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QUANTIFYING DROPLET CLUSTERING IN CLOUDS

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A great deal of work has been devoted in the last decade to identify whether or not the spatial positions of cloud particles are statistically correlated. If, in fact, spatial correlations do exist there could be consequences in radiation attenuation (Kostinski (2001) and Shaw *et al.* (2002a)) and collision rates (note the assumption of independent and identically distributed spatial positions of droplets in Saffman & Turner (1956)). Although quite varied theoretical tools have been utilized in attacking this problem, many of them are fundamentally related in ways that have not been explicitly noted. In studying the analytic connections between these methods, a great deal of physical intuition and knowledge about the limitations of the statistics in question can be acquired.

More specifically, the tools of the pair-correlation function [$\eta(r)$], clustering index [$CI(r)$], volume-averaged pair-correlation function [$\overline{\eta(r)}$], autocorrelation function [$\rho(r)$], and power spectral density [$P(k)$] are analyzed here. Most of these methods have been used recently to characterize correlations in spatial positions of cloud droplets. In the interests of brevity, the relative merits of these statistics will not be addressed here. A more thorough literature review and careful analysis can be found in Shaw *et al.* (2002b). Representative papers are: Kostinski & Jameson (2000); Kostinski & Shaw (2001) (pair-correlation function), Baker (1992); Chaumat & Brenguier (2001) (clustering index), Jaczewski & Malinowski (2000) (volume averaged pair-correlation function), and Gerber *et al.* (2001) (power spectral density). The pair-correlation function, $\eta(r)$, (or in the fluids literature, the radial distribution function $g(r) \equiv \eta(r) + 1$) is an ideal measure to identify scale-localized correlations because of its memoryless character. Each of the other variables, with the exception of $\rho(r)$, is fundamentally defined through an integration process, thus introducing cumulative scale memory to these statistics. The auto-correlation function, $\rho(r)$, does not fall to this criticism, but technically is only well-defined for continuous variables. Since cloud particle existence is inherently a point process (either a droplet is present or it is not), the language of the pair-correlation function is more natural.

In addition to the benefits mentioned above, there exist simple relationships between the pair-correlation function and the other mentioned cloud clustering statistics Shaw *et al.* (2002b). Briefly, $\eta(r)$ is related to $\overline{\eta(r)}$ and $CI(r)$ through the use of the Ornstein-Zernike equation (or correlation-fluctuation theorem); $\eta(r)$ is related to $\rho(r)$ through a multiplicative parameter; and $\rho(r)$ is related to $P(k)$ via the Wiener-Khinchin theorem. Because of the fact that $\eta(r)$ has no memory, the value of η at a given spatial scale has a clear physical meaning. The pair-correlation function is *defined* as the scale-localized deviation from an uncorrelated (Poisson) stationary process.

Specifically, consider volume elements dV_i of magnitude small enough such that the number of particles given in the volume is merely $\bar{n}dV_i$ where $\bar{n}$ is the mean droplet number density. Then if no correlations exist on scale r, one would expect the probability that there exist droplets in both dV_1 and dV_2 (without correlations) to be given by

$$P_r(1,2) = (\bar{n}dV_1)(\bar{n}dV_2). \quad (1)$$

We then define $\eta(r)$ through

$$P_r(1,2) = (\bar{n}dV_1)(\bar{n}dV_2)[1 + \eta(r)]. \quad (2)$$

so that $\eta(r)$ is a *direct* measure of the scale deviation from pure (Poisson-distributed) randomness. Perhaps more simply, $\eta(r)$ can be computed as the number of detections observed distance r apart, divided by the number expected for a completely random distribution (with unity subtracted off of this quantity) (for a more detailed explanation, see Kostinski & Jameson (2000), Kostinski & Shaw (2001), and Shaw *et al.* (2002b)). Operationally, then, $\eta(r)$ can be written as

