# P1.22 MASSFLUX BUDGETS OF SHALLOW CUMULUS CLOUDS *

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## 1. Introduction

The parameterization of vertical transport due to cumulus clouds is often performed by a massflux approach (Tiedtke, 1989). In such schemes it is assumed that the cumulus cloud field can be well represented by a top-hat distribution. An important variable that needs to parameterized is the convective massflux $(M_c)$,

$$M_c \equiv \sigma_s(w_s - \overline{w}) = \sigma_s(1-\sigma_s)(w_s - w_e), \qquad (1)$$

which is usually diagnosed from the continuity equation for mass,

$$\frac{\partial M_c}{\partial z} = -\frac{\partial \sigma_s}{\partial t} + E - D, \qquad (2)$$

with $\sigma_s$ the cloud fraction, and $E$ and $D$ the lateral entrainment and detrainment rates, respectively. Another approach would be to include a prognostic equation for the vertical velocity in the cloud, which offers the advantage that it links the thermodynamic state of the atmosphere to the dynamics by the buoyancy term. In this paper we will analyze the budgets of the massflux by means of a large-eddy simulation (LES) of shallow cumulus based on observations during BOMEX (Siebesma and Cuijpers, 1995). Moreover, we will present the vertical velocity variance budget in the massflux approach to assess how well the top-hat approach represents the Reynolds-averaged budget.

## 2. The conditionally sampled vertical velocity equation

In our simulations and analyses, we use the Boussinesq equations and their LES implementation. The filtered prognostic equation for the resolved part of the momentum equation reads

$$\frac{\partial u_i}{\partial t} = \frac{g}{\theta_0}(\theta_v - \overline{\theta_v})\delta_{i3} - \frac{\partial u_i u_j}{\partial x_j} - \frac{\partial \pi}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j}. \qquad (3)$$

The velocity components $u_i = (u, v, w)$ are the components in $x_i = (x, y, z)$ directions, respectively, $\pi$ is the modified pressure (Deardorff, 1973), $t$ is the time, $g$ the gravitational acceleration, $\theta_v - \overline{\theta_v}$ the perturbation of the virtual potential temperature with respect to its horizontally-averaged mean value, $\theta_0$ the reference state potential temperature, $\delta_{ij}$ the Kronecker delta, and $\overline{u_j''\psi''}$ and $\tau_{ij}$ are the subgrid flux terms that arise from the filtering procedure.

Conditionally sampling Eq. (3) gives (Young, 1988; Schumann and Moeng, 1991)

$$\left[\frac{\partial w}{\partial t}\right]_s = \frac{g}{\theta_0}([\theta_v]_s - \overline{\theta_v}) - \left[\frac{\partial w^2}{\partial z}\right]_s - \left[\frac{\partial u_h w}{\partial x_h}\right]_s - \left[\frac{\partial \pi}{\partial z}\right]_s - \left[\frac{\partial \tau_{3j}}{\partial x_j}\right]_s \qquad (4)$$

where $u_h = (u, v)$, $\partial x_h = (\partial x, \partial y)$ and the the operator '$[\ ]_s$' denotes the conditionally sampled mean. An identical equation as (4) can be written for the environment simply by replacing the subscript '$_s$' by '$_e$'. In the remainder of the paper the square brackets are, for notational convenience, replaced by subscripts 's' and 'e', except when the operator is applied on a derivative. Note that if we move the sampling operator inside a derivative, we must apply Leibniz rule, for example (Young, 1988)

$$\left[\frac{\partial w^2}{\partial z}\right]_s = \frac{\partial [w^2]_s}{\partial z} + \frac{[w^2]_s}{\sigma_s}\frac{\partial \sigma_s}{\partial z} + \left\{\frac{\partial w^2}{\partial z}\right\}_{b,s}. \qquad (5)$$

We compute the entrainment $(E_w)$ and detrainment $(D_w)$ rates for the vertical velocity from

$$\frac{\partial \sigma_s w_s}{\partial t} = \sigma_s \frac{g}{\theta_0}(\theta_{v,s} - \overline{\theta_v}) - \frac{\partial M_c w_s}{\partial z} - \frac{\partial \sigma_s [w'' w'']_s}{\partial z} + E_w w_e - D_w w_s - \sigma_s \left[\frac{\partial \pi}{\partial z}\right]_s, \qquad (6)$$

and the continuity equation (2) with $E$ and $D$ replaced by $E_s$ and $D_s$, respectively. The double primes indicate the 'subplume' perturbations.

