P1.12 CAPACITANCE OF BULLET ROSETTE ICE CRYSTALS

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1. Introduction

It has recently become clear that cirrus clouds significantly affect the global energy balance and climate, due to their great radiative impact on atmospheric thermal structure. Variations in the assumed cirrus radiative properties can significantly alter the results of climate models (Ramanathan et al. 1983, Liou, 1992). The radiative properties of cirrus depend on their microphysical properties such as ice crystal habit, ice water content and number concentration. Recently, Liu et al. (2001a,b,c) used a two-dimensional cirrus model to study the effects of cloud microphysical parameters on the cirrus development. They found that the cirrus development and its radiative property are sensitive functions of the ice crystal habit. Their results show that while the habit itself has a direct impact on the scattering of radiation, it asserts its influence on the overall cirrus radiative property mainly via its control on the diffusional growth process. During this process, ice crystals of different habits may grow at different rates by vapor condensation. Thus, for example, under the same initial and boundary conditions, a cirrus cloud consisting of columnar ice crystals may develop different ice concentration and ice water content that result in a different radiative property than a cirrus of ice plates.

In most cirrus models, the ice crystal growth rates is parameterized by utilizing the classical ice growth theory, called the electrostatic analog theory. In this theory, the diffusional growth rate of ice crystals depends on a quantity called capacitance, which is a function of both ice crystal size and habit. In order to determine the ice crystal growth rates in cirrus cloud models, it is necessary to know the values of the capacitance.

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One of the most important ice crystal habits in cirrus is bullet rosettes (Heymsfield 1975; Parungo 1995). Heymsfield and Ioquinta (2000) reported high occurrence frequency of rosettes in many midlatitude cirrus, making the bullet rosette one of the dominant habits for the cirrus clouds they have investigated. Thus it is obviously important to know the capacitance of rosettes. Yet the capacitance of rosettes has never been determined in a rigorous way, mainly due to their complicated shapes that render precise mathematical treatment difficult. McDonald (1963) and Heymsfield (1975) suggested that the capacitance of particles with more intricate and spatial branches, such as rosettes, could be approximated as that for spherical particles of equal radii. Liu et al. (2001a,b,c) used this sphere approximation and showed that a cirrus cloud consisting of bullet rosettes would have much larger radiative effect than cirrus clouds of other types of ice crystals. For example, starting with the same initial environmental conditions, the rosette cirrus would have a peak heating rate due to infrared radiation two and half and two times that for cirrus of ice columns and ice plates, respectively, and six times that for spheres. Its heating rate due to short-wave solar radiation is also substantially greater than other habits. Obviously, it is important to determine the capacitance of the bullet rosettes more precisely in order to accurately assess their impact on the radiative heating.

This paper is devoted to the task of calculating the capacitance of bullet rosette ice crystals. In the following sections, we will first review briefly the electrostatic analog theory of ice crystal diffusional growth to clarify the role of the capacitance. Then we will describe the techniques of simulating the shapes of these rosettes and the methods of determining their capacitance. This will be followed by the discussion of the results and their implications to the ice cloud development and radiative impacts.

2. The Electrostatic Analog Theory of Ice Crystal Growth

The classical theory of ice crystal growth is called the electrostatic analog because it dwells on the similarity between the equations governing the water vapor distribution around an ice crystal and the electrostatic potential distribution around an electric conductor of the same shape as the ice crystal. A detailed discussion can be found in standard textbooks of cloud physics (e.g., Pruppacher and Klett, 1997; Hobbs, 1976; Young, 1993) and will not be repeated here.

3. Mathematical Determination of Capacitance

We will determine *dm/dt* explicitly and then use the electrostatic analog to obtain the capacitance *C*. To determine dm/dt, we need to solve the water vapor density distribution first.

