3.4 RE-EVALUATION OF EXTREME RAINFALL AREAL REDUCTION FACTORS

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1. INTRODUCTION

The estimation of flood frequency values, important for the design of hydraulic structures such as dams, bridges and canals, has traditionally been done using statistically derived curves that are fit to stream flow annual maximums (Water Resources Council, 1967). With the advent of sophisticated rainfall-runoff models, as well as the fact that stream flow data is temporally and spatially limited, precipitation values are now generally used instead. Specifically, intensity-duration-frequency (IDF) curves are used to estimate the depth of rainfall at a point for a specified duration and return interval. Since IDF curves are based on point rainfall, it is necessary to have a means of converting these point values to areal values. The standard procedure for converting point depths to a mean areal depth is to multiply the average of the point depths for a given duration, frequency and area by a depth-area correction, or an areal reduction factor (ARF).

Perhaps the most common source of ARF for the U.S. is Technical Paper 29 (TP-29) (U.S. Weather Bureau, 1957). TP-29 defines ARF as the ratio of the mean annual maximum areal rainfall to the mean annual maximum point rainfall. Their analysis shows that area, as well as storm duration, has a large influence on ARF. Accordingly, TP-29 provides ARFs for basin areas ranging up to approximately 1000 km^2 and for storm durations of 1, 3, 6 and 24 hours. Due to the relatively short record length of precipitation data available (at most, 15 years), frequency considerations could not be accurately determined. TP-29 also assumes the area-depth relationship is independent of geographic location. Therefore, TP-29 presents a single, geographically averaged ARF-area curve based on the 2.33-year return period, which is often used to extrapolate up to 100 years.

As defined and developed in TP-29, the 24-hour depth-area ratio has been re-evaluated based on 47 years (1949-1995) of rainfall data from the Cooperative Observer (COOP) network. ARF is calculated for two diverse locations in the eastern U.S. wihich maximize station density; northeastern New Jersey and southwestern North Carolina. The effect of the longer period of record on ARF, as well as its variation over frequency and geographic location is explored.

2. METHODOLOGY

The general methodology of ARF calculation used in TP-29 is retained and is mathematically represented by Equation 1. The daily areal rainfall is calculated by averaging each station's daily point rainfall. The highest of these in each year is selected and an overall mean is computed by averaging the maximum annual rainfall in each year. This average constitutes the numerator in Eq. 1. The largest point measurement at each station in each year is subsequently recorded. The grand mean over all stations and over all years of record is calculated and constitutes the denominator in Eq. 1.

$$ARF_{2.33} = (\Sigma\Sigma\, R_{ij}^* / yn) / (\Sigma\Sigma\, R_{ij} / yn) \qquad (1)$$

where R_{ij} = annual maximum point rainfall for year j at station i
R_{ij}^* = point rainfall for station i on the day the annual maximum areal rainfall occurs in year j
y = number of year of record
n = number of stations in the area

In addition to TP-29's averaging of point rainfall depths to produce an areal estimate, Thiessen polygons and inverse distance weights (squared grid unit distances) have also been used to re-evaluate ARF. Such spatial interpolation routines were obtained from the Mean Areal Precipitation Program (MAP3) developed by the National Weather Service Hydrologic Research Laboratory (Lindsey, 1993). The grid system used in MAP3 is identical to that used in the Hydrologic Rainfall Analysis Project (HRAP) 4 km x 4 km grid.

Since daily precipitation totals at COOP stations are typically recorded at different times, all COOP stations have been adjusted to a standard observation time of 0800 using nearby, hourly reporting COOP stations (DeGaetano, 1995). This observation time was selected to minimize the number of adjustments, as over 50% of all station-years (for both New Jersey and North Carolina) reported between the hours of 0700 and 1000, with 25% reporting at 0800 in the New Jersey study area and 30% in the North Carolina area.

