P3.9 MEASURING THE VISCOUS DISSIPATION OF TURBULENT KINETIC ENERGY WITH A HOTWIRE AND A SONIC ANEMOMETER

Jan Slager, Peter G. Duynkerke and Arjan van Dijk[1]
IMAU, Utrecht University, The Netherlands

INTRODUCTION

We consider the budget of turbulent kinetic energy (TKE) in the atmospheric boundary layer (ABL). TKE is produced at large scales ($\mathcal{L} \sim$ kilometers) and the generated eddies decay into smaller eddies until their TKE is dissipated at small scales ($\lambda \sim$ millimeters). It is difficult to measure the dissipation in the field, since the available anemometers either are too coarse to see the dissipating eddies, or their lab-calibration cannot be used in the field. In this study we solve the calibration problem by combining a sonic- and a hotwire anemometer. The combination is used at the Cabauw-site (4 m and 100 m) to estimate dissipation and other contributions to the TKE-balance for different stabilities. Different methods for estimation of the dissipation will be compared.

1. HYBRID ANEMOMETER

A commonly used anemometer to measure turbulence in the ABL is the sonic anemometer. It can measure the three-dimensional velocity vector with a frequency of about 60 Hz. However, the sonic anemometer has a disadvantage: it averages velocities over a path of about 10 cm. This path-averaging implies that the smallest structures in the flow are not measured. Its calibration is robust, but one cannot use it to measure all terms of the turbulent kinetic energy budget. An anemometer often used in the laboratory is the hotwire anemometer, which can detect velocity fluctuations at a scale of about 1 mm. This is small enough to measure the dissipative scales in the atmosphere. Its use in the field is restricted for two reasons: it is fragile and its calibration drifts due to e.g. crystallisation or dust. We solve the latter problem by constructing a hybrid anemometer, consisting of a sonic and a hotwire anemometer (see figure 1). The stable calibration of the sonic is used to *in situ* calibrate the hotwire. This hybrid anemometer enables us to measure at both the production- and the dissipative scales in the ABL.

To measure wind velocity a sonic anemometer measures the difference in travel-times of two acoustic pulses along the same path, but with opposite directions. With a tri-axial sonic one can estimate the full velocity-vector. The mean travel-time of the acoustic pulses is a measure for temperature. The transfer-function (spectral sensitivity) of our sonic is drawn in figure 2. From this figure it is clear that eddies with a length scale of about one-tenth of the sonic path-length are not detected.

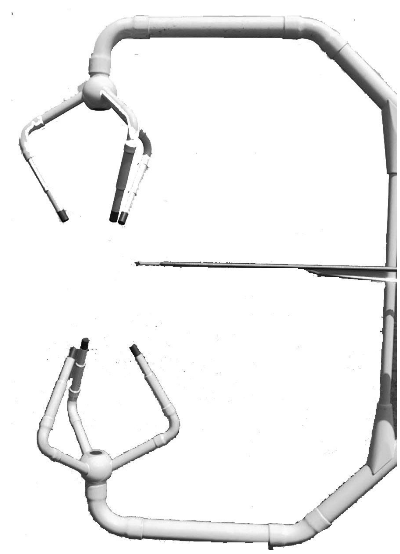

Figure 1: Picture of our hybrid anemometer. The hotwire anemometer is placed near the center of the sonic anemometer

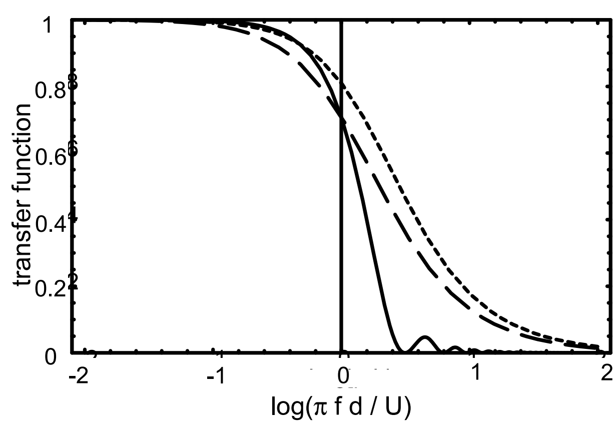

Figure 2: Transfer function for sonic path-averaging as a function of dimensionless frequency for three different cases: velocity in mean wind direction (line), vertical velocity (small dashes) and temperature (long dashes).

