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1. INTRODUCTION

The Canadian mesoscale model (Benoit et al., 1997) provided daily forecasts across the Alps at 3-km resolution during the entire MAP field phase of 1999. The model had been extensively optimized in previous years for efficiency on various computer architectures (Thomas et al., 1997) and accuracy inside as well as at the model boundaries (Thomas et al. 1998). An overview of its performance during MAP is given in Benoit et al. (2002). Following the experiment, it became more and more evident that there was a problem related to finescale orography forcing in the model (Schaer et al., 2002). The problem is soon to be explained (Klemp et al., 2002) and no doubt corrected. Here we describe a modification to the model dynamics kernel which in particular greatly reduces its spurious sensitivity to finescale orography.

2. THE MODEL

The main feature of the MC2 (Mesoscale Compressible Community) model is its absolutely stable semi-implicit semi-Lagrangian numerical time integration scheme developed by the late Andre Robert (Robert, 1969; Kwizak and Robert, 1971; Robert, Henderson and Turnbull, 1972, for the semi-implicit part and Robert, 1981; Robert, 1982; Robert, Yee and Ritchie, 1985; Tanguay, Robert and Laprise, 1990, for the semi-Lagrangian part). The scheme here is applied to time discretize the non-hydrostatic meteorological equations, the Euler exact equations of motion.

In the horizontal the equations are written in a so-called invariant form readily admitting, with the specification of a single scaling parameter S , a choice of orthogonal coordinates: cartesian, spherical and even cylindrical (rotating annulus experiments) as well as all conformal mappings (stereographic, mercator, ...) of the spherical earth. In the vertical a terrain-following oblique coordinate system of the height-variety is available. Recently the metric has been generalized to allow for maximum flexibility in the choice of vertical coordinate definition. In particular, we may use the SLEVE coordinate of Schaer et al. (2002).

Space discretization is done using second order finite differences with variables distributed on a set of staggered grids (Arakawa-C type in the horizontal,

Charney-Phillips type in the vertical). This grid system is particularly well suited for deriving the elliptic-type numerical equation that characterizes the semi-implicit scheme. Since the time discretization scheme is absolutely stable, a balance can easily be achieved between space and time truncation errors, so higher order schemes in space are not felt necessary.

Horizontal resolution is taken to be uniform in the chosen coordinate (e.g. in spherical coordinates it is uniform in latitude and longitude while in mercator projection it is uniform in map coordinates). Vertical resolution may be varied at will.

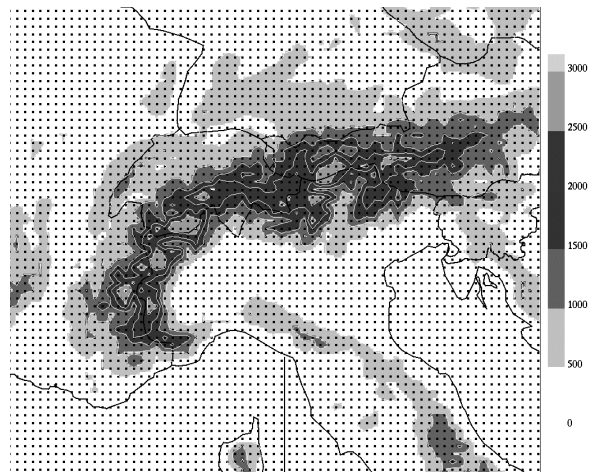


Figure 1. Orography of the Swiss Model at 14-km resolution. Partial domain; dots are the SM grid points.

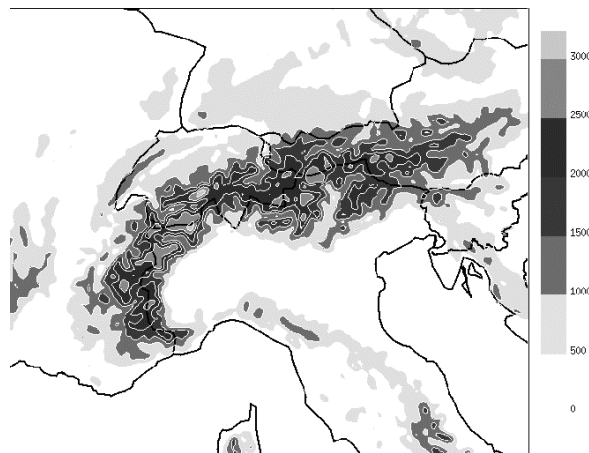


Figure 2. Orography of MC2 at 3-km resolution. Entire domain.

