1.1 REPRESENTATIONS OF MICROPHYSICAL PROCESSES IN MESOSCALE MODELS: APPLICATIONS FOR TROPICAL CYCLONES

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1. INTRODUCTION

Model representations of the lifetime of clouds, of the production and evolution of precipitation, and of radiative fluxes and heating rates are sensitive to the rate at which populations of ice-phase particles fall. Using a single column model, Petch et al. [1997] showed that relatively small variations in ice crystal velocities could produce large differences in cloud mass and radiative properties. Jakob [2002] found that the climate of the European Centre for Medium-Range Weather Forecast (ECMWF) model, particularly in the Tropics, could be changed dramatically through the modification of assumed fall speed of ice particles. Starr et al. [2000] noted that ice fallout processes have dominant effects on the vertical distribution of ice water and on the intensity of circulation within cirrus. Parameterizations of fallout also influence the dynamics and horizontal structure of systems. Lord et al. [1984] used an axisymmetric, nonhydrostatic hurricane model to show that cooling associated with melting ice particles initiates and maintains model downdrafts, the magnitude and extent of which are sensitive to velocities of snow and graupel particles.

In many mesoscale models, an expression for mass-weighted fall speed, V_m, for a population of ice-phase particles is determined using assumptions about particle size distributions and particle fall velocities. The velocity, V, of an individual particle is typically written as $V = aD^b$, where D represents the particle's dimension. The size distribution is typically represented by an exponential function. This approach is convenient because an analytic expression for V_m is obtained through integration, thereby reducing computational time for numerical simulations.

In this paper, a new parameterization scheme for V_m is derived that accounts for a generalized mass-diameter relationship, not based on the assumption of spherically equivalent particles that many other schemes use. This expression is then applied to different categories of ice commonly used in mesoscale models, such as snow, graupel, pristine ice, and hail. The main advantages of this approach over existing schemes are that consistent definitions of diameter for the calculations of mass and fall speed are used to develop the scheme and its form allows for easy sensitivity studies on the effects of varying the coefficients that describe fall velocities and masses of individual particles. This parameterization is then applied for typical size distributions observed in tropical cyclones.

2 .SIZE DISTRIBUTION REPRESENTATION·

Exponential distributions are frequently assumed in the development of microphysical parameterization schemes. In the majority of papers describing these schemes, it is not explicitly stated whether ice-phase exponential distributions are expressed in terms of melted-equivalent diameter (D_m-diameter of a melted water sphere having the same mass as particle), volume-equivalent diameter (D_v-diameter of a sphere having same volume as particle), or maximum dimension, D. Reference must be made to the paper with the original observations to determine which diameter is used. For example, Lin et al. [1983] used intercept parameters from Gunn and Marshall [1958], which characterize melted equivalent snow particles, and Reisner et al. [1998] used melted equivalent size distributions from Sekhon and Srivastava [1970]. On the other hand, Rutledge and Hobbs [1983] used intercept parameters from Houze et al. [1979], which appear to be based on maximum dimension.

For smaller ice crystals, classified as pristine ice, monodisperse size distributions are frequently used to characterize them in mesoscale models [e.g., Reisner et al. 1998]; hence the following discussion applies to snow and graupel. The assumption of an exponential distribution in terms of D does not imply the existence of an exponential distribution in terms of D_v, and vice-versa. Locatelli and Hobbs [1974], hereafter LH74, and Mitchell [1996], hereafter M96, derived relationships for the mass of ice particles of the form $m(D) = \alpha D^\beta$, where α and β are coefficients that depend on particle shape and maximum dimension. The mass of a volume-equivalent particle with diameter D_v is given by $\pi\rho D_v^3/6$, where ρ is the bulk density of the particle (0.1 g cm^{-3}, 0.4 g cm^{-3}, and 0.5 g cm^{-3} are typical values for snow, graupel, and pristine ice respectively). Assuming constant ρ , an exponential distribution in terms of maximum dimension implies a relationship of the form

$$N(D_v) = N_0 \exp\left(-\lambda \left(\frac{\rho\pi}{6\alpha} \right)^{1/\beta} D_v^{3/\beta} \right) \quad (1)$$

where N_0 and λ represent the intercept and slope of the exponential distribution in terms of maximum dimension.