The sensing element of a hotwire anemometer is a thin tungsten wire of about 5μm thick and 1 mm long. It is heated to a temperature of about 200°C. When a flow passes the wire, then the wire looses heat. With an electrical circuit the temperature can be kept constant. The voltage required to maintain a constant temperature is a

[1]*corresponding author address:* Arjan van Dijk, Institute for Marine and Atmospheric Research, Utrecht University, Princetonplein 5, NL - 3584 CC Utrecht, The Netherlands `A.vanDijk@phys.uu.nl`

measure for the wind velocity. A hotwire anemometer is in good approximation only sensitive to wind normal to the wire (van Dijk 1999). We did some static hotwire calibrations (velocity and temperature sensitivity) in the Laboratory for Aerodynamics and Hydrodynamics of TU Delft. These calibrations appeared to be of little value in the field because of their drift.

To circumvent lab-calibration we developed a procedure to calibrate the hotwire anemometer with the sonic anemometer. The final measurement in the ABL itself is used to extract a calibration for the hotwire. First, we computed the horizontal velocity from the three dimensional sonic velocity. Then the hotwire voltage (taken at 2000 Hz) and the horizontal velocity from the sonic (taken at 20 Hz) were made of comparable frequency-range by filtering both records with a Gaussian filter with a cut-off frequency of 1 Hz. The resulting time-series are used to fit a calibration curve.

The two estimates for the mean velocities during various time-series, one via the sonic and the other via the now calibrated hotwire, matched (as expected: the fast fluctuations do not contribute to the mean wind velocity). In the frequency-region which is measured well by both the sonic and the hotwire anemometer (below 1 Hz), the two power spectra were the identical. We conclude that our procedure for combining the hotwire and the sonic anemometer is sufficient to perform measurements at both the production and the dissipative scales in the ABL.

2. DISSIPATION

We want to test the TKE-balance in the ABL. The new contribution that can be directly estimated with the just described hybrid anemometer is dissipation ε:

$$\varepsilon \equiv \nu \overline{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i}}. \tag{1}$$

The scale at which dissipation of TKE takes place is the Taylor micro-scale λ. It is defined as:

$$\lambda \equiv \left(\frac{\overline{u'^2}}{\overline{(\partial_x u')^2}}\right)^{1/2}. \tag{2}$$

This scale is a measure of the average dimension of the smallest eddies in a turbulent flow. Not much TKE resides in eddies smaller than the Taylor micro-scale. We estimate viscous dissipation in two different ways.

The first estimate uses (1), which follows directly from the Navier-Stokes equation. Assuming an infinite Reynolds-number, Taylor's hypothesis of frozen turbulence and isotropy, the dissipation can be written as:

$$\varepsilon = \frac{15\nu}{U^2}\overline{\left(\frac{\partial u}{\partial t}\right)^2}. \tag{3}$$

This estimate requires measuring at the small length scales. In practice equation 3 overestimated viscous dissipation: a too low AD-resolution gave unrealistically large values for the spectrum at high wavenumbers, producing large non-existing time derivatives of the velocity-field. This problem was solved by inserting one-point variance-spectrum E_u of velocity u in equation 3: and replacing the upper integration boundary by a finite wavenumber k_d, at which all viscous dissipation is implied:

$$\varepsilon = 30\nu\int_0^\infty k^2 E_u(k)dk \simeq 30\nu\int_0^{k_d} k^2 E_u(k)dk \tag{4}$$

The indirect method for estimating the viscous dissipation is Kolmogorov's four-fifths law, which is described in detail by Frisch (1995). Under assumption of infinite Reynolds number and isotropic turbulence, one can derive the following *exact* relation from the Navier-Stokes equation

$$\overline{(\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{l})-\boldsymbol{u}(\boldsymbol{x}))^3} \equiv \overline{\delta u(\boldsymbol{x},l)^3} = -\frac{4}{5}\varepsilon l \tag{5}$$

where l is a distance increment. The differences between the two methods are clear from figures 3 and 4. In the first a measured dissipation spectrum is shown, indicating that most of the viscous dissipation occurs at the small wavelengths, which consequently will have to be measured. In the second figure it can be seen that for lengthscales larger than the Taylor micro-scale ($\sim$ 7 cm) expression 5 converges to a constant value, demonstrating that this method does not require measuring of the smallest scales. Both methods will be used in practice in the following section.