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Coupled to this dynamical kernel, in a *splitting up* mode, is an extensive library of parameterization schemes for physical effects pertinent for a variety of situations and scales.

3. THE PROBLEM

From the beginning we were well aware of the sensitivity of MC2 to finescale orography. And this is well illustrated in Figures 1 and 2, first presented at a previous MAP meeting (Benoit et al., 1999). Figure 1 shows the orography at 14-km resolution of the Swiss model. Figure 2 shows the orography of MC2 at 3-km resolution as used during the MAP field phase. It is obvious that the orography prepared at the time for MC2 is smooth (4- Δx filtering), considering that it has five times the resolution of the SM. The grid size of MC2 is 350 x 300.

The problem related to finescale orography forcing in MC2 was exposed by Leuenberger et al. (2001). This is further confirmed by Schaer et al. (2002) which show in particular that a substantial reduction of small-scale noise at intermediate and upper model levels - noise related to small-scale topography features - is achieved in MC2 by adopting a terrain-following coordinate system with a scale-dependent decay of the terrain features with height. This way, the need to pre-smooth the orography is being substantially reduced. Examples of the problem

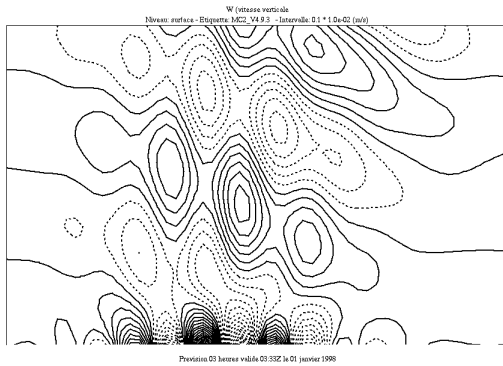


Figure 3. ISOthermal basic state.

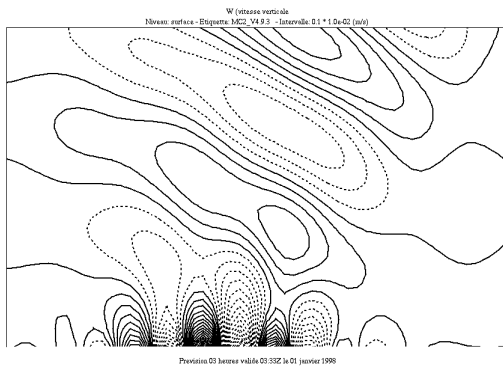


Figure 4. VARIABLE basic state. are shown and further discussed in the next section.

4. NOT THE SOLUTION BUT...

Schaer et al (2002) describe an idealized experiment of stably stratified flow over a two-dimensional mountain ridge having a bell-shaped structure with superposed 8- Δx small-scale variations, the basic-state flow being uniquely defined by the Brunt-Vaisala frequency $N=0.01 \text{ s}^{-1}$ and the horizontal wind velocity $U=10 \text{ m/s}$. With surface temperature $T_0=288 \text{ K}$, this corresponds to a realistic tropospheric temperature gradient. For the model, which uses an isothermal basic state, this leads to fairly large temperature and pressure perturbations. This is the situation where the model stationary solution shows considerable distortions in the small scales (Figure 3). On the other hand, with $N=0.01871$, the modeled atmosphere is isothermal just like its basic state and the simulation becomes nearly perfect. This suggested to us the possible benefits of introducing a non-isothermal basic state. In particular we could have the option of a basic state based on constant N . This is what was done. The result when $N_0=N=0.01$ is shown in Figure 4.

For real cases, it is not suitable to use a basic state with constant N . We have adopted the following hyperbolic tangent profile for temperature:

$$T_* = T_{surf} + (T_{surf} - T_{top}) \tanh(z/H)$$

with $T_{surf}=288 \text{ K}$; $T_{top}=220 \text{ K}$; $H=10000 \text{ m}$. Figures 5 and 6 compare cross-sections across the Rocky mountains near Vancouver showing vertical motion (contours every 5 cm/s) for 6-hour 10-km resolution forecasts of MC2, respectively with ISOthermal and VARIABLE basic states. Only a 2- Δx filtering is applied on the orography. Noise is reduced in the VAR integration, especially below 5 km. As a measure of success in eliminating the noise, we show the same forecast done with the SLEVE coordinate and the isothermal basic state in Figure 7. The SLEVE integration clearly wins at high altitudes in particular since we are not able to fit the atmospheric mean profiles everywhere and especially near the tropopause. Combining the SLEVE and VAR features (not shown) slightly reduces the noise again. While obviously not solving the problem, the option of a variable basic state, which we describe in the next section, is a valuable addition to the dynamics kernel of our model.

