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## 1. INTRODUCTION

Ever since the introduction of differential reflectivity ( $Z_{dr}$ ) measurement, one of the long-standing goals of polarimetric radar has been the estimation of the raindrop size distribution (DSD). Seliga and Bringi (1976) showed that  $Z_{dr}$ , for an exponential DSD, is directly related to the median volume diameter ( $D_o$ ). Careful intercomparisons between radar measurements of  $Z_{dr}$  and  $D_o$  derived from surface disdrometers and airborne imaging probes have shown that  $D_o$  can be estimated to an accuracy of about 10-15% (see, for example, Aydin et al., 1987; Bringi et al., 1998). A general gamma distribution model was suggested by Ulbrich (1983) to characterize the natural variation of the DSD. The specific differential propagation phase ( $K_{dp}$ ) is a forward scatter measurement whereas  $Z_{dr}$  is a backscatter measurement. The weighting of the DSD by  $Z_{dr}$  and  $K_{dp}$  is controlled by the variation of mean raindrop shape with size. A combination of the three radar measurements ( $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$ ) can be utilized to estimate the DSD, specifically a parametric form of the DSD such as the gamma DSD. This paper presents algorithms for the estimation of parameters of a gamma DSD from polarimetric radar measurements at various frequency bands.

## 2. RAINDROP SIZE DISTRIBUTION

The raindrop size distribution describes the probability density/distribution function of raindrop sizes. In practice, the normalized histogram of raindrop sizes (normalized with respect to the total number of observed raindrops) converges to the probability density function of raindrop sizes. A gamma distribution model can adequately describe many of the natural variations in the shape of the raindrop size distribution (Ulbrich, 1983). The gamma raindrop size distribution can be expressed as (Chandrasekar and Bringi, 1986),

$$N(D) = n_c f_D(D) \quad (m^{-3} mm^{-1}) \quad (1)$$

where  $N(D)$  is the number of raindrops per unit volume per unit size interval ( $D$  to  $D+\Delta D$ ),  $n_c$  is the number concentration and  $f_D(D)$  is the probability density

function (pdf). When  $f_D(D)$  is of the gamma form it is given by,

$$f_D(D) = \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} e^{-\Lambda D} D^\mu, \quad \mu > -1 \quad (2)$$

where  $\Lambda$  and  $\mu$  are the parameters of the gamma pdf. Any other gamma form such as the one introduced by Ulbrich (1983),

$$N(D) = N_0 D^\mu e^{-\Lambda D} \quad (3)$$

can be derived from this fundamental notion of raindrop size distribution. It must be noted that any function used to describe  $N(D)$  when integrated over  $D$  must yield the total number concentration, to qualify as a DSD function. This property is a direct consequence of the fundamental result that any probability density function must integrate to unity. The relation between  $D_o$ ,  $\mu$  and  $\Lambda$  is given by

$$\Lambda D_o \equiv 3.67 + \mu \quad (4)$$

Similarly, a mass-weighted mean diameter  $D_m$  can be defined as

$$D_m = \frac{E(D^4)}{E(D^3)} \quad (5)$$

where  $E$  stands for the expected value. Using (4),  $f_D(D)$ , the gamma pdf described by (2), can be written in terms of  $D_o$  and  $\mu$  as,

$$f_D(D) = \frac{(3.67 + \mu)^{\mu+1}}{\Gamma(\mu+1) D_o} \cdot \left(\frac{D}{D_o}\right)^\mu e^{-\left[\frac{(3.67 + \mu) D}{D_o}\right]} \quad (6)$$

The above form makes the normalized diameter ( $D/D_o$ ) as the variable rather than  $D$ . Several measurables such as water content ( $W$ ) and rainfall rate ( $R$ ) can be expressed in terms of the DSD as,

$$W = \frac{\pi}{6} \rho_w n_c E(D^3) \quad (7)$$

and

$$R = \frac{\pi}{6} n_c E[v(D) D^3] \quad (8a)$$

where  $R$  is the still-air rainfall rate and  $v(D)$  is the terminal velocity of raindrops (Gunn and Kinzer, 1949). The conventional unit of rainfall rate is  $mm h^{-1}$ . Converting to this unit, rainfall rate is expressed as,

$$R = 0.6\pi \times 10^{-3} n_c \cdot E[v(D) D^3] \quad (mm h^{-1}) \quad (8b)$$

where  $n_c$  is in  $m^{-3}$ ,  $v(D)$  in  $m s^{-1}$  and  $D$  in  $mm$ .