Figure 1 shows number concentration plotted as a function of D and D_v for the snowflake size distributions observed by Braham [1990] in light and heavy snow, and for graupel size distributions parameterized by Rutledge and Hobbs [1984] for a low (.5 g kg^{-1}) and a high (5 g kg^{-1}) graupel mixing ratio (q_g). These distributions will henceforth be used to define light and heavy snowfall and

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light and heavy graupel showers; all these distributions represented in terms of maximum dimension. There are large differences in the curves plotted as functions of D (thin lines) and D_v (medium lines), and the curves in terms of D_v are not true exponential distributions. If exponential distributions in terms of D_v were originally assumed, then the distributions in terms of maximum dimension would not be true exponential distributions.

The departures of curves in terms of D_v (medium lines) from exponential distributions are not that significant when compared to deviations of observed distributions from exponential behavior. However, the N_0 and λ for the light and heavy snow distributions change from (1.34×10^6 m^{-4}, 20.1 cm^{-1}) and (6.4×10^6 m^{-4}, 11.4 cm^{-1}) for the maximum dimension distributions to (4.54×10^6 m^{-4}, 45.3 cm^{-1}) and (1.28×10^7 m^{-4}, 25.7 cm^{-1}) for best-fit exponential distributions in terms of D_v. The thick lines represent exponential distributions in terms of D_v that conserve the total number and mass of the distribution, an important constraint when performing numerical studies. The N_0 and λ characterizing these distributions are (2.02×10^6 m^{-4}, 30.3 cm^{-1}) and (1.2×10^7 m^{-4}, 20.4 cm^{-1}) for light and heavy snow. For light and heavy graupel showers, the N_0 and λ vary from (4.0×10^6 m^{-4}, 16.4 cm^{-1}) and (4.0×10^6 m^{-4}, 8.9 cm^{-1}) for the maximum dimension distributions to (4.47×10^6 m^{-4}, 24.8 cm^{-1}) and (4.25×10^6 m^{-4}, 13.5 cm^{-1}) for best fit exponentials in terms of D_v, to (5.85×10^6 m^{-4}, 23.9 cm^{-1}) and (6.1×10^6 m^{-4}, 13.6 cm^{-1}) for distributions in terms of D_v conserving mass and number. Because of these differences, it is important to be aware of the observations used to develop a parameterization scheme. The slopes and intercepts can change by more than 2 depending upon which definitions are used.

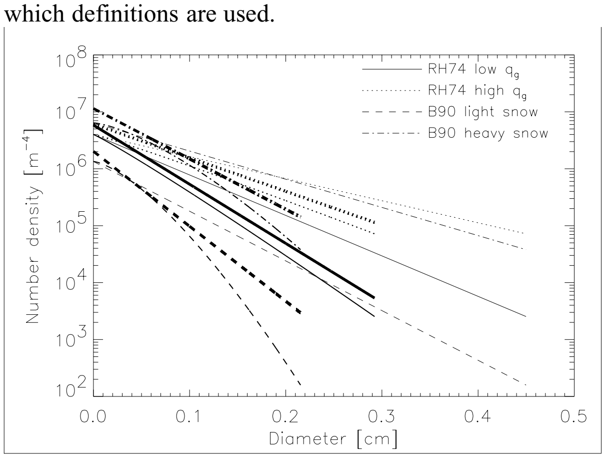

Figure 1: Number concentration as function of maximum diameter (thin lines) or volume-equivalent diameter (thick lines) for light and heavy graupel and snow distributions as indicated in the legend. The thickest lines represent volume-equivalent distributions conserving number and mass compared to distributions in terms of maximum dimension.

3. REPRESENTATION OF FALL VELOCITY

To determine how these differences impact calculations of V_m, a commonly used equation (e.g., Dudhia 1989),

$$V_m = \frac{a\Gamma(4+b)}{6\lambda^b} \quad , \qquad (2)$$

was used. For light and heavy snow, differences in N_0 and λ lead to changes in V_m from 94.7 and 116.9 cm s^{-1}, to 70.2 and 86.5 cm s^{-1} for the best fit exponential distributions, and to 81.4 and 94.3 cm s^{-1} for the number/mass conserving distributions. For light and heavy graupel, V_m varies from 191.0 and 245.4 cm s^{-1} to 161.2 and 206.9 cm s^{-1} and to 163.7 and 206.2 cm s^{-1} for the same three types of distributions. Since differences between V_m are up to 35% for snow and up to 20% for graupel, and vary systematically between representations, their impacts on numerical simulations may need to be considered.