5. THE MODIFIED EQUATIONS

If we consider the model equations written for simplicity in cartesian coordinates in an absolute frame of reference and not considering moist processes:

$$\frac{dV}{dt} + RT\nabla q + \mathbf{g} = F_V; \quad \frac{dT}{dt} - \frac{RT}{c_p} \frac{dq}{dt} = \frac{Q}{c_p}; \quad \frac{c_v}{c_p} \frac{dq}{dt} + \nabla \cdot \mathbf{V} = 0$$

with $q = \ln p$, from which one subtracts a non-isothermal basic state characterized by $T_*(z), q_*(z)$ and defining the perturbation quantities, $T' = T - T_*, q' = \ln(p/p_*)$, one obtains:

$$\begin{aligned} \frac{d\mathbf{V}}{dt} + RT_* \nabla q' - \mathbf{g} \frac{T'}{T_*} &= F_V - RT_* \nabla q' \\ \frac{dT'}{dt} - \frac{RT_*}{c_p} \frac{dq'}{dt} + \left(\frac{\partial T_*}{\partial z} + \frac{g}{c_p} \right) w &= \frac{Q}{c_p} + \frac{RT_*}{c_p} \frac{dq}{dt} \\ \frac{c_v}{c_p} \frac{dq'}{dt} - \frac{g}{c_*^2} w + \nabla \cdot \mathbf{V} &= 0 \end{aligned}$$

with $c_*^2 = (c_p / c_v) RT_*$. Besides the fact that the basic state parameters are variable, the only difference between this and our original model system is the presence of the gradient $\partial T_* / \partial z$. For the implementation of the semi-implicit semi-Lagrangian scheme, it is necessary to eliminate variable coefficients in front of material derivatives. This happens once here in front of dq'/dt in the thermodynamic equation. All considered, it seemed advantageous to define buoyancy $B = g T' / T_*$ and generalized pressure $P = R T_* q'$, to obtain at the end:

$$\begin{aligned} \frac{d\mathbf{V}}{dt} + (\nabla - \beta_* \mathbf{k}) P - B \mathbf{k} &= F_V - \frac{B}{g} (\nabla - \beta_* \mathbf{k}) P \\ \frac{d}{dt} (B - \gamma_* P) + w N_*^2 &= \left(\gamma_* Q - \frac{R}{c_v} B \nabla \cdot \mathbf{V} \right) - \beta_* B w \\ \frac{d}{dt} \left(\frac{P}{c_*^2} \right) - \frac{g}{c_*^2} w + \nabla \cdot \mathbf{V} &= 0 \end{aligned}$$

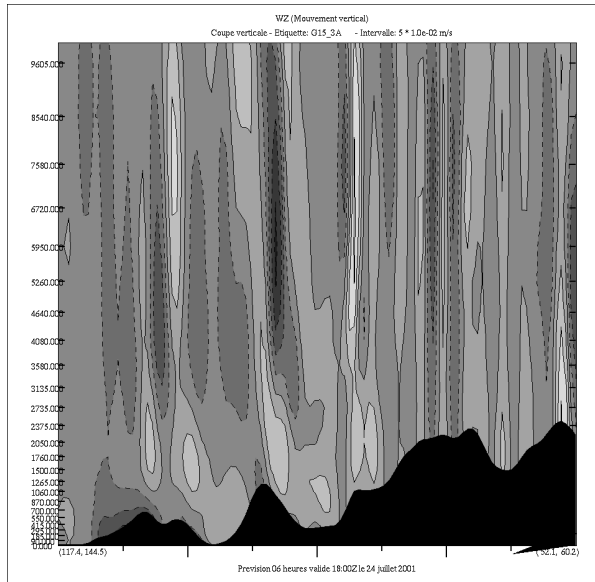


Figure 5. A noise problem. ISOthermal basic state.