In order to compare the pdf of  $D$  (or,  $f_D(D)$ ) in the presence of varying water contents, the concept of scaling the DSD has been used by several authors (Sekhon and Srivastava, 1971; Willis, 1984; and Testud et al., 2000). The corresponding form of  $N(D)$  can be expressed as,

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$$N(D) = N_w f(\mu) \left( \frac{D}{D_0} \right)^\mu \exp \left[ - (3.67 + \mu) \frac{D}{D_0} \right] \quad (9)$$

where  $N_w$  is the scaled version of  $N_0$  defined in (3),

$$N_w = \frac{N_0}{f(\mu)} D_0^\mu \quad (10a)$$

and

$$f(\mu) = \frac{6}{(3.67)^4} \cdot \frac{(3.67 + \mu)^{\mu+4}}{\Gamma(\mu + 4)} \quad (10b)$$

with  $f(0) = 1$  and  $f(\mu)$  is a unit less function of  $\mu$ . One interpretation of  $N_w$  is that it is the intercept of an equivalent exponential distribution with the same water content and  $D_0$  as the gamma DSD. (Bringi and Chandrasekar, 2001). Thus  $N_w$ ,  $D_0$  and  $\mu$  form the three parameters of the gamma DSD.

### 3. RAINDROP SHAPE AND IMPLICATION FOR POLARIMETRIC RADAR MEASUREMENTS

The equilibrium shape of raindrops is determined by a balance of hydrostatic, surface tension and aerodynamic forces. The commonly used model for raindrops assumes oblate spheroidal shapes, with the axis ratio  $b/a$ , where  $b$  and  $a$  are the semi-minor and the semi-major axis lengths, respectively. Pruppacher and Beard (1970) give a simple model for the axis ratio ( $r$ ) based on a linear fit to wind tunnel data as,

$$r = 1.03 - 0.062D; \quad 1 \leq D \leq 9 \text{ mm}. \quad (11)$$

Rotating linear polarization data in heavy rain (Hendry et al., 1987) has indicated that raindrops fall with the mean orientation of their symmetry axis in the vertical direction. The large swing in the crosspolar power in their data implies a high degree of orientation of drops with the standard deviation of canting angles estimated to be around  $6^\circ$  assuming a Gaussian model. It is reasonable to assume that the standard deviation of canting angles is in the range  $5-10^\circ$  (Bringi and Chandrasekar, 2001).

#### 3.1 Differential Reflectivity

The differential reflectivity can be written as (Seliga and Bringi, 1976),

$$10 \log_{10} \frac{E[\sigma_{hh}(D)]}{E[\sigma_{vv}(D)]} = 10 \log_{10} (\xi_{dr}^{-1}) \quad (12)$$

where the symbol  $E$  represents expectation and  $\sigma_{hh}$  and  $\sigma_{vv}$  are the cross sections at horizontal and vertical polarizations, respectively.

Seliga and Bringi (1976) showed that for an exponential distribution and axis ratio given by (11),  $Z_{dr}$  can be expressed as a function of the median volume diameter  $D_0$ . This microphysical link between a radar measurement and a parameter of the DSD is important. More fundamentally,  $\xi_{dr}^{-1}$  may be related to the reflectivity factor weighted mean of  $r^{7/3}$  (Jameson, 1985). For a more general gamma form an approximate power law fit can be derived assuming  $-1 \leq \mu \leq 5$ ,  $0.5 < D_0 < 2.5 \text{ mm}$ , and  $N_w$  chosen to be consistent with thunderstorm rain rates. Using the fit recommended

by Andsager et al. (1999) for the Beard and Chuang (1987) equilibrium shapes power law fits to  $D_0$  and  $D_m$  can be derived as,

$$D_0 = 1.619 (Z_{dr})^{0.485} \quad (\text{mm}) \quad (13a)$$

$$D_m = 1.529 (Z_{dr})^{0.467} \quad (\text{mm}) \quad (13b)$$

where  $Z_{dr}$  is in decibels and the fits are valid at S band frequency (near 3 GHz, Bringi and Chandrasekar, 2001).

#### 3.2 Specific Differential Phase

The relation between specific differential phase ( $K_{dp}$ ) and the water content and raindrop axis ratio was described by Jameson (1985).