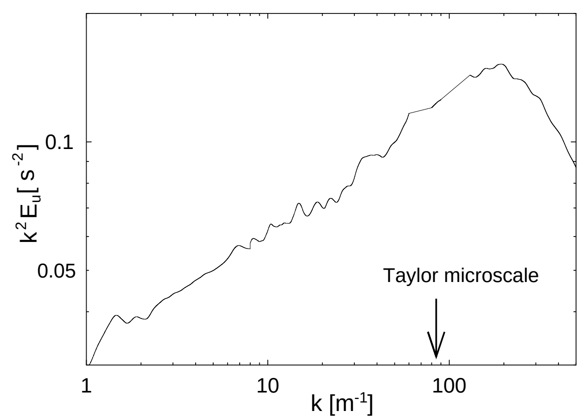

Figure 3: Spectrum of the dissipation. The peak at $k \approx 200m^{-1}$ corresponds with a length scale of 3 cm, which is half of Taylor micro-scale λ.

3. OBSERVATIONAL RESULTS

Our measurements were performed near and at the 213 m Cabauw tower of the KNMI in the period March-May 2001. The Cabauw tower is unique: The surroundings

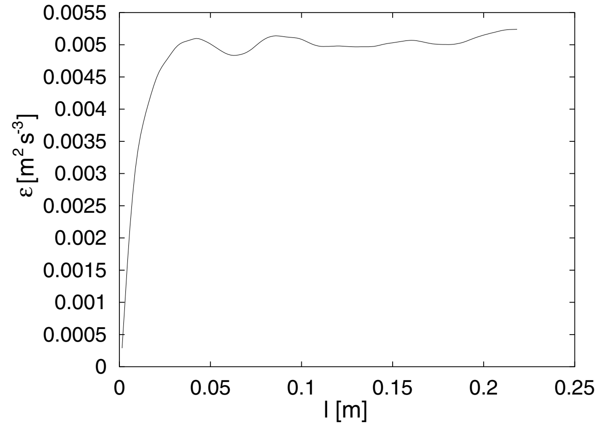

Figure 4: Plot of $\varepsilon = \overline{\delta u(\boldsymbol{x},l)^3}/(\frac{4}{5}l)$ versus separation l. For $0.03 < l < 0.2$ the flat behaviour indicates the validity of Kolmogorov's four-fifths law and represents ε.

of Cabauw are very flat and homogeneous (short grass). Therefore we don't have to worry about disturbances or internal boundary layers. Furthermore, the mast has booms of about 9.5 m length in three directions. These booms enable one to perform measurements undisturbed by the mast, independent of wind direction. We chose the boom in the SE-direction, because fully developed convective boundary layers usually occur during nice spring weather, when the wind is from the East or South-East. From 9 to 23 May we performed measurements at a height of 100 m. The other measurements were performed on a separate mast in the field at a height of about 4 m.

We measured under various stability regimes (some full daily cycles were recorded at 2 kHz with the hybrid anemometer). We computed three terms of the turbulent kinetic energy equation, which can be written as:

$$\begin{aligned} \frac{d\overline{e}}{dt} &= -\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z} + \frac{g}{\overline{\theta_v}}\overline{w'\theta_v'} \\ &\quad - \frac{\partial}{\partial z}\left(\overline{w'e} + \frac{\overline{w'p'}}{\rho}\right) + \nu\frac{\partial^2\overline{e}}{\partial x_j^2}. \end{aligned} \tag{6}$$

To estimate the transport term, we have to know vertical derivatives of third-order correlations. These were not available and this term is consequently not measured. We thus consider only shear production, buoyancy and viscous dissipation. The time derivative can usually be neglected (stationary situation). In dimensionless form, which can be obtained by division by $\kappa z/u_*^3$, this gives:

$$\Phi_M - \frac{z}{L} + \Phi_\varepsilon = 0 \tag{7}$$

where L is the Monin-Obukhov length. We will use symbol ζ for stability z/L. Both the direct and the indirect method were used to estimate dissipation. The results of the two methods corresponded within measuring accuracy.