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If we multiply Eq. (4) times a factor $\sigma_s(1-\sigma_s)$ and subtract the conditionally sampled prognostic velocity equation for the environment multiplied times the same factor, we obtain a prognostic equation for the massflux

$$\frac{\partial M_c}{\partial t} + \sigma_s(1-\sigma_s)\left(\left\{\frac{\partial w}{\partial t}\right\}_{b,s} - \left\{\frac{\partial w}{\partial t}\right\}_{b,e}\right) = \sigma_s(1-\sigma_s)\frac{g}{\theta_0}(\theta_{v,s}-\theta_{v,e}) - \sigma_s(1-\sigma_s)\left(\left[\frac{\partial w^2}{\partial z}\right]_s - \left[\frac{\partial w^2}{\partial z}\right]_e\right) - \sigma_s(1-\sigma_s)\left(\left[\frac{\partial u_h w}{\partial x_h}\right]_s - \left[\frac{\partial u_h w}{\partial x_h}\right]_e\right) - \sigma_s(1-\sigma_s)\left(\left[\frac{\partial \pi}{\partial z}\right]_s - \left[\frac{\partial \pi}{\partial z}\right]_e\right) - \sigma_s(1-\sigma_s)\left(\left[\frac{\partial \tau_{3j}}{\partial z}\right]_s - \left[\frac{\partial \tau_{3j}}{\partial z}\right]_e\right), \tag{7}$$

where we used the notation $_b$ to indicate the net effect of the boundary terms which follow from the application of Leibniz' rule. Lastly, we show the vertical velocity variance equation in the massflux approach which we have derived from (6) and its counterpart for the environment,

$$\frac{\partial \sigma_s(1-\sigma_s)(w_s-w_e)^2}{\partial t} = 2\frac{g}{\theta_0}M_c(\theta_{v,s}-\theta_{v,e}) - \frac{\partial(1-2\sigma_s)M_c(w_s-w_e)^2}{\partial z} - 2M_c \times \left[\frac{1}{\sigma_s}\frac{\partial \sigma_s[w''w'']_s}{\partial z} - \frac{1}{1-\sigma_s}\frac{\partial(1-\sigma_s)[w''w'']_e}{\partial z}\right] - 2M_c(\left[\frac{\partial \pi}{\partial z}\right]_s - \left[\frac{\partial \pi}{\partial z}\right]_e) - (E_w+D_w)(w_s-w_e)^2. \tag{8}$$

Note that in the Reynolds-averaging approach the vertical velocity variance budget equation reads

$$\frac{\partial \overline{w'^2}}{\partial t} = 2\frac{g}{\theta_0}\overline{w'\theta_v'} - \frac{\partial \overline{w'^3}}{\partial z} - 2\overline{w'\frac{\partial \pi'}{\partial z}} - 2\overline{w'\frac{\partial \tau_{3j}'}{\partial x_j}}. \tag{9}$$

where the primes indicate perturbations of the resolved variables with respect to the horizontal slab-mean value.

# 3. Results

The large-eddy simulation has been performed with the IMAU/KNMI model (VanZanten, 2000). The simulation was done with a central-difference scheme (64 x 64 x 75 points). The horizontal and vertical grid spacings were 100 m and 40 m, respectively. The initialization was based on the BOMEX field experiment. We performed a simulation of 6 hours, and used the results of the last 4 hours for our analysis by averaging over all output fields during this time period. To illustrate the dynamics of shallow cumulus clouds the budget for the vertical velocity variance $\overline{w'^2}$, computed according to (9), is shown in Figure 1. The buoyancy flux is the primary production source of $\overline{w'^2}$. Except for a shallow layer around the cloud base the buoyancy flux is positive from the surface up to the inversion layer. At the top of the mixed layer, where the buoyancy flux is negative, saturated air parcels can reach their level of free convection by the upward vertical momentum they have gained. At these levels the turbulence transport term is the major term that is producing vertical velocity variance. In addition, the pressure term gives a positive, albeit small contribution, near the cloud base as well. The turbulent transport term becomes positive again above about 1100 m. In the bulk of the cloud layer the dissipation and the pressure gradient term act to destroy vertical velocity variance. The pressure term redistributes vertical momentum into the horizontal directions, whereas the dissipation of the resolved vertical motions produces subgrid scale turbulence motions.

