1. INTRODUCTION

Onboard the core satellite, the centerpiece of the newly authorized Global Precipitation Measurement (GPM) mission, will be a dual-frequency radar transmitting at 13.6 and 35 GHz frequencies in addition to a TRMM-like radiometer with expanded capabilities. The dual-frequency capability of the radar is expected to take precipitation measurement beyond the accomplishments of the Tropical Rainfall Measurement Mission (TRMM). Although multi-frequency methods in general improve our rainfall retrievals over the single-frequency methods, their success is still limited. This paper intends to investigate the cause(s) of such underachievement and to embark on, hopefully, finding a remedy.

2. BACKGROUNDS

In the following discussion we limit our scope to liquid precipitation only. We assume that the only material reflecting or attenuating a radar pulse is the water droplets in the forms of rain and/or cloud. Moreover, we assume these droplets to be spherical. These assumptions should and will be relaxed in future investigations.

The radar retrieval of rainfall is based upon the return signal from the reflection of the radar pulse by hydrometeors in the illuminated volume, which is normally expressed as equivalent radar reflectivity factor in mm$^3$ m$^{-6}$, $Z_e = \frac{10^6 \lambda^4}{\pi^3} \int_{r_{\text{min}}}^{r_{\text{max}}} \sigma_s(r) n(r) dr$, (1)

where $\lambda$ is the wavelength (cm) used by the radar, $K_e$ is the dielectric factor of water, $\sigma_s(r)$ is the backscatter cross section (cm$^2$) of a raindrop with radius $r$ (cm), and $n(r)$ (m$^3$ cm$^{-1}$) is the droplet size distribution (DSD) in the volume illuminated by the radar pulse. Since the reflected signal is attenuated by the medium between the illuminated volume and the radar antenna, a closely related quantity is the specific attenuation (dB km$^{-1}$), $k = 0.434 \int_{r_{\text{min}}}^{r_{\text{max}}} \sigma_s(r) n(r) dr$, (2)

whereas the mathematical expression for rainfall rate in mm/hr is $R = 4.8\pi \int_{r_{\text{min}}}^{r_{\text{max}}} v(r) r^3 n(r) dr$, (3)

where $v(r)$ is the fall speed of the raindrop with radius $r$. All three quantities are integrals involving the DSD. Therefore many of the radar precipitation retrieval methods seek to relate $Z_e$ and $k$ to $R$ in simple analytical expressions. This results in many $Z - R$ relations reported in Battan (1973). However, there is no information in either $Z_e$ or $k$ about $v(r)$, which is usually a complicated function of air motion, hence most methods assume it to be the same as the fall speed in stagnant air, $v_i(r)$. The authors believe this is one of the causes in the limited success achieved by $Z - R$ relations.

Next, we turn our attention to a similar integral quantity also involving the DSD, the liquid water content (g/m$^3$), $W = \frac{4}{3} \pi \rho_w \int_{r_{\text{min}}}^{r_{\text{max}}} r^3 n(r) dr$, (4)

which we have better hope of accurate retrieval without knowing anything concerning $v(r)$. Since the DSD, $n(r)$, is the underlining thread connecting (1), (2) and (4), many researchers (e.g. Marshall and Palmer, 1948; Ulbrich, 1983) therefore assume a certain distribution model for the DSD (e.g. exponential or gamma) and carry out precipitation retrieval based on it. However, the operands in the integrals of (1), (2) and (4) have different dependencies. Consequently different DSD’s giving the same $Z_e$ or $k$ may yield different $W$ values and, vice versa. It is this non-uniqueness that has plagued the progress in radar rainfall retrieval in general. This non-uniqueness, the authors believe, is another major cause to the myriad of different $Z - R$ relationships.

In as early as the 1950s, Atlas and colleagues (Atlas, 1954; Atlas and Chmela, 1957 and references thereof) find that, when Rayleigh approximation is valid, the radar reflectivity factor $Z$ (mm$^6$ m$^{-3}$) can be expressed in terms of liquid water content, $W$ (g/cm$^3$), median volume diameter, $D_e$ (cm), and $G$ as
\[ Z = \frac{6 \times 10^6}{\pi \rho_w} D_0^6 \]  

where
\[
G = \int_0^{D_{\text{max}}} D' N(D)dD
\]
\[
D_0 = \int_0^{D_{\text{max}}} D' N(D)dD
\]
is a dimensionless measure of the breadth of the DSD. One can easily verify that the above relation holds regardless of the distribution model assumed, i.e. it is true for any DSD. In other words, the three parameters: \( W, \) \( D_0, \) and \( G \) uniquely characterize the DSD in determining \( Z. \) In this investigation we venture to find such characteristic parameters suitable for the GPM radar frequencies where Rayleigh approximation may no longer be valid.