For stable conditions the different terms of the TKE-equation and their sum (the inbalance) are plotted as a function of stability z/L in the plot on the right in figure 5. The lines are regression curves:

$$\begin{aligned} \Phi_M &= \beta_{M0} + \beta_{M1}\cdot\zeta \\ \Phi_\varepsilon &= \beta_{\varepsilon 0} + \beta_{\varepsilon 1}\cdot\zeta \end{aligned} \tag{8}$$

We fitted these curves through the points in the range $0 < \zeta < 1$, with the following results:

$$\begin{aligned} \beta_{M0} &= 1.0 \pm 0.06 \\ \beta_{M1} &= 4.0 \pm 0.17 \\ \beta_{\varepsilon 0} &= 0.95 \pm 0.08 \\ \beta_{\varepsilon 1} &= 4.39 \pm 0.23 \end{aligned} \tag{9}$$

The parametrisation of Φ_M agrees with earlier measurements at the same location. Duynkerke (1999) for example found a parametrisation of $\Phi_M = 0.9 + 4.0\zeta$, which differs from the commonly used parametrisation of $\Phi_M = 1 + 5.3\zeta$ (Högström 1996). More interesting is the parametrisation of Φ_ε, which is in good approximation balanced by shear production Φ_M, implying an inbalance term which is equal to minus the buoyancy. All measurements show a positive imbalance term, indicating that more energy is dissipated than produced. The most plausible physical process behind this is a term that was neglected in (7): transport of TKE into the stable ABL. This is in agreement with measurements done by Högström (1996). From the measurements it becomes also clear that the linear curves can only be used for low-stability ranges. For larger values of ζ, non-linear corrections have to be added to the lines. This is in agreement with measurements by Nieuwstadt (1984).

For convective conditions we present the results of the TKE-balance in figure 5 in the plot on the left. The dashed lines are commonly used parametrisations: $\Phi_M = (1-19\zeta)^{-1/4}$ wheras the other dashed line represents bouyancy, which is by definition equal to $-\zeta$ (equation 7). As with the stable cases, we see in this figure a close balance between production Φ_M and dissipation Φ_ε. The imbalance term is however negative, which means that more energy is produced than dissipated. This can be explained from the character of a convective ABL: the large eddies transport TKE away from the surface (measurement height) to dump it at greater heights. This transport of TKE leads to a negative term in the local TKE budget, as was found. The ratio between the required transport terms and viscous dissipation is in the order of 25 - 35 percent, which implies that viscous dissipation is still the dominant sink of energy. But this also indicates that the contribution of transport terms to the local kinetic energy-budget may not be neglected!

4. DISCUSSION

We successfully constructed a hybrid anemometer from a sonic- and a hotwire anemometer to measure even at the