The Laplace equation will be non-dimensionalized first for the convenience of analysis:

$$\nabla^2 \rho_v' = 0 \qquad (1)$$

where the primed quantity in the integrand are non-dimensionalized according to the following relations:

$$\rho_v' = \frac{\rho - \rho_\infty}{\rho_s - \rho_\infty}, \quad \vec{r}' = \frac{\vec{r}}{a} \qquad (2)$$

where *a* is the radius of the ice crystal and *r* represents the radial distance from the origin, which is defined as the center of the ice crystal in the present study. The radius of the rosette considered here is defined as the distance from the center of the rosette to the tip of one of the lobes. We assume that all lobes have the same length in this study.

Once the vapor density distribution ρ_v' is determined, the capacitance is obtained by

$$C = \frac{a}{4\pi} \oint_s \nabla' \rho_v' \cdot \vec{ds}' \qquad (3)$$

Smythe (1956, 1962) and Wang et al. (1985) performed similar calculations to determine the capacitance of right circular cylinders of finite lengths but they solved the Lapalce equation analytically. In the present study, we will use numerical techniques due to the complicated shapes of the rosettes.

We need first to define appropriate boundary conditions for our numerical problem. Since the capacitance is a function of the positions of these boundaries, the capacitance of an isolated conductor will be different from the same conductor when it is placed near another charged body with finite potential. In atmospheric clouds, the mean distance between individual cloud particles is rather large relative to the particle size, typically on the order of many tens to hundreds particle radii (Prupppacher and Klett 1997). This means that the ice particles can often be considered as isolated individual particles, and hence the most relevant capacitance for our purpose here will be that of an isolated ice crystal. Thus the appropriate boundary conditions for the present situation are:

$$\rho_v' = \begin{cases} 1 \text{ on the surface of the rosette} \\ 0 \text{ at a distance far away from the rosette.} \end{cases}$$

(4)

4. Treatment of the Boundary Conditions

The inner boundary

We use numerical methods to solve Eq. (1) subject to the boundary conditions (4). Specifically, we will use the finite element techniques for solving the Lapalce equation. This requires the setup of a grid system. The first step is to prescribe the boundary points.

The inner boundary is the surface of the bullet rosette. The shape of a bullet rosette is highly complicated and it is not easy to determine the coordinates of the boundary surface. To simplify this problem, we use the mathematical expression given by Wang (1998) to approximate such a shape:

$$r = \{\alpha[\cos^2(m\theta)]^\beta + \gamma\}^\delta \{\alpha'[\sin^2(n\varphi)]^{\beta'} + \gamma'\}^{\delta'} \qquad (5)$$

This equation is expressed in spherical coordinates so that *r* is the radial, θ the zenithal, and φ the azimuthal coordinates. The parameters $\alpha, \beta, \gamma, \delta, \alpha', \beta', \gamma',$ and δ' are freely adjustable so as to fit the shape of a particular rosette. The shape generated by this expression will have 2*mn* lobes or branches. For example, a four-branch combination of bullets can be generated by the following expression:

$$r = [1 - \cos(2\theta)^4]^{20}[1 - \sin(\varphi)^4]^{20} \qquad (6)$$

where *m*=2 and *n*=1 here. The width of the branch is controlled by β and β' in Eq.(5). We understand that some of these are not good approximations of the real rosettes. Obviously it is impossible that such a simple expression will reproduce all the intricate structures of bullet rosettes, but at least it captures the essential multi-lobe feature that is the characteristic of these ice crystals.

The outer boundary

Although the inner boundary of the problem is not spherical symmetric, the outer boundary of the present problem, if set at infinity, will be a sphere, as the distribution of any field whose source is finite (such as the rosette considered here) will become spherically symmetric when the distance approaches infinity. However, in numerical calculations, the distance of the outer boundary has to remain finite and hence the field distribution here may deviate from a true sphere. In the present study we assume that this finite outer boundary is also spherical. In order to assess the impact of this assumption on the accuracy of the results, we performed sensitivity tests for the outer boundary distance. It turns out that the results are not very sensitive to the outer boundary distance as long as it is a few radii away from the center of the crystal.