with $\gamma_* = g / c_p T_*$ and $\beta_* + \gamma_* = N_*^2 / g$. In effect, the elliptic problem to be solved is then essentially unchanged except for the variability of the coefficients. After space and time discretization, we get:

$$\frac{P}{c_*^2} - \Delta t^2 \nabla_H^2 P - \Delta t^2 \left(\frac{\partial}{\partial z} - \frac{g}{c_*^2} \right) v_* \left(\frac{\partial}{\partial z} - \frac{N_*^2}{g} \right) P = Q_q^*$$

with $v_* = (1 + \Delta t^2 N_*^2)^{-1}$, the cartesian derivative operators having been first transformed to terrain-following Z-coordinates:

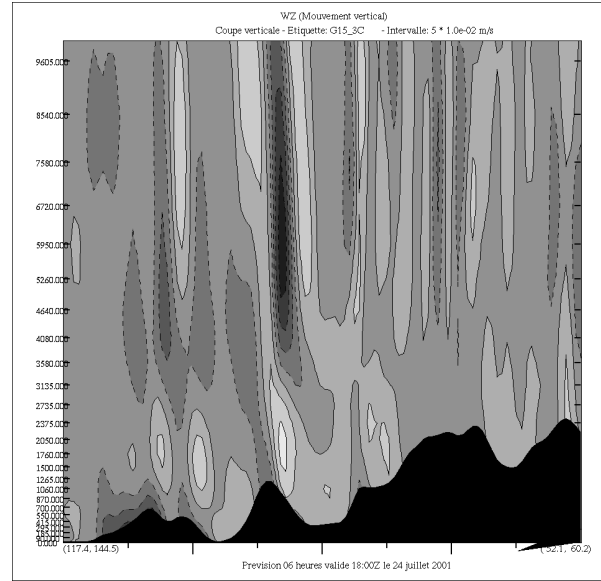


Figure 6. Reduced noise. VARIABLE (tanh) basic state.

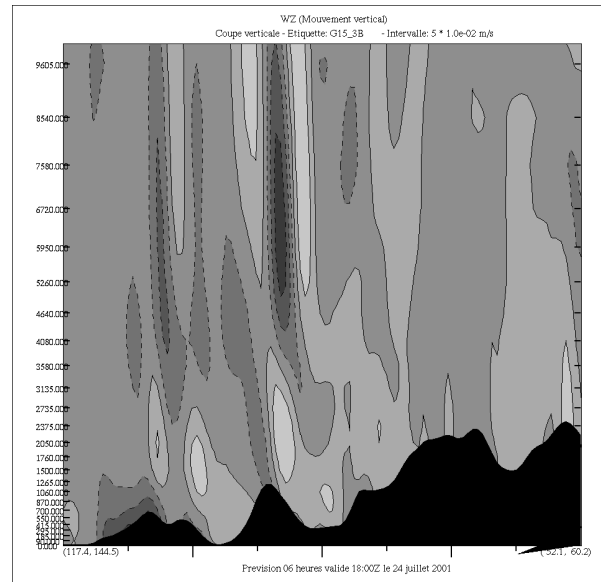


Figure 7. Reduced noise. SLEVE coordinate.

$$\nabla_H = \left(\frac{\partial}{\partial x} - \frac{\partial z}{\partial x} \frac{\partial}{\partial z} \right) \mathbf{i} + \left(\frac{\partial}{\partial y} - \frac{\partial z}{\partial y} \frac{\partial}{\partial z} \right) \mathbf{j}; \quad \frac{\partial}{\partial z} = \frac{\partial Z}{\partial z} \frac{\partial}{\partial Z}$$

and then translated to finite difference and averaging operators.

6. CONCLUSION

The spurious sensitivity of MC2 to finescale orography is being actively investigated and ways to reduce this sensitivity have been found here and elsewhere. Following the study of Klemp et al. (2002), it is clear that in our case the problem is related to numerical inconsistencies between the semi-Lagrangian scheme and other parts of the code. We have recently coded and tested a consistent Eulerian scheme. The distortions present in the stationary mountain-wave study when using the semi-Lagrangian scheme disappears when using the Eulerian scheme. The Eulerian scheme will therefore become our control scheme as far as this noise problem is concerned.

Our contribution in this respect has been to study the importance of the choice of basic state used in the implementation of semi-implicit semi-Lagrangian scheme. Traditionally, mostly isothermal basic states have been used not only because of the simplicity of the resulting elliptic problem but also for stability reasons. It is to be noted that the basic state temperature adopted here varies as a function of height only, meaning that, in model coordinates, it is a general function of position.

When the model is initialized in hydrostatic equilibrium, if that state corresponds to the chosen hydrostatic basic state, no matter what the underlying terrain is, all model variables vanish and for all times: this is the *exact noflow condition*.

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