To investigate the functional dependence of ARF on return period, several probability distributions, including the beta-P, Gumbel and log-Pearson type 3, were evaluated to determine which most accurately estimates extreme point and areal precipitation. Distribution parameters were estimated using the method of maxi-

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mum likelihood following Wilks (1993). Using a standard bootstrapping procedure, the beta-P distribution best represented the most extreme rainfall amounts at a point, as well as over an area, and is the distribution adopted for this study. For example, the average 50-year areal rainfall (based on 1000 bootstraps) over the 3500 km^2 New Jersey basin was estimated to be 14.61 cm using the beta-P distribution; 13.69 cm using the log-Pearson type 3; and 12.29 cm using the Gumbel, while the empirical 50-year areal rainfall is 16.97 cm.

To calculate ARF associated with a given return period, the general form of Eq. 1 is still used. The j-year areal precipitation estimated with the beta-P distribution is substituted into the numerator. Similarly, the average j-year point precipitation over all stations in the basin is substituted into the denominator of Eq. 1.

Due to the more mountainous nature of the North Carolina study area (elevation ranges from 200 to 2000+ meters), a "topographical bias" correction factor has been developed that is applied to Thiessen and inverse distance weighted HRAP grid point estimates. Statistical relationships between several topographic characteristics and the bias of the median estimated daily rainfall annual maximum (RMED) were investigated akin to Prudhomme and Reed (1998). Among the topographical variables tested are the actual station elevation and the elevation from a digital elevation model (DEM) at 5-, 10- and 15-minute resolution. Furthermore, for each of the 8 cardinal directions, distance to the sea (i.e. moisture source), slope (at 10, 20 and 30 km) and exposure, were investigated.

Using univariate, least-squares regression, the slope of the terrain is best able to account for the variation in RMED. For inverse distance weights, the product of the eastern slope at 20 km and the southern slope at 10 km ($SLOPE20_E * SLOPE10_S$) describes the largest percentage of variance in RMED (R^2=46%). For Thiessen polygons, the southeastern slope at 10 km ($SLOPE10_{SE}$) is best able to describe the variance in RMED (R^2 = 26%). Both regressions are significant at the 99% level.

A corresponding and significant ($p < 0.05$) correction factor was also identified for New Jersey. The product of the northern slope at 20 km and the average of the western slope at 10 and 20 km ($SLOPE20_N * (SLOPE10_W + SLOPE20_W) / 2$) best accounts for the variance in RMED (R^2 = 30%) using inverse distance weights. No significant relationship ($p > 0.10$) could be found for New Jersey using Thiessen polygons.

3. RESULTS

Figure 1 shows the ARF-area curves for New Jersey, which were calculated for the initial 3500 km^2 basin, as well as 50 subcatchments. These subcatchments are binned based on their area and an average ARF is calculated for each bin. Binning helps to reduce the relatively large variability in ARF for a given basin area and ensures that ARF for a specific basin size is representative of several unique basins of approximately the same size.

ARF-area points were fit using a nonlinear least squares fit to an exponential model with coefficients of determination, R^2, between 74% (the 50-year ARF) and 86% (the 2-year ARF). The re-evaluated NJ ARF is shown using inverse distance weights with the bias correction, though the difference between the adjusted and unadjusted NJ ARF is relatively small (maximum difference of 1-2% at 4000 km^2). ARF generated using Thiessen polygons and standard averages are not shown as all three spatial interpolation methods produced similar ARF-area curves for New Jersey. TP-29's ARF-area curves are conveniently represented using Leclerc and Schaake's (1972) equation:

$$\begin{aligned} ARF = 1 - \exp(-1.1t^{0.25}) + \\ \exp(-1.1t^{0.25} - 0.003863A), \end{aligned} \qquad (2)$$

where t = duration time, in hours,
and A = area, in square kilometers

Figure 1 shows that for basins < 500 km^2, the difference between ARF at the various return periods is small. For basins larger than 500 km^2, however, ARF clearly depends on the return period. For a fixed area,