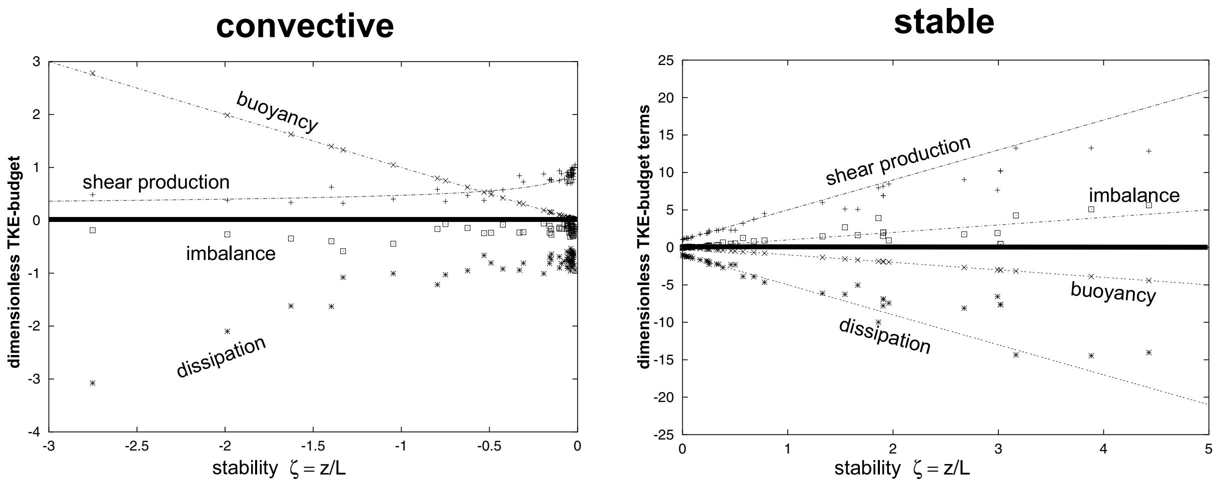

Figure 5: Various terms of the dimensionless TKE equation plotted as a function of stability z/L. Left: convective situations, right: stable cases. In the plots, + signs denote shear-production Φ_M, $\times$ signs denote buoyancy -ζ, the stars denote viscous dissipation, and the squares denote their sum: the imbalance. In the convective plot, the commonly used parametrisation for Φ_M is plotted, whereas the other dashed line represents dimensionless buoyancy, which is by definition equal to $-\zeta$. The horizontal line indicates the 0-level. The lines drawn in the stable plot are regressions to the regime $\zeta \leq 1$: from the top downward $1+4\zeta, \zeta, -\zeta$ and $1-4\zeta$ respectively.

smallest scales in the ABL. Our approach is supported by the result that two totally different methods to estimate viscous dissipation of TKE (direct and via Kolmogorov's four-fifths law) gave the same estimates within measuring accuracy. We used the method to analyse TKE-balances in the ABL, where the transport term was estimated as a residual term to close the balance. Our study showed that, for all stabilities, there is 'almost' a balance between shear, buoyancy and dissipation. In stable conditions the dissipation is slightly larger than the production, indicating a positive transport term. In convective conditions production is slightly larger than dissipation, indicating a negative transport term. Both indications are supported by the review written by Högström (1996).

Some recommendations for further research follow from this study. It is advisable to use hotwire probes with four wires to measure the complete three-dimensional velocity field (or more wires for the full gradient tensor, Tsinober et al. (1992)). A problem may then be the procedure to calibrate a probe with four wires (van Dijk 1999). We advocate measurement at greater heights, so that more information can be obtained about *very* (un)stable conditions. We will have a better understanding of the atmospheric boundary layer when we know how much turbulent kinetic energy is transported in convective boundary layers. The problem of fine AD-resolution needs to be solved: Measurements can be degraded when some samples are beyond the voltage range of the datalogger. Therefore one needs a datalogger system that has both a wide range of voltages and also a fine resolution.

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5. REFERENCES

Duynkerke, P., 1999: Turbulence, radiation and fog in dutch stable boundary layers. *Boundary-Layer Meteorology*, **90**, 447–477.

Frisch, U., 1995: *Turbulence; The legacy of A.N. Kolmogorov*. Cambridge University Press, Cambridge.

Högström, U., 1996: Review of some basic characteristics of the atmospheric surface layer. *Boundary-Layer Meteorology*, **78**, 215–246.

Nieuwstadt, F., 1984: The turbulent structure of the stable, nocturnal boundary layer. *J. Atm.Sci.*, **41**, 2202–2216.

Tsinober, A., E. Kit, and T. Dracos, 1992: Experimental investigation of the field of velocity gradients in turbulent flows. *J.Fluid Mech.*, **242**, 169–192.

van Dijk, A., 1999: *Aliasing in one-point turbulence measurements; theory, DNS and hotwire experiments*. Ph.D. thesis, Delft University of Technology, The Netherlands.