N = 0$ in the lower layer, and $N = constant$ in the upper layer. Equation (1) then reduces to,

$$\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} = 0 \qquad (2)$$

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in the lower layer. In this layer the Scorer parameter is zero and gravity waves are not supported. Following Smith (1979, page 101), the analytical solution for the flow in the lower layer can be written in terms of the vertical velocity as,

$$w(x,z) = \frac{-2HU_o ax(a+z)}{((a+z)^2+x^2)^2} \tag{3}$$

or the streamline displacement field from $w = U\frac{\partial \eta}{\partial x}$ as,

$$\eta(x,z) = \frac{Ha(a+z)}{(a+z)^2+x^2} = \left(\frac{Ha}{a+z}\right)\left[\frac{(a+z)^2}{(a+z)^2+x^2}\right] \tag{4}$$

for $0 < z < D$. For the flow in the upper layer, scale analysis for appropriate values of the mountain halfwidth a and atmospheric parameters N and U_o reduce the governing linear equation, (1) to the hydrostatic form,

$$\frac{\partial^2 w}{\partial z^2} + \frac{N^2}{{U_o}^2} w = 0 \text{ or } \frac{\partial^2 \eta}{\partial z^2} + \frac{N^2}{{U_o}^2}\eta = 0 \tag{5}$$

This is now solved with the interfacial boundary condition of $\eta(x,D) = H'A^2/(A^2+x^2)$ $(H' = Ha/A,\ A = a+D)$ acting as the bottom boundary for the upper layer flow, resulting in the flow field,

$$\eta(x,z) = H'A\left(\frac{A\cos l(z-D) - x\sin l(z-D)}{A^2+x^2}\right) \tag{6}$$

for $z \geq D$ (where $l = N/U_o$). The flow response given by (6) represents a hydrostatic, vertically propagating gravity wave field in the upper layer. The vertical flux of horizontal momentum in the upper layer wave field can be calculated and compared to the result for a linear hydrostatic wave field in an inviscid 'single' layer atmosphere with $\overline{u} = U_o$ and $N = constant$. It can be seen that $M_{twolayer} = \beta M_{onelayer}$ with β being the reduction factor given by,

$$\beta = \left(\frac{\frac{a}{D}}{(1+\frac{a}{D})}\right)^2 ,$$

and M denoting the momentum flux in the x-direction calculated as, $M = \rho_o \int_{-\infty}^{\infty} u'w\, dx$.

4. LINEARLY SHEARED LOW LEVEL WIND

The second situation considered in this contribution is that of a basic state wind profile which is linearly sheared in the lower boundary layer air. The static stability is still considered to be neutral so that $N = 0$, and the wind profile is given by $\overline{u}(z) = U_b + Cz$ for $0 < z < D$. The windspeed at the ground ($\overline{u}(0) = U_b$) is chosen to be non-zero for ease of mathematical analysis, but can be chosen to be arbitrarily close to zero to closely mimic a true viscous bottom boundary condition within the atmospheric boundary layer. The vertical velocity in the lower layer flow is once again found from the governing equation, (2), and is given by eq. (3). However, the streamline displacement field is now

$$\begin{aligned}\eta(x,z) &= \frac{HU(0)}{U(z)}\left(\frac{a}{a+z}\right)\left[\frac{(a+z)^2}{(a+z)^2+x^2}\right] \\ &= \frac{U_b}{U_b+Cz}\left(\frac{Ha}{a+z}\right)\left[\frac{(a+z)^2}{(a+z)^2+x^2}\right]\end{aligned} \tag{7}$$

for the linearly sheared wind profile. In the upper layer the basic state atmospheric conditions of constant N and U_o are used with the resulting hydrostatic governing linear equation given by (5). The Fourier integral solution to (5) with the interfacial/bottom boundary condition of $\eta(x,D) = U_b h(x)/U_o$ results in the streamline displacement field,

$$\eta(x,z) = H''A\left(\frac{A\cos l(z-D) - x\sin l(z-D)}{A^2+x^2}\right) \tag{8}$$

for the vertically propagating wave field in the upper layer ($z \geq D$) with $H'' = H\left(\frac{U_b}{U_o}\right)\left(\frac{a}{A}\right)$. The same comparison can be made between the momentum flux for a linear hydrostatic gravity wave field from a 'single' layer inviscid atmosphere forced by the same mountain profile. Then

$$M_{twolayer} = \beta\gamma M_{onelayer}$$

with $\gamma = \left(\frac{U_b}{U_o}\right)^2$ and β defined from the previous case.

In this situation two non-dimensional parameters control the gravity wave flow response in the upper layer. The second, new parameter is γ, the ratio squared of the ground level windspeed to the windspeed at the top of the considered boundary layer air mass. The windspeed at the interfacial boundary ($\overline{u}(D) = U_o$) is taken to be greater than the windspeed at the ground ($\overline{u}(0) = U_b$) giving rise to an additional dependence of the momentum flux decrease and wave amplitude decrease on the ground to boundary top windspeed ratio.

5. DISCUSSION

A simple theory has been presented of the leading order perturbations resulting from a mountain completely contained within the boundary layer. The nonlinear turbulent nature of the flow within the neutral boundary layer is not considered, only the leading order effect of the neutral stratification. The theory highlights two important non-dimensional parameters controlling the gravity wave field in the upper layer. These are a/D, the ratio of the scale of the mountain half-width to the boundary layer depth, and U_b/U_o, the ratio of the near surface windspeed to the windspeed at the top of the boundary layer. The two situations analysed show that the lower boundary layer acts to reduce the wave amplitude compared to inviscid, single layer theory. In a more realistic, atmosphere this reduction in wave amplitude will effect the height at which the gravity wave field overturns, moving it to greater heights. With more complex and realistic wind profiles this effect of the boundary layer may remove potential breaking which would otherwise have occurred in the absence of the boundary layer. Olafsson and Bougeault (1997b) have provided persuasive evidence that this was the case during the PYREX observational campaign of October 1990.

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