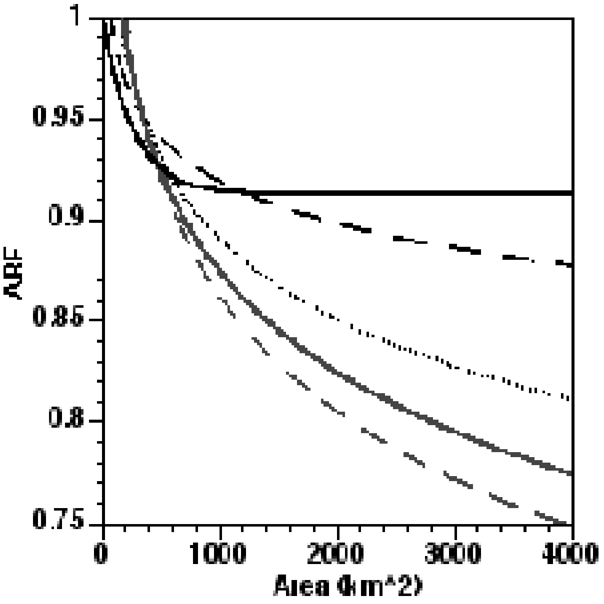

Figure 1. New Jersey area-depth curves for the 2-year (black dashed); 10-year (black dotted); 25-year (gray solid) and 50-year (gray dashed) return interval. TP-29 ARF (black solid) is shown using Eq. 2.

as return period increases, ARF decreases. This dependence of ARF on return period qualitatively agrees with several other studies, including Stewart (1989), Niemczynowicz (1982) and Myers and Zehr (1980).

TP-29 ARF most closely resembles the 2-year, re-evaluated ARF, as would be expected. They become more dissimilar, however, for areas > 1000 km^2, where the slope of the TP-29 ARF-area curve approaches zero. TP-29 ARF decreases to only 91% of the average point value for a basin size of 4000 km^2, whereas the re-evaluated ARF decreases to 88%. The difference between TP-29 ARF and the re-evaluated ARF becomes even greater for larger return periods. The newly calculated ARF for return periods of 25 and 50 years decreases to 79% and 74%, respectively.

The adjusted ARF-area curve for North Carolina is shown in Figure 2. ARF is calculated for the initial 18,000 km^2 basin, as well as 47 subcatchments. The sub-basins are then grouped according to their area and an average ARF is computed for each group. R^2 for the NC ARF-area curve ranges between 72 and 87%, with the better R^2 again associated with the lower return intervals.

Similar to New Jersey, the bias adjustment did not have a large effect on the ARF-area curve. The maximum difference between the adjusted and unadjusted ARF using inverse distance weights is between 1-2%, depending on the return period. An even smaller difference exists when comparing adjusted and unadjusted ARF based on Thiessen polygons. There is, however, a qualitative agreement between the difference in adjusted and unadjusted ARF at NJ and NC. Adjusted ARF (using inverse distance weights) does not decrease as rapidly with increasing area as the corresponding unadjusted ARF.

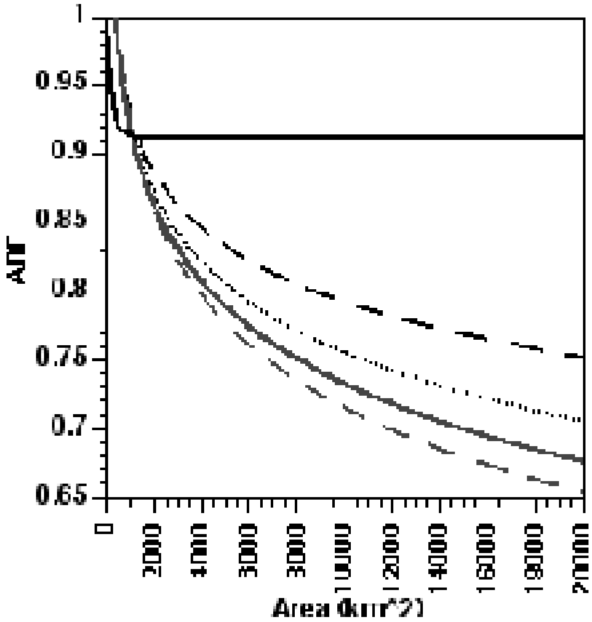

Figure 2. As in Fig. 1, except for North Carolina.