3. METHODOLOGY

In the optical wavelengths, Hansen and Travis (1974) propose using effective radius and effective variance, defined as
\[
e_r = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} r^2 n(r)dr}{\int_{r_{\text{min}}}^{r_{\text{max}}} r^3 n(r)dr}
\]
and
\[
v_e = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} r^4 n(r)dr}{r_{\text{max}}^3 \int_{r_{\text{min}}}^{r_{\text{max}}} r^3 n(r)dr} - 1
\]
respectively, for cloud microphysical retrievals. Since the size parameter, \( 2\pi r / \lambda, \) is comparable (especially at the 35 GHz frequency) these are the first candidates in our search for characteristic DSD parameters. The results, not surprisingly, turn out to be very encouraging.

We start with a very general formulation for droplet size distributions, a combination of two modified gamma distribution, which is capable of producing the bimodal feature often observed in rain,

\[
n(r) = \eta_{\mu, k, z_{\text{min}}, z_{\text{max}}} \left( \frac{r}{r_{e,1}} \right)^\mu \exp \left[ -\left( \frac{r}{r_{e,1}} \right)^\kappa \right] \left( \frac{fN_r}{r_{e,1}} \right)
+ \eta_{\mu, k, z_{\text{min}}, z_{\text{max}}} \left( \frac{r}{r_{e,2}} \right)^\mu \exp \left[ -\left( \frac{r}{r_{e,2}} \right)^\kappa \right] \left( \frac{1-f)N_r}{r_{e,2}} \right)
\]

where \( \mu \) and \( \kappa \) are dimensionless parameters of the distribution, \( N_r \) is the total number concentration, \( f \) is a fraction between 0 and 1, \( r_{e,1} \) and \( r_{e,2} \) (\( r_{e,1} < r_{e,2} \)) are the characteristic radii for the first and second modified gamma distributions respectively.

\[
\eta_{\mu, k, z_{\text{min}}, z_{\text{max}}} = \eta(\mu, k, z_{\text{min}}, z_{\text{max}})
= \eta \left[ \mu, k, \left( \frac{r_{\text{min}}}{r_{e,1}} \right), \left( \frac{r_{\text{max}}}{r_{e,1}} \right) \right]
= \gamma \left( \frac{\mu + 1}{k}, z_{\text{min}} \right) - \gamma \left( \frac{\mu + 1}{k}, z_{\text{max}} \right)
\]
and \( \gamma \) is the incomplete gamma function.

\[
\gamma(x, \alpha) = \int_0^x r^{\alpha-1}e^{-r}dr.
\]

In our investigation we set \( r_{\text{min}} = 0. \) Even with this constraint, such a DSD model allows no fewer than six adjustable parameters (excluding \( N_r \), for reasons that become clear later): \( \mu, k, f, r_{\text{min}}, r_{e,1}, \) and \( r_{e,2} \), so that one may have confidence that it will approximate any natural DSD well.

With (8) as our model for DSD we may evaluate \( W, \) \( r_e \) and \( v_e \) to be

\[
W = \frac{4}{3} \pi \rho_w H(3)N_r r_{e,1}^3
\]
\[
r_e = \frac{H(3)}{H(2)} r_{e,1} \quad \text{and} \quad v_e = \frac{H(2)H(4)}{[H(3)]^2} - 1,
\]

where

\[
\eta_m(m, \mu, k, f, r_{e,1}, r_{e,2}, r_{\text{min}}, r_{\text{max}})
= \int \eta_{\mu, k, z_{\text{min}}, z_{\text{max}}} \left( \frac{r_{e,2}}{r_{e,1}} \right)^\mu \exp \left[ -\left( \frac{r_{e,2}}{r_{e,1}} \right)^\kappa \right] \left( \frac{1-f)N_r}{r_{e,2}} \right)
\]

We notice that \( N_r \) is not present in the formulation of either \( r_e \) or \( v_e \). In other words, when a distribution is chosen by fixing the other parameters, \( N_r \) only serves to modify the value of \( W \). The value of \( r_e \) or \( v_e \) is not

<table>
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<th>( r_{e,1} )</th>
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<td>0.3</td>
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Table 1. Distribution parameters for the five categories of distributions.
affected by \( N_f \) at all.

In our simulations we systematically vary \( \nu_e \) from 0.1 to 0.5 with a 0.1 increment. For each value of \( \nu_e \) we vary \( r_e \) among the values of 0.25, 0.5, 1, 2 and 4 mm. For each paired values of \((\nu_e, r_e)\), a parameter-finding procedure finds, in each of the five categories listed in Table 1, the distribution parameters: \( \mu_e \), \( \kappa_e \), \( r_{e1} \) and \( r_{e2} \) for a number of distributions yielding the desired \( r_e \) and \( \nu_e \) values. Three distributions are then randomly selected in each distribution category from those found by the parameter-finder. This results in fifteen (15) distributions for each pair of \((\nu_e, r_e)\). For the reason mentioned above, the total number concentration, \( N_f \), is adjusted so that the liquid water content is held constant at 1 g/m\(^3\) in our later discussions unless explicitly stated otherwise.