$$\eta(r) = \frac{\overline{N(R)N(R+r)}}{\overline{N}^2} - 1 \quad (3)$$

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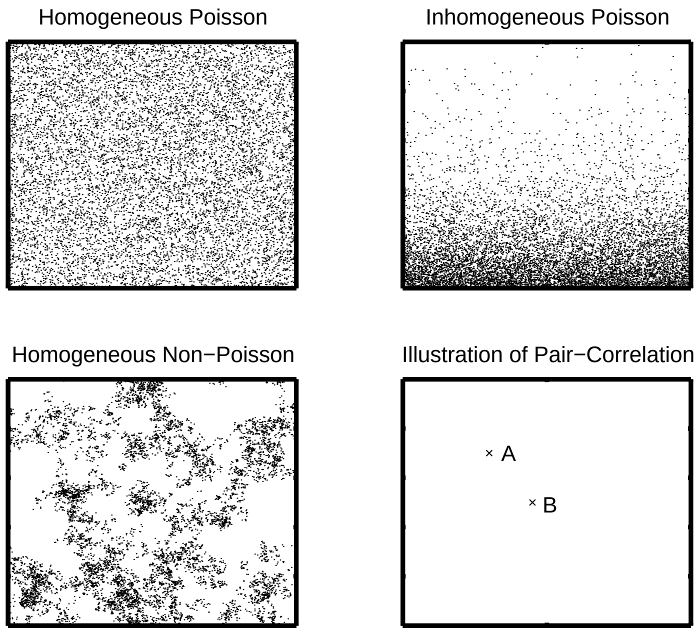

Figure 1: *Upper Left* Statistically homogeneous Poisson process. Particle positions are independent of any correlations. *Lower Left* Statistically homogeneous but correlated random process. Particle positions are correlated (i.e. not independent). *Upper Right* Statistically inhomogeneous Poisson process. A physical example would be an ideal gas of molecules subject to gravitation. *Lower Right* An introduction to the notion of pair-correlation; see text for description.

which is directly linked to the autocorrelation function, though care must be taken in remembering that detection of a cloud droplet is a point process.

Due to the fact that the pair correlation function provides a direct measure of the deviation from pure randomness, $\eta(r)$ carries with it physical meaning while avoiding ad hoc assumptions about the origin or magnitude and shape of correlations; this point, though subtle, is one of the reasons that the pair-correlation function is best suited to classification of stationary point processes and is applicable here. Because of the scale-localized character of the measurement, it seems likely that the pair-correlation function will be able to be incorporated directly into analytic expressions for physical processes. Some work in applying this notion to collision rates in a dilute medium has already been completed by Sundaram & Collins (1997) and in coagulation of aerosol particles by Kasper (1984).

We have noted that this method of correlation is fundamentally different than some other commonly used approaches. Figure 1 gives a simple demonstration how this is so. The upper-left hand panel shows a small cell out of a "perfectly random" distribution. As such, there are no correlations on all scales (i.e. particle positions are independent), the statistical moments are independent of the origin, and the particle positions are uniformly distributed throughout the volume.

The lower-left hand panel demonstrates the notion of a homogeneous, but not Poisson, distribution. In this distribution, moments are still independent of the origin, and the distribution is still uniform with the caveat that the particle positions are not independent; the placement of each particle is correlated to the other particles in the distribution. In making such a distribution, the particles will (for positive correlations) inevitably form clumps, but the position of the clumps are unpredictable in space, thus assuring statistical homogeneity (or stationarity).

The upper right panel represents the underlying model for an alternate approach to classifying correlations – the notion of an inhomogeneous Poisson process. In such a distribution, the local number density is fundamentally a deterministic (hence predictable) process, and not independent of the origin. In order to use this approach, it must be possible to model the large-scale variations deterministically. Pawlowska *et al.* (1997) and Pinsky & Khain (2001) are representative papers that utilize such a model of an inhomogeneous Poisson process.

The lower right panel of the figure is a visual tool for better understanding the pair-correlation function. The first point (A) is placed completely at random within a volume. The following particles then have an 'biased' probability of being at point B, a distance r away from A, enhanced (or repressed) by a factor of $(1 + \eta(r))$ multiplied by the probability of being at point B devoid of any correlations. If $\eta(r)$ is, for example, a positive but monotonically decreasing function of r, clumps like those in the lower left panel of figure 1 result.

To further illustrate the points above, numerical examples and data-analysis were carried out in both Kostinski & Shaw (2001) and Shaw *et al.* (2002b). In the former, a numerical simulation is used to demonstrate how the use of statistics with memory, especially when coupled with a finite-resolution probe, can suppress detection of real coherence. In both of these studies, data analysis on data gathered by the Fast-FSSP (forward scattering spectrometer probe) was analyzed, revealing evidence of mild, but existent, correlations on some scales (even in regions of the cloud that visually appeared stationary and completely random). Whether or not the magnitude of these correlations is negligible has not yet been fully determined. At this point, more careful investigation as to the implications of short-scale droplet clustering on the theories of cloud evolution and radiative transfer are warranted; it seems quite likely that further investigation will yield non-negligible short-scale correlations, and preliminary studies indicate that the effects of even mild clumping can be quite extreme when incorporated into rate-governed processes.

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