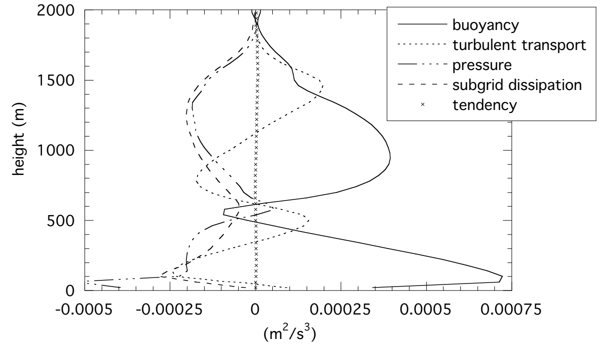


Figure 1: The vertical velocity variance budget. Linestyles are according to the legend.

The massflux budgets as computed according to Eq. (7) are shown in Fig. 2. They have similar features as the vertical velocity variance budget in Fig. 1. The buoyancy term in the cloud massflux budget has a negative value at the upper part of the cloud layer, whereas $\overline{w'\theta_v'}$ is positive in this part. Therefore, the positive horizontal-mean buoyancy flux in this layer must be due to turbulence in the dry environment and to subplume perturbations within the cloud. The turbulent transport term is an important production term for the convective

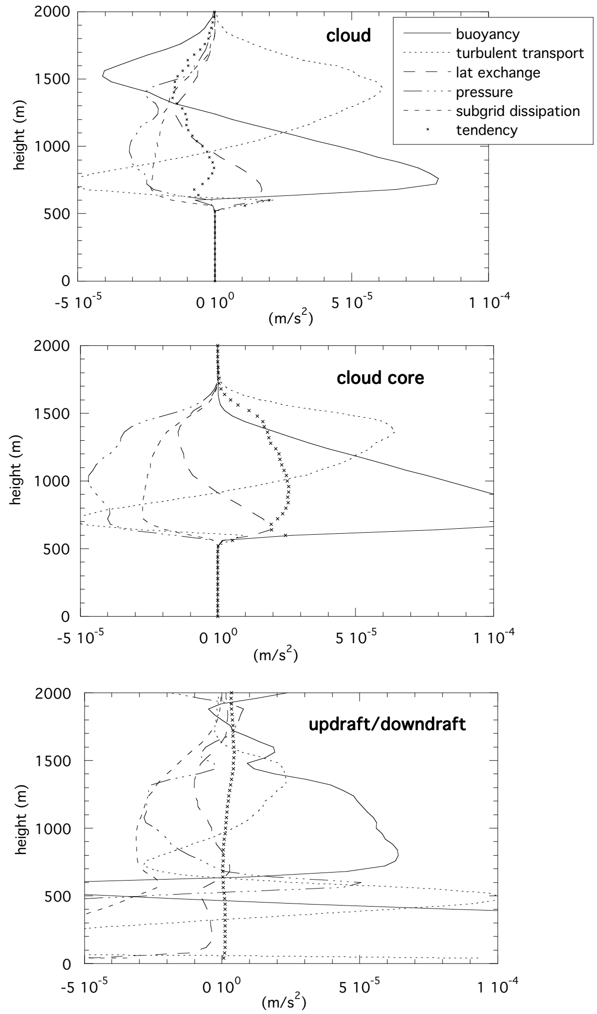


Figure 2: Massflux budgets for the cloud-environment, cloud core-environment and updraft-downdraft decomposition, in the upper, middle and lower panel, respectively. The linestyles are according to the legend shown in the upper plot.

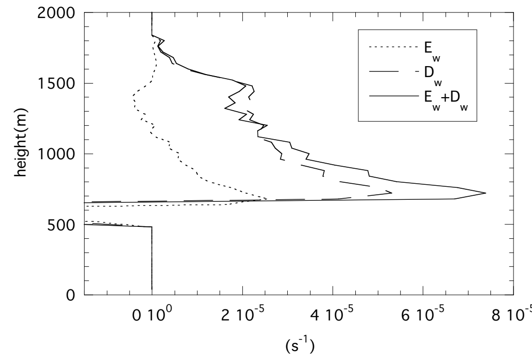


Figure 3: Lateral entrainment ($E_w$) and detrainment ($D_w$) rates, and their sum, for the vertical velocities sampled in the cloud core.

massflux in the upper part of the cloud layer. The conditionally sampled horizontal advection of vertical velocity, which formally represents the lateral exchange of massflux, acts to produce massflux at the lower part of the cloud layer and diminishes the massflux above. The role of the pressure and subgrid flux terms are similar to the ones in the vertical velocity variance budget in the sense that they both tend to destroy massflux. In that respect the subgrid flux term is analogous to the dissipation term in the vertical velocity variance budget, and this result might be somewhat controversial. Scaling considerations lead to the conclusion that dissipation by molecular viscosity can be neglected for motions on scales typical for cumulus convection, and that it is only of importance at the largest wavenumbers of the velocity spectra, the Kolmogorov scales. The 'dissipation' in the massflux budget, however, arises from the subgrid term in (3). In the vertical velocity variance equation it is exactly this term that causes the dissipation. However, the amount of the resolved kinetic energy that is lost is not dissipated into heat, but acts as a production term in the prognostic equation for the subgrid TKE equation, and therefore the subgrid term can be interpreted as a mechanism to convert resolved motions into subgrid perturbations. Hence, in the massflux budgets the subgrid parameterization term removes vertical momentum from the sampled eddies to feed the turbulent motions of small-scale eddies which have sizes smaller than the grid size of the LES.