The details of the discretization technique using the finite element method can be found in standard textbooks on finite element analysis.

5. Results and Discussions

The capacitances of seven bullets rosette ice crystals are calculated by using the numerical techniques as outlined in the previous section. These rosettes consist of two, three, four, six, eight, twelve and sixteen lobes. The choice of the rosette cases was based on the simplicity for the calculations because of their geometrical symmetry. The mathematical expressions for these rosettes and their shapes are shown in Fig. 1. The values of the adjustable parameters are chosen so that the thickness of the lobes looks visually reasonable. Due to the limitation of the formulas, it is difficult to obtain a symmetric shape for each lobe. Thus sometimes the dimension of the lobe in the θ-direction is substantially different than that in φ-direction. But the multi-lobe characteristic of the rosettes is well reproduced. The crystals generated in this way have the lobes of equal length *a*. Also shown in Fig. 1 are the total surface areas, volumes, and the cross-sectional areas projected in the x-, y-, and z-axis.

Capacitances of Rosettes

The computed capacitances of the rosettes as a function of the number of lobes are shown in Fig. 2. It is unclear at the moment whether the slight scatter in the results is due to the numerical errors or the results of the shape parameters in Eq. (5) chosen or both. However the general trend of the curve is fairly clear and the scatter should not influence the conclusions to any significant extent.

It is seen in Fig. 2 that the rosette capacitance increases from about 0.5 to near 0.9 (in unit of *a*) as the number of lobes increases from 2 to 16. The capacitance of a conducting sphere is its radius *a*. Thus it appears that the capacitance of a rosette will approach that of a sphere if the number of lobes approaches infinity, i.e., as its shape approaches a sphere. The following power relation can fit the rosette capacitance curve in Fig. 2:

$$C = 0.434N^{0.257} \tag{7}$$

where *C* is the capacitance in unit of *a* and *N* the number of lobes.

Fig. 2 thus shows that the capacitance of a rosette is smaller than that of a sphere of equal radius. This implies that calculations of the rosette growth rate based on the spherical capacitance assumption overestimate. The overestimation is the most serious for rosettes with fewer lobes but becomes less so if the number of lobes is large.

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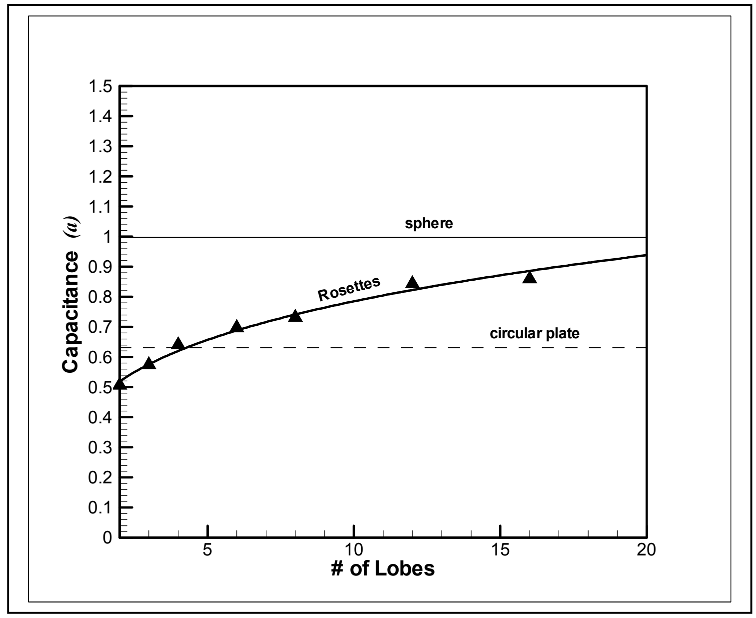