Natural cloud variability will cause differences in V_m greater than those noted above. For example, Brown and Swann [1997] noted that observed values in the intercept parameter for graupel could vary by three orders of magnitude; similar variations in the intercept parameter for snow exist. However, the nature of the ice-phase size distributions must still be carefully categorized to avoid systematic errors in V_m of the nature above. This distinction between D and D_v based distributions must also be made when comparing slope and intercept parameters from different parameterization schemes.

For a population of ice-phase particles, V_m is given by

$$V_m = \frac{\int_0^\infty V(D')m(D')N(D')dD'}{\int_0^\infty m(D')N(D')dD'} \qquad (3)$$

where D' can either be D or D_v depending on the nature of the observations used to develop the parameterization. In Eq (3), V is the velocity of an individual ice crystal and m its mass. For integrations over D, $m(D) = \alpha D^\beta$ should be used in Eq. (3). LH74 and M96 have derived α and β coefficients for a variety of crystal shapes and sizes. For integrations over D_v, $m(D_v) = \rho\pi/6 D_v^3$ should be used.

LH74, M96, Heymsfield and Iaquinta [2000] and others have derived expressions of the form $V(D) = aD^b$ to describe fall speeds of ice-phase particles, where the coefficients a and b depend on particle shape and size. For fall speed relationships introduced by LH74, D represents the diameter of a sphere of equivalent cross-sectional area to the observed ice crystal for aggregate crystals, the diameter of the blunt end for conical graupel, and the average value of the length of the branches for branched particles (dendrites, hexagonal graupel); for all other ice particles and for the studies of M96 and HI00, D represents the particle's maximum dimension. Published mass-diameter relationships (LH74, M96) have diameter defined in a similar manner.

To more accurately calculate V_m, separate relationships must be derived depending upon whether the exponential distribution is based upon maximum dimension D or volume-equivalent diameter D_v. For size distributions parameterized in terms of D, substituting for V(D) and m(D) gives

$$V_m = \frac{a\Gamma(1+b+\beta)}{\lambda^b \Gamma(1+\beta)} \quad (4)$$

where Eq. (4) differs from Eq. (2) in form, in addition λ is defined in a different manner to ensure mass conservation.

For number distributions defined in terms of D_v, expressions for V(D) must be converted to expressions for $V(D_v)$ using the mass-diameter relationships of LH74 and of M96. Substituting these expressions into Eq. (3) gives

$$V_m = \frac{a\Gamma(4+3b/\beta)}{6\lambda^{3b/\beta}} \left(\frac{\rho_x \pi}{\alpha 6} \right)^{b/\beta} \quad (5)$$

An additional complication arises when V_m is derived using velocity-diameter relationships derived by LH74 for aggregate crystals, since LH74 defined the dimensions of these crystals to be those of equivalent projected area spheres for both mass and velocity relationships. Similar expressions for V_m may be derived for that case. Because exponential distributions are easy to integrate, no extra resources are needed to use these equations in a model.

To see the importance of this new parameterization, V_m is calculated using Eq. (2) and compared against V_m obtained following Eq (5) or an equivalent equation for typical mass densities, and N_0 values for snow (4.0×10^6 m^{-4}) and graupel (4.0×10^6 m^{-4}) used in bulk microphysical schemes. In such schemes, Reisner et al. [1998], for example, define (a,b) to be (351.2 $cm^{1-b}s^{-1}$, .37) for graupel and (177.4 $cm^{1-b}s^{-1}$, .41) for snow; M96 define (α,β) as (.049 g $cm^{-\beta}$, 2.8) for graupel and (.0028 g $cm^{-\beta}$, 2.1) for snow. For light and heavy graupel, V_m varies from 182.1 and 225.7 cm s^{-1} to 198.2 and 248.5 cm s^{-1}, and for light and heavy snow, V_m varies from 69.9 and 98.2 cm s^{-1} to 63.4 and 78.8 cm s^{-1}. There are several different effects that cause V_m values to differ, such as variation of λ between the new and old scheme, the use of a different equation for V_m, and the impact of (α,β) coefficients. Because these different effects can either lower or raise V_m, and because the impacts of the different effects varies depending on the coefficients describing the properties of the different ice-phase particles, it is hard to determine in advance how V_m will vary between the new and old parameterization schemes. Hence, to investigate the importance of this new scheme it is necessary to calculate differences in V_m produced over a wide range of a, b, α and β values.