$K_{dp}$  can be related to the water content as (Bringi and Chandrasekar, 2001)

$$K_{dp} = \left( \frac{180}{\lambda} \right) \cdot 10^{-3} \cdot c \cdot W (1 - \bar{r}_m) \quad (\text{deg. km}^{-1}) \quad (18)$$

where  $c \equiv 3.75$  is both dimensionless and independent of wavelength. This result links the specific differential phase with parameters of the DSD. If the equilibrium axis ratio model given in (11) is used in (18) then  $K_{dp}$  is given by,

$$K_{dp} = \left( \frac{180}{\lambda} \right) \cdot 10^{-3} \cdot c \cdot W (0.062) D_m (\text{deg. km}^{-1}) \quad (19)$$

Thus  $K_{dp}$  is related to the product of  $D_m$  and water content. Though the above result was obtained using the Rayleigh-Gans approximation, it is valid up to 13 GHz (Bringi and Chandrasekar, 2001).

#### 3.3 Mean Raindrop Shape Derived From Polarimetric Radar Measurements

Gorgucci et al. (2000) assumed a simple linear model for axis ratio versus size of the form,

$$r = 1 - \beta D \quad (20)$$

and derived radar-based estimators of  $\beta$ .

It was shown in section 3a that  $\xi_{dr}^{-1}$  is related to the reflectivity weighted axis ratio. Similar dependence on  $K_{dp}$  can be derived from (18). Let  $p(r)$  be the probability density function of the axis ratio for a given diameter. The expression for  $K_{dp}$  can be generalized as (Bringi and Chandrasekar, 2001),

$$K_{dp} = \frac{2\pi c'}{k_0} \int D^3 N(D) \int (1-r) p(r) dr dD \quad (21)$$

$$= \frac{2\pi c'}{k_0} \int D^3 N(D) [1 - E(r)] dD \quad (22)$$

where  $E(r)$  is the mean value of  $r$ , and  $c'$  is a constant. The functional dependence of  $E(r)$  versus  $D$  may be modeled as in (20). Using the linear model in (20), Gorgucci et al. (2000) showed the variations of  $Z_{dr}$  and  $K_{dp}$  with respect to  $\beta$ , and in turn derived an estimator for  $\beta$  based on polarimetric radar measurements. This can be used subsequently in algorithms relating  $Z_{dr}$  and  $K_{dp}$  to the parameters of the DSD, which gives rise to a methodology for estimating the gamma DSD parameters based on radar measurements.

#### 4. ESTIMATORS OF THE GAMMA DSD PARAMETERS

Seliga and Bringi (1976) showed that for an exponential distribution, the two parameters of the DSD, namely  $N_w$  and  $D_0$ , can be estimated using  $Z_{dr}$  and  $Z_h$ . They used a two-step procedure where they estimated  $D_0$  using an equilibrium raindrop shape model and subsequently used that in the expression for  $Z_h$  to estimate  $N_w$ . This procedure can essentially be applied for a gamma DSD, and generalized to account for raindrop oscillations using the linear model in (20). The procedure for estimating the gamma DSD parameters is as follows: first estimate  $\beta$  using the algorithm described by Gorgucci et al. (2000), and subsequently, estimate  $D_0$ ,  $N_w$  and  $\mu$  recognizing the prevailing  $\beta$  value. Using simulations, an estimator for  $\hat{D}_0$  can be derived as

$$\hat{D}_0 = a_1 Z_h^{b_1} (-\xi_{dr})^{c_1} \quad (23)$$

These coefficients for S-band are

$$a_1 = 0.56, \quad b_1 = 0.064, \quad c_1 = 0.024 \beta^{-1.42} \quad (24)$$

Similar estimates can be derived for C and X band as with the corresponding coefficients are given in table 1.

Coefficients	$a_1$	$b_1$	$c_1$
S	0.56	0.064	$0.024 \beta^{-1.42}$
C	0.526	0.0973	$0.0118 \beta^{-1.31}$
X	$0.195 \beta^{-0.55}$	0.0498	$0.0344 \beta^{-0.471}$

Table 1 The coefficients of  $D_0$  estimate parameters at S, C, and X band

Simulations can also be utilized to evaluate the performance of the estimator of  $D_0$  in (23). Figure 1 shows a scatter plot of  $\hat{D}_0$  versus true  $D_0$  at S band for widely varying  $\beta$  and gamma DSD parameters as given by (23). Quantitative analysis of the scatter gives a correlation coefficient of 0.963. It can be seen from Fig. 1 that  $D_0$  is estimated fairly well with negligible bias over a wide range. Figure 2 shows the normalized standard deviation (NSD) of  $\hat{D}_0$  as a function of  $D_0$ . Figure 2 shows that  $D_0$  can be estimated to an accuracy of about 10% when  $D_0 > 1$  mm. The corresponding parameters at C and X band are as follows. The correlation coefficients are 0.93 and 0.95, whereas the normalized standard errors are 15% and 13% respectively.