Fig. 2. Capacitance of rosettes vs. # of lobes

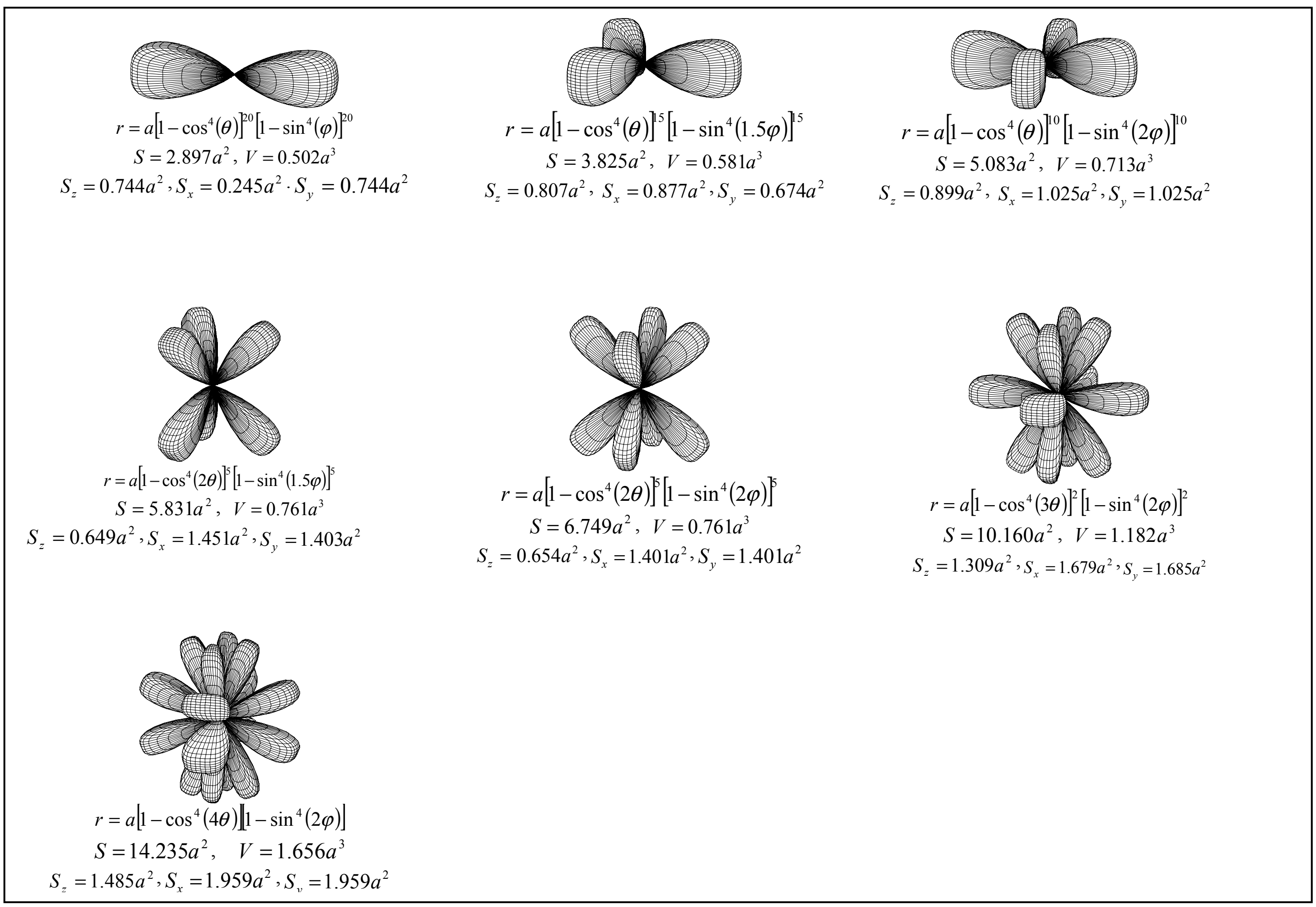

Fig. 1. The seven bullets rosette ice crystals considered and their generating formulas .