Figure 2 compares V_m calculated using the new and old approaches. The four panels represent comparisons for light and heavy snowfall and light and heavy graupel showers with mixing ratios previously defined. The data points represent V_m calculated with different (a,b) and (α,β) coefficients for different shapes of ice-phase particles as tabulated by M96 and LH74. A large number of different ice-phase particles are included as snow and graupel. For bulk microphysical models, any particle where riming plays an important role is assumed to be graupel, from rimed aggregates to hail. Snow crystals are typically aggregates of other particles that are not rimed (plates, side planes, bullets, columns, aggregates), but can also represent individual ice crystals that have grown large. Coefficients describing mono-habit pristine crystals with typical sizes smaller than 100 μm, assumed to be pristine ice, are not included in the sample used to create Fig. 2.

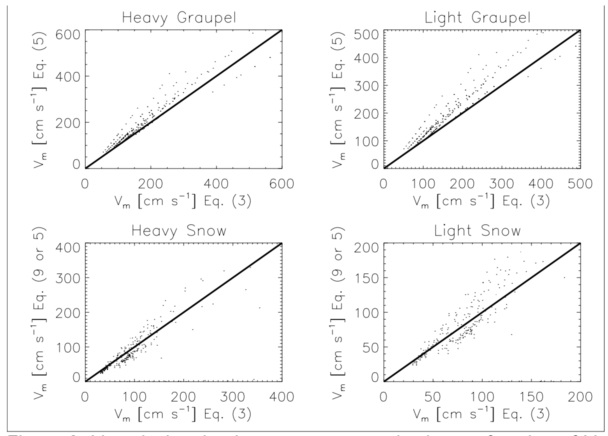

Figure 2: V_m calculated using new parameterization as function of V_m calculated using standard schemes (Eq. (2)). Different points correspond to different (a,b), (α,β) and (γ,σ) coefficients which categorize snow and graupel.

The differences between V_m calculated between the new and old parameterization schemes are not as large as might be expected, given the large number of changes that have been made to the parameterization coefficients. There is on average 24% difference between the mean velocities for the snow crystals and on average 15% difference for the graupel velocities. However, when certain rimed or aggregate particles represent either graupel or snow, the differences can be larger. The results are important in showing that extra levels of detail may lead to less accurate calculations of V_m, if all such extra levels of detail are not included in the equations.

4. DISCUSSION

A new parameterization scheme describing the mass-weighted terminal velocity of distributions of snow and graupel particles, suitable for use in numerical models with bulk microphysical schemes, has been developed. It has been shown that careful consideration is needed as to whether the exponential distributions, used in parameterization development, are based in terms of volume-equivalent diameter or in terms of maximum dimension. The use of a distribution based on one diameter definition in a microphysical scheme versus the use of another based on a different diameter definition can result in systematic errors in the calculation of V_m of over 35% for snow and of over 20% for graupel.

A new parameterization scheme that accounts for the variation of both the mass and velocity of different ice-phase particles was developed. Use of this new parameterization scheme varies mass-weighted terminal velocities by approximately 25% for snowflake distributions and by approximately 15% for graupel distributions.

The real strength of the new parameterization is that it allows for different mass and velocity coefficients to be consistently included in the development of new expressions for V_m. The scheme is so general that modifications can be made on a case-by-case basis if more information about the composition of particles or the mixtures of different particles is available. This is important because the majority of past parameterization schemes for microphysical clouds have been developed for mid-latitude systems. Current work is focusing on applying this parameterization to tropical cyclones by choosing the appropriate parameterization coefficients.

5. ACKNOWLEDGEMENTS

The support of the NASA CAMEX program is acknowledged. The comments of B. Rauber and M. Ramamurthy helped focus the study.

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