3.6 EVALUATION OF RELATIONSHIPS BETWEEN BOUNDARY_LAYER HEIGHT AND HEAT FLUX

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1. INTRODUCTION

The height of the convective boundary layer depends on a number of factors, particularly surface heat flux, temperature profile, mechanical turbulence production due to wind shear and surface roughness, vertical velocity, and horizontal advection. Under summertime continental conditions, a reasonable hypothesis is that z_i depends on the temperature profile (lapse rate) and on the integrated surface heat flux. In general it is not *a priori* reasonable to neglect advection and subsidence, but these processes are difficult to measure and we will show empirically that they can be neglected for some data sets. Here we evaluate three relationships against measurements from a measurement period in the SOS99 field project (30 June 1999 – 10 July 1999). One of the relationships we examine includes mechanical production. Our evaluation includes careful attention to uncertainties and their propagation through the relationships.

1.1 Boundary-Layer Height, Heat Flux, and Lapse Rate Relationships

The boundary-layer height z_i is in general a function of surface heat flux, the temperature profile, mechanical turbulence production due to shear and surface roughness, vertical velocity, and horizontal advection. Various relationships have been proposed to diagnose or predict z_i based on combinations of these parameters (Batchvarova and Gryning 1994, Yi et al. 2001, Siebert et al. 2000). Unfortunately many of the parameters are difficult to measure. Here we first use a simple relationship and show that it is adequate at least within the uncertainties of our measurements.

From a 0-order (slab) model of the boundary layer we derive a simple relationship between surface buoyancy flux (H), lapse rate above the inversion (γ), and z_i :

$$z_i = \sqrt{\frac{2(2c+1)}{\gamma}}\sqrt{\int_0^t H dt'} \qquad (1)$$

In this form, both the lapse rate and the flux are under square roots, making the diagnosis of z_i less uncertain (in a fractional sense) than the measurements of H and γ. For this study we assume that c, the entrainment parameter, is a constant. In fact this value is not well known and not constant (see Angevine [1999] and references therein). Values in the range 0.1-0.5 have appeared in the literature. Here we take its value to be 0.3, keeping in mind the uncertainties.

Yi et al. (2001) arrives at a similar relationship using a general linear relation between the height and the square root of the integral of the flux:

$$z_i = b + a\sqrt{\int_0^t H dt'} \qquad (2)$$

where a and b are constants to be empirically determined by fitting to measurements.

Batchvarova and Gryning (1994) propose a comprehensive relationship for BL height. When recast in a similar form to (1), it appears as:

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$$z_i = \sqrt{2 z_i \left[\frac{\gamma \cdot z_i^2}{(1+2c) z_i - 2BL\kappa} + \frac{c_T \cdot u_*^2}{\beta (1+c) z_i - BL\kappa} \right]^{-1}} \sqrt{\int_0^t H dt} \qquad (3)$$

where L is the Obukhov length, κ is the vonKarman constant, and B is a constant. The other parameters are described by Batchvarova and Gryning (1994). This relationship has two main parts. Both terms take into account the effect of the production of mechanical turbulence by wind-shear (the subtracted term in each denominator). The right hand term accounts for the effect of ’spin-up’, this effect describes the fact that air which is entrained from the free atmosphere into the boundary layer must adjust to the mean energetic level within the layer (White et al., 1999; Siebert et al. 2000). If the mechanical turbulence term and the ‘spin-up’ terms are neglected, (1) reappears.

1.2 SOS99 Project

The measurements we use to evaluate the above relationships were taken at the Southern Oxidants Study 1999 Nashville Summer Intensive (SOS99), which took place in the Nashville/Middle Tennessee region in June and July. The project as a whole was aimed at improving the understanding of the processes that control the formation and distribution of fine particles and ozone. The three study themes were local and regional contrasts, ozone and fine particle formation in plumes, and the diurnal cycle in chemistry and meteorology. Data from two sites of the ground-based network are used here. This study uses results from days 181-190 (30 June - 9 July 1999).

The Dickson site was located in an area of mixed deciduous forest and pastureland approximately 53 km (33 miles) WNW of the center of Nashville. This site has primarily rural characteristics and is rarely impacted by the Nashville urban plume.

2. MEASUREMENTS

2.1 Boundary-Layer Height

During SOS99, z_i at Dickson was measured using a 915 MHz wind profiling radar (profiler) deployed by NOAA’s Environmental Technology Laboratory (Carter et al., 1995; Ecklund et al., 1988). Profilers are designed to respond to fluctuations of the refractive index in clear air. The height of the boundary layer can be found because the intensity of the backscattered radar signal is enhanced by the humidity gradient at the top of the boundary layer. The range resolution of the profiler was 60 m, while the minimum range is approximately 150 m. The averaging time of the data is 30 minutes. The technique is well established and comparisons with other instruments show good agreement (Grimsdell and Angevine 1998; White et al. 1999; Cohn and Angevine, 2000). For purposes of the uncertainty calculations below, we take the uncertainty of the hourly z_i measurement to be 50 m.

2.2 Sensible Heat Flux

The sensible heat flux was measured atop a fire tower located at Montgomery Bell State Park, near Dickson, TN. The sonic anemometer (R2, Gill Instruments, Lymington, England), was located 35 m AGL, about 10 m above the mean canopy height. The horizontal and vertical components of the instantaneous wind vector along with the sonic temperature, were transmitted at a sampling rate of 10 Hz to a laptop computer, located at the base of the tower. Mean variance and covariance statistics were generated every 