The diagnosed entrainment and detrainment rates for the cloud core decomposition are shown in Figure 3. The entrainment and detrainment rates for the vertical velocity have slightly smaller values compared to the ones found for conserved variables by Siebesma and Cuijpers (1995). The entrainment rate $E_w$ becomes negative above 1000 m. For other sampling criteria we also find negative values for $E_w$. However, it should be reminded that the net effect of lateral mixing in the vertical velocity variance budget equation (8) is given by $-(E_w + D_w)(w_s - w_e)^2$. If $(E_w + D_w) > 0$, this term acts to dissipate the vertical velocity variance, which is the case for the bulk of the cloud layer except in a shallow layer near the cloud base. This seems to violate the concept that $D(E)$ represent the lateral mass exchange from the sampled cloud (environment) into the environment (sampled cloud), which rate is uniquely determined from the conditionally sampled continuity equation for mass. The $E_w$ and $D_w$ are the effective bulk

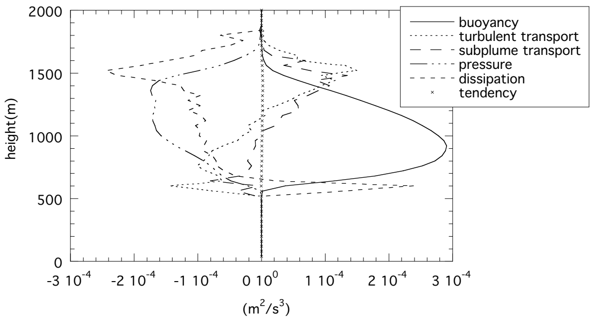


Figure 4: The cloud core vertical velocity variance budget in the massflux approach according to (8), the dissipation is represented by the term $-(E_w + D_w)(w_s - w_e)^2$. Linestyles are according to the legend.

entrainment and detrainment rates that would make the vertical velocity budgets balance after the assumption that the entrained or detrained air properties are the averaged conditionally sampled air properties. The obtained results for $E_w$ and $D_w$ should therefore be rather interpreted as tuned, reciprocal time scales for the conditionally sampled vertical velocity equation.

Figure 4 presents the vertical velocity variance budget in the massflux approach (8) for the cloud core. The dissipation is computed with the values for $E_w$ and $D_w$ shown in Fig. 3. Although they are not identical, the physical interpretation of the budget is similar to the vertical velocity variance budget. Since cloud core points are partly selected on the basis of a positive buoyancy excess, the buoyancy flux is producing vertical velocity variance throughout the cloud layer. As a consequence, the cloud core decomposition cannot represent overshooting clouds that rise due to their inertia despite a negative buoyancy excess. The turbulent transport and the subplume contribution are both important in the redistribution of vertical velocity variance from the lower part to the upper part of cumulus cloud layer.

Note that the scale of this budget is much smaller than for the Reynolds-averaged variance budget (Fig. 1). This indicates, for example, that the virtual potential temperature flux in the top-hat approach, $M_c(\theta_{v,s} - \theta_{v,e})$ is much smaller than $\overline{w'\theta_v'}$. For the cloud-environment the correspondence is even weaker. This can be explained by the generation of negatively buoyant cloud parcels due to mixing and a subsequent evaporative cooling at the cloud boundaries.

# 4. Conclusions

In a model in which the entrainment and detrainment rates are prescribed, and in which the continuity equation for mass (2) is used to determine the massflux, the vertical massflux gradient is fully constrained. The advantage of any prognostic equation for the vertical velocity in the massflux approach is that it links the thermodynamic state of the atmosphere to the dynamics by the buoyancy term. However, it requires the parameterization of pressure effects and lateral mixing which both act to diminish the upward vertical velocities in the cumuli. An example of a massflux model that includes a prognostic equation for the vertical velocity is presented by Lappen and Randall (2001), who developed this model to simulate clear and cloudy atmospheric boundary layers.

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