##### 4.1 Estimation of $N_w$

Once  $D_0$  is estimated,  $N_w$  can be easily estimated using one of the moments of the DSD such as  $Z_h$  or  $K_{dp}$ . For example,  $Z_h$  can be written in terms of the gamma DSD parameters as,

$$\frac{Z_h}{N_w} = F_z(\mu) D_0^7 \quad (25)$$

Thus it can be seen that  $N_w$  can be estimated in terms of  $D_0$ . However, the estimate of  $D_0$  can be obtained in terms of  $Z_h$  and  $Z_{dr}$  (or  $K_{dp}$  and  $Z_{dr}$ ). Therefore, a direct estimate of  $N_w$  can be pursued of the form,

$$\log_{10}(N_w) = a_2 Z_h^{b_2} \xi_{dr}^{c_2} \quad (26)$$

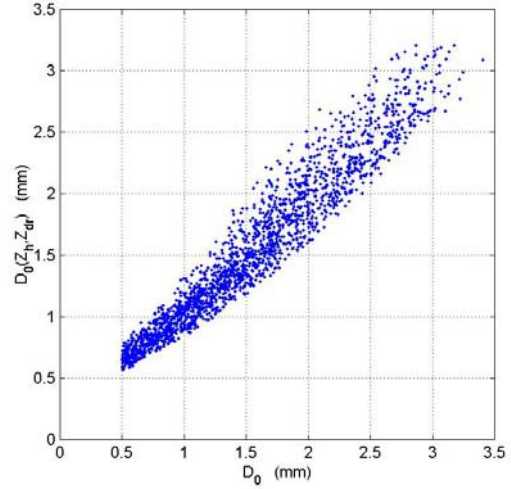


Figure 1. Scatterplot of  $D_0(Z_h, Z_{dr})$  versus the true value of  $D_0$  for widely varying RSD, at S band.

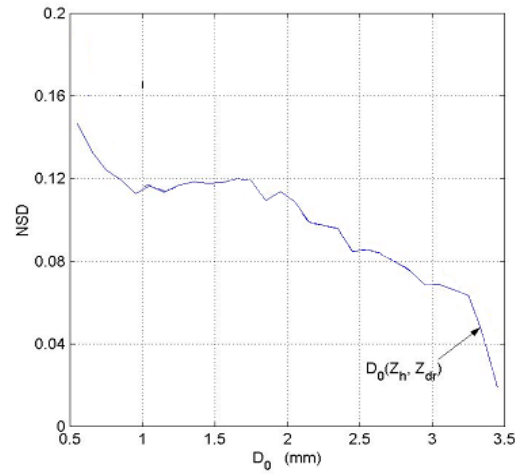


Figure 2. Normalized standard deviation (NSD) in the estimates of  $D_0$  as a function of the true value of  $D_0$  at S band.

The variability of  $a_2$ ,  $b_2$ ,  $c_2$  can be parameterized in terms of  $\beta$  at S band as

$$a_2 = 3.29, \quad b_2 = 0.058, \quad c_2 = -0.023 \beta^{-1.389} \quad (27)$$

In summary, the estimator for  $N_w$  is obtained as follows. Using  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$  first estimate  $\beta$  as given in (26). Subsequently, calculate the coefficients in (27) and use in (26) to estimate  $N_w$ . Figure 3 shows a scatter plot of  $\log_{10} \hat{N}_w$  versus true  $\log_{10} N_w$ , where  $\log_{10} \hat{N}_w$  is estimated using (26). It can be seen from Fig. 3 that  $\log_{10} \hat{N}_w$  is estimated fairly well. Quantitative analysis of the scatter yields a correlation coefficient of 0.831. Fig. 3 shows the NSD of  $\log_{10} \hat{N}_w$  as a function of  $\log_{10} \hat{N}_w$ . It can be seen, from Fig. 4, that  $\log_{10} N_w$  is estimated to a normalized standard deviation of better than 7% when  $\log_{10} N_w > 3.5$ . Note that due to the wide variability of  $N_w$ ,  $\log_{10} N_w$  is the preferred scale of comparison (similar to dB scale for reflectivity). At C and X bands the parameterization for  $N_w$  can be obtained in a similar manner and the results are summarized in table 2.

Coefficients	$a_2$	$b_2$	$c_2$
S	3.29	0.058	$-0.023 \beta^{-1.389}$
C	$3.62 \beta^{-0.0622}$	0.054	$-0.03 \beta^{-1.12}$
X	2.97	0.072	$-0.0294 \beta^{-1.26}$

Table 2 One coefficients of  $N_w$  estimate parameters at S, C, and X band.