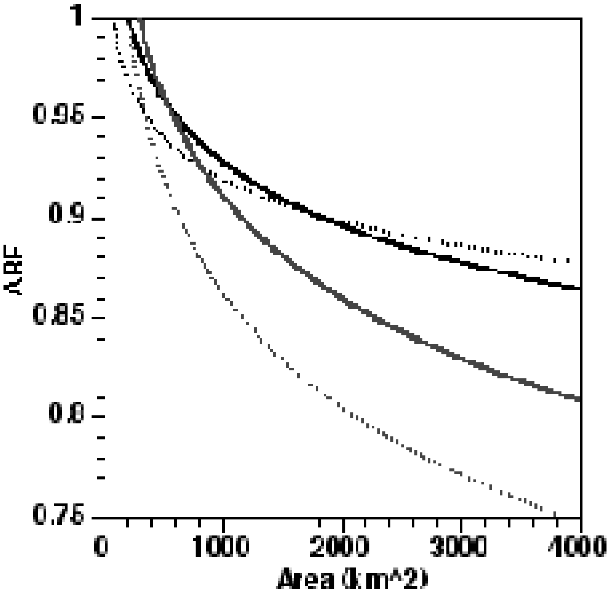

Figure 3. Geographical variation of the area-depth curve for the 2-year (black) and 50-year (gray) return period for New Jersey (solid) and North Carolina (dotted).

For North Carolina, there is more dependence on the spatial interpolation method used and the ARF-area curve. In particular, ARF generated by averaging stations to obtain MAP does not decrease as rapidly as ARF generated by inverse distance weights or Thiessen polygons. This amounts to up to an 8% difference at 20,000 km^2, depending on the return period. Since inverse distance weights and Thiessen polygons result in similar ARF curves, as well as the fact these interpolation methods are nonuniformly weighted, the ARF-area curve in Fig. 2 is shown using inverse distance weights. Again, return period has a substantial effect of the ARF-area relationship. The 50-year ARF decreases to 65% of the average point rainfall depth at 20,000 km^2, while the 2-year ARF decreases to 76%. The original TP-29 ARF is calculated once again using Eq. 2. As can be seen, there is no distinguishable change in the original TP-29 ARF for basins > 1000 km^2.

Figure 3 compares the re-evaluated ARF for North Carolina and New Jersey. For the 2-year return period, New Jersey ARF decreases at a faster rate than North Carolina ARF for basins < 1500 km^2. For basins larger than 1500 km^2, the relationship is reversed and NC ARF decreases at a faster rate. This is a subtle distinction, however, as the maximum difference between the NJ and NC 2-year ARF is < 2%. For all other return periods investigated, the relationship between NJ and NC ARF is clearer; North Carolina ARF decreases at a slower rate than New Jersey ARF. For the 25-year return period, NC ARF decreases to 83% at 4000 km^2, while NJ ARF decreases to 77%. Similarly, the 50-year NC ARF at 4000 km^2 is 81%, whereas the corresponding New Jersey ARF is 75%.

4. SUMMARY

The 24-hour areal reduction factor based on 47 years of COOP data has been re-evaluated for two diverse geographic locations in the eastern US. Similar to ARF published nearly 45 years ago in TP-29, the updated ARF exponentially decays with increasing basin area. The re-evaluated ARF-area relationship, however, varies with return period, unlike TP-29 ARF. For a given basin area, the more extreme rainfall events are associated with a lower, re-evaluated ARF. Therefore, TP-29 ARF provides a conservative, upper estimate of the reduction of point precipitation for a given area. The re-evaluated ARF also varies between study areas, with more of a geographical difference at higher return intervals. The dependence of the re-evaluated ARF on the spatial interpolation method used, as well as topography, does not appear to be as strong of a relationship.

5. REFERENCES

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