4. RESULTS

Figure 1 displays fifteen DSDs each in logarithmic scale for \((\nu_e, r_e)\) of (1 mm, 0.2) in the upper panel and (2 mm, 0.2) in the lower panel. The difference among the distributions is quite obvious from the figure.

The upper panel of Figure 2 shows the reflectivity factors at 13.6 GHz (thick line) and 35 GHz (thin line) for the fifteen distributions with \((\nu_e, r_e)\) of (0.25 mm, 0.5). It is rather difficult to detect variations in these lines, therefore their deviations from the means are plotted in the lower panel with thick lines connecting solid circles for 13.6 GHz and thin lines connecting open squares for 35 GHz. We pick this figure out of the 25 available because it exhibits the greatest variation in \( Z_e \) among all \((\nu_e, r_e)\) combinations. Yet, the entire range of variation is only about 1 dB for 13.6 GHz and less than 0.5 dB for 36 GHz.

The variations of \( Z_e \) among distributions is depicted in a more comprehensive manner in the contour plot of Figure 3. The solid lines contour the average \( Z_e \) of the fifteen distributions as a function of \((\nu_e, r_e)\) while the dotted lines contour the minimum \( Z_e \) of the distributions and the dashed lines the maximum. The closeness of the dotted and dashed contours to their corresponding solid ones demonstrates that, when liquid water content is held constant, \((\nu_e, r_e)\) is a good predictor of \( Z_e \). (The large separation for the 13.6 GHz near \( r_e = 4 \) mm, i.e., the boundary of our \( r_e \) domain, is mainly due to sparseness in data points (25 total), the slow change in \( Z_e \) values in this region, and an artifact of the contouring procedure used.)

The variations in specific attenuation, \( k_e \), is even less. The greatest range of variation among the fifteen distributions occurs at \((\nu_e, r_e)\) of (2 mm, 0.2) and it is less than 0.2 dB. In a similar figure as Fig. 3 (not shown), the tightness among the contours of different line styles is more pronounced that that shown in Fig. 3. This means that the variations caused by differences in
distributions are less in $k$ than in $Z_w$ across the entire range of $(r_o, v_e)$ values.

Now, we take a look at the influence of $W$. Figure 4 shows the combined influence of $r_o$ and $W$ simultaneously on $Z_w$ for both frequencies at a constant $v_e$ of 0.1. Five diagonal lines are isopleths of $r_o$ equal to 0.25, 0.5, 1, 2 and 4 mm from left to right respectively. (The isopleths 0.25 and 0.5 mm are too close together for distinction.) Each of these lines connects the $Z_w$ values for $r_o$ of 0.1, 0.266, 0.707, 1.88 and 5 g/m$^3$ from lower-left to upper-right respectively. The constant $W$ values are also connected by line segments to make the grid-like appearance in the figure. The closeness of the isopleths for $r_o \leq 0.5$ mm may explain the difficulties of radar remote sensing encountered in light rain situations. Because a DSD with a smaller $W$ and a larger $r_o$ may be interpreted as one with a larger $W$ and a smaller $r_o$, it leads to error in the $R$ estimate. Although the isopleths of smaller $r_o$ values grow apart as $v_e$ increases (not shown), remembering that $v_e$ is a measure of the breadth of the DSD one expects a smaller $v_e$ for light rains.

The non-uniqueness in $k$ for smaller $r_o$ and $v_e$ values is worse than that in $Z_w$. However, it appears that, within the errors shown in Fig. 2, there is almost a one-to-one correspondence between $(Z_w(13.6,35), W)$ for $r_o > 1$ mm when $v_e$ is known.

5. CONCLUSIONS

We find in this study that, as far as equivalent radar reflectivity factor and specific attenuation are concerned, the three parameters $r_o$, $v_e$, and $W$ characterize the droplet size distribution more adequately than parameters currently used in radar precipitation retrievals. In other words, it doesn’t matter what DSD models one assumes; as long as the distributions have the same $r_o$, $v_e$, and $W$ they yield roughly the same $Z_w$ and $k$ in the two frequencies of 13.6 and 35 GHz. Such a framework also sheds light on the causes for the usual difficulties encountered in radar rainfall retrieval, such as the copious $Z - R$ relations and the particular inaccuracies in light rain situations. Furthermore, it facilitates better characterization of retrieval errors.

6. REFERENCES