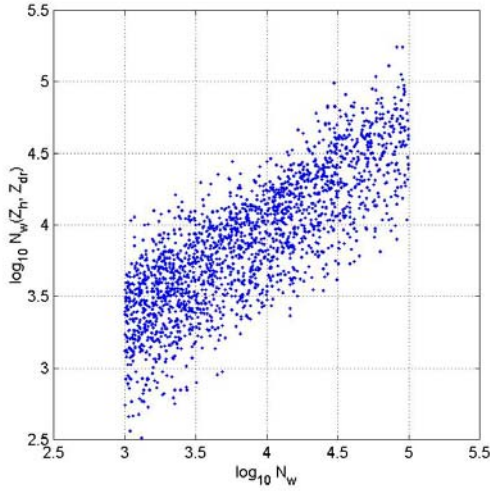


Figure 3. Scatterplot of  $\log_{10} N_w(Z_h, Z_{dr})$  versus the true value of the  $\log_{10} N_w$  for widely varying RSD at S band.

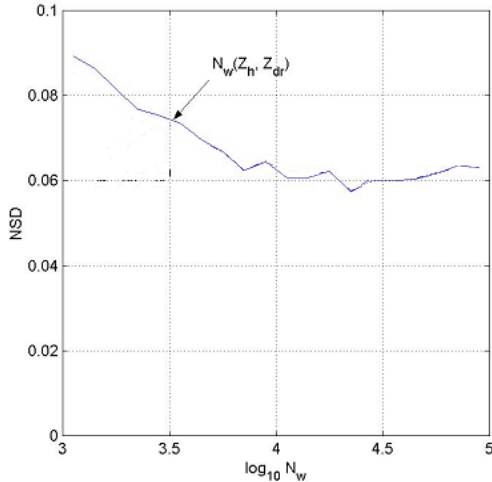


Figure 4. Normalized standard deviation in the estimates of  $\log_{10} N_w$ , as a function of  $\log_{10} N_w$  at S band.

#### 4.2 Parameterization of $\mu$

The parameter  $\mu$  describes the overall shape of the distribution. Once  $D_0$  is estimated,  $\mu$  can be estimated from the following parameterization which was constructed empirically as,

$$\hat{\mu} = \frac{a_5 D_0^{b_5}}{(\xi_{dr} - 1)} - c_5 (\xi_{dr})^{d_5} \quad (28)$$

The variability of  $a_5$ ,  $b_5$ ,  $c_5$  and  $d_5$  can be parameterized in terms of  $\beta$  as

$$a_5 = 200 \beta^{1.89} \quad (29a)$$

$$b_5 = 2.23 \beta^{0.039} \quad (29b)$$

$$c_5 = 3.16 \beta^{-0.046} \quad (29c)$$

$$d_5 = 0.374 \beta^{-0.355} \quad (29d)$$

$D_0$  calculated from (23) can be utilized in (28) to estimate  $\mu$ . However, in practice  $D_0$  has to be estimated using  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$ . Estimating  $\mu$  under such conditions will result in large errors than that. Estimating  $\mu$  accurately under practical conditions, especially in the presence of measurement errors is very difficult using the procedures discussed here.

#### 5. IMPACT OF MEASUREMENT ERROR ON THE ESTIMATES OF $D_0$ AND $N_w$ .

Estimators of  $D_0$  use measurements of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ . Any error in the measurement of these three parameters will directly translate into errors in the estimates of  $D_0$  and  $N_w$ . The three measurements  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  have completely different error structures.

The  $Z_h$  is based on absolute power measurement and has a typical accuracy of 1 dB. The  $Z_{dr}$  is a relative power measurement which can be estimated to an accuracy of about 0.2 dB.  $K_{dp}$  is the slope of the range profile of the differential propagation phase  $\Phi_{dp}$ , which can be estimated to an accuracy of a few degrees. The subsequent estimate of  $K_{dp}$  depends on the procedure used to compute the range derivative of  $\Phi_{dp}$  such as a simple finite-difference scheme or a least squares fit. Using a least squares estimate of the  $\Phi_{dp}$  profile, the standard deviation of  $K_{dp}$  can be expressed as (Gorgucci et al. 1999),

$$\sigma(K_{dp}) = \sqrt{3} \frac{\sigma(\Phi_{dp})}{N\Delta r} \sqrt{\frac{N}{(N-1)(N+1)}}, \quad (30)$$

where  $\Delta r$  is the range resolution of the  $\Phi_{dp}$  estimate and  $N$  is the number of range samples along the path. For a typical 150 m range spacing, and with  $2.5^\circ$  accuracy of  $\Phi_{dp}$ ,  $K_{dp}$  can be estimated over a path of 3 km, with a standard error of  $0.32^\circ \text{ km}^{-1}$ .

The measurement errors of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are nearly independent. In the following, simulations are used to quantify the error structure of the estimates of  $D_0$  and  $N_w$ .

The normalized standard deviation in the estimates of  $D_0$  and  $N_w$  including the effect of measurement error are evaluated and shown in Figs. 5 and 6, respectively. Fig. 5 shows the NSD in the estimates of  $D_0$ . Comparing Fig. 5 to Fig. 2 it can be seen that in general, there is about a 10% increase in the NSD of  $D_0$  estimate due to measurement error. The NSD of the  $N_w$  estimates in the presence of measurement errors, are shown in Fig. 6. Again comparing these to the NSD computations without measurement error (Fig. 4), a 4% to 16%

increase is noted depending on the value of  $N_w$ . Thus,  $D_0$  and  $N_w$  can be estimated fairly well from radar measurements at least for convective rainfall with  $R \geq 5-10 \text{ mm h}^{-1}$ . These errors can be further reduced using other techniques such as spatial averaging whenever possible. The following section presents evaluation of the algorithms developed here using disdrometer observations.

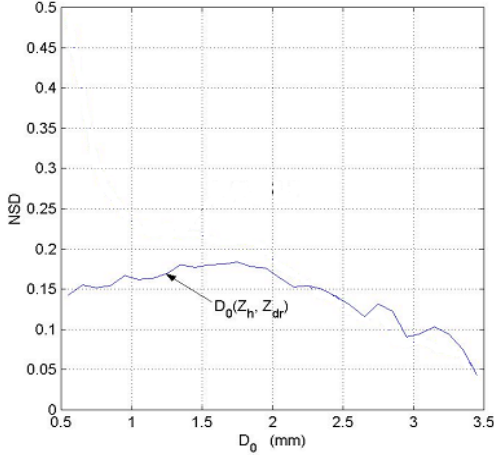


Figure 5. Normalized standard deviation in the estimates of  $D_0$  as a function of  $D_0$  in the presence of radar measurement errors at S band.

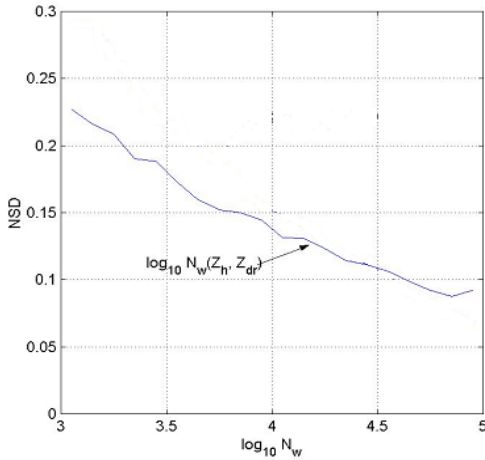


Figure 6. Normalized standard deviation (NSD) in the estimates of  $\log_{10} N_w$  as a function of  $\log_{10} N_w$  in the presence of radar measurement errors at S band.

## 6. EVALUATION OF THE ALGORITHMS USING DISDROMETER DATA

The algorithms developed in this paper to estimate  $D_0$  and  $N_w$  are applied to data collected with a J-W impact disdrometer (Joss and Waldvogel 1967) during a rainfall season (covering about three months) from Darwin (Australia). This data set was collected by the Bureau of Meteorology Research Center (BMRC) and includes a variety of rainfall types from a tropical regime with rain rates between 1 to  $150 \text{ mm h}^{-1}$ . The disdrometer data

consists of measurements of  $N(D)$  in discrete intervals of  $\Delta D$  at 30 seconds intervals which are subsequently averaged over 2 minutes. While several methods are available to fit the measured  $N(D)$  to a gamma form (e.g., Willis 1984), the method used here is based on Bringi and Chandrasekar (2001).

Once the set of  $(N_w, D_0, \mu)$  parameters are obtained, the radar observables  $Z_h, Z_{dr}$  and  $K_{dp}$  are simulated based on the following assumptions:

- 1) Axis ratio versus  $D$  relation based on the fit proposed by Andsager et al. (1999).
- 2) Gaussian canting angle distribution with mean of  $0^\circ$  and standard deviation  $10^\circ$ .
- 3) Truncation of the gamma DSD at  $D_{max}=3.5 D_m$ .

The simulated set of radar observables ( $Z_h, Z_{dr}$  and  $K_{dp}$ ) when used in (23) gives an “effective”  $\beta$  of 0.0475 (for comparison the equilibrium  $\beta$  is 0.062).

Note that the algorithms for  $D_0$  and  $N_w$  are constructed to be insensitive to the actual value of  $\beta$ , so that the details of the assumptions used in simulating the set of radar observables are not of particular relevance, and this fact is indeed the power of the proposed  $D_0$  and  $N_w$  algorithms. In order to evaluate these algorithms using disdrometer measurements, the simulated values of  $Z_h, Z_{dr}$  and  $K_{dp}$  are used to calculate  $\hat{D}_0, \hat{N}_w$  and  $\hat{\mu}$  which are then compared against  $D_0, N_w$  and  $\mu$  estimated by gamma fits to the set of measured  $N(D)$ . Figure 7a shows the  $D_0$  comparisons while Fig. 7b shows the NSD. Note that the  $\hat{D}_0$  algorithm can retrieve the “true”  $D_0$  quite accurately (NSD<7%) especially for  $D_0 > 1 \text{ mm}$ . As expected the  $D_0$  estimates get very accurate for higher values. The  $\log_{10}(N_w)$  comparisons are shown in Fig. 8a while 8b shows the NSD. The scatter in Fig. 8a shows that the accuracy in the retrieval of  $\log_{10} N_w$  is quite high (<5 %) for  $N_w > 1000 \text{ mm}^{-1} \text{ m}^{-3}$  (for reference the Marshall-Palmer value for  $N_w$  is  $8000 \text{ mm}^{-1} \text{ m}^{-3}$ ).

## 7. SUMMARY AND CONCLUSION

One of the long-standing goals of polarimetric radar has been the estimation of the parameters of the raindrop size distribution. Estimators for the parameters of a three parameter gamma model, namely  $D_0, N_w$  and  $\mu$  are developed in this paper based on the radar observations  $Z_h, Z_{dr}$  and  $K_{dp}$ . The behavior of the three radar observations  $Z_h, Z_{dr}$  and  $K_{dp}$  are influenced by the underlying DSD, and the mean shape of raindrops.  $Z_{dr}$  is proportional to the reflectivity weighted axis ratio whereas  $K_{dp}$  is proportional to the volume weighted deviation of the axis ratio from unity. In addition, reflectivity is proportional to the sixth moment of the DSD, with corresponding variability due to polarization. Thus, the different polarimetric radar observations weight the DSD differently. It should be noted that the DSD estimates computed here correspond to radar measurements from the radar resolution volume. Among the three measurements ( $Z_h, Z_{dr}$  and  $K_{dp}$ ),  $Z_{dr}$  is the most directly related to a parameter of the DSD, namely  $D_0$ . Gorgucci et al. (2000) described a procedure to estimate the mean shape-size relation of raindrops based on a simple linear model. Therefore, after the prevailing shape-size relation is established,  $Z_{dr}$  can be

used to estimate  $D_0$  directly. This concept is implemented in this paper as an algorithm to estimate  $D_0$  from  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$ . Statistical analysis of the estimator of  $D_0$  indicates that it can be estimated to an accuracy of 10% when  $D_0$  is 2 mm (and similar accuracies at the other  $D_0$  values). Once  $D_0$  is estimated, other measurements such as  $Z_h$  or  $K_{dp}$  can be used to estimate  $N_w$ , to a normalized standard deviation of about 6.5 % when  $N_w=8000 \text{ mm}^{-1} \text{ m}^{-3}$  and similar order at the other values. The estimation of  $\mu$  is not easy because of the least influence of this parameter on the three measurements  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$ . Therefore, the parametric estimates of  $\mu$  derived are not as accurate. Measurement errors in  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$  play a key role in the final accuracy of DSD estimates.  $Z_{dr}$  is a differential power measurement between two correlated signals, and can be measured accurately.

This high degree of accuracy in  $Z_{dr}$  translates to high accuracy in  $D_0$ . However, to estimate the prevailing mean shape-size relation,  $K_{dp}$  is needed which is relatively noisy at low rain rates. A hybrid approach is implemented in this paper such that when  $K_{dp} \leq 0.2 \text{ deg. km}^{-1}$  the equilibrium shape model is used to estimate  $D_0$ . Bringi et al. (2002) have extended this procedure to low rain rates. The algorithms developed here were applied to one rainy season of disdrometer data collected in Darwin, Australia. The disdrometer analysis indicates that the algorithms work fairly well for the estimation of  $D_0$  and  $N_w$ . In summary, the algorithms presented in this paper can be used to estimate the parameters of the raindrop size distribution, from polarimetric radar data at a frequency near 3 GHz (S-band), directly. However attenuation correction needs to be introduced for C and X band.

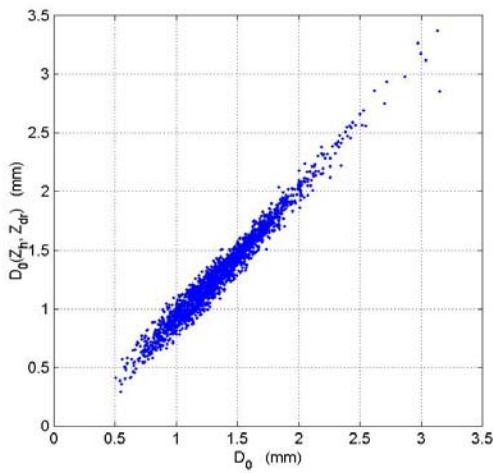


Figure 7a. Scatterplot of the estimate of  $D_0$  computed from simulations of  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$ , versus the direct estimate of  $D_0$  for RSD obtained from a disdrometer located near Darwin, Australia.

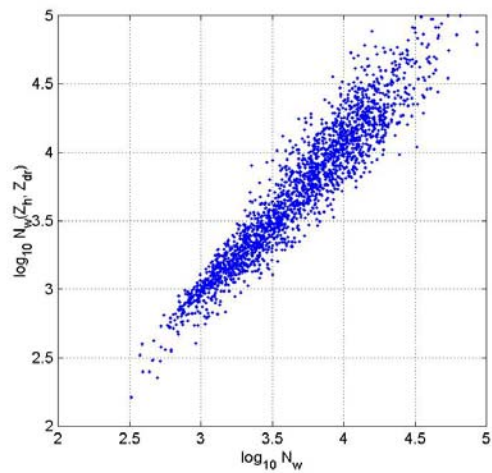


Figure 8a. Scatterplot of  $\log_{10} N_w(Z_h, Z_{dr})$ , versus the direct estimate of  $\log_{10} N_w$  for RSD obtained from a disdrometer located near Darwin, Australia.

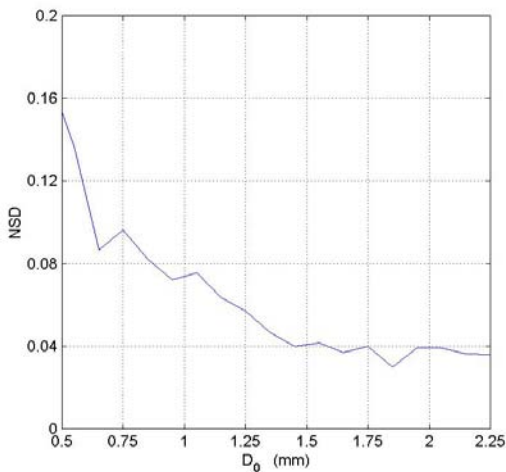


Figure 7b. Normalized standard deviation in the estimate of  $D_0$ , computed from simulations of  $Z_h$ ,  $Z_{dr}$  and  $K_{dp}$ , versus the direct estimate of  $D_0$  for RSD obtained from a disdrometer located near Darwin, Australia.

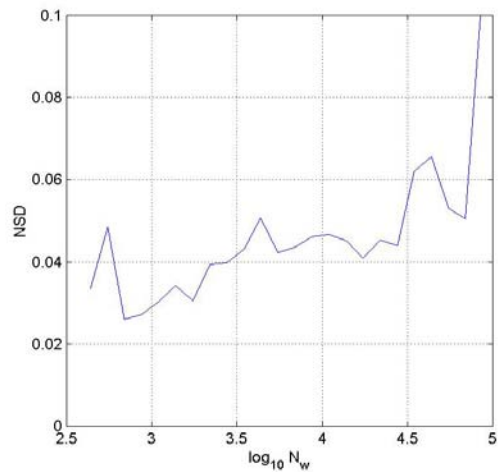


Figure 8b. Normalized standard deviation of  $\log_{10} N_w(Z_h, Z_{dr})$ , versus the direct estimate of  $\log_{10} N_w$  for RSD obtained from a disdrometer located near Darwin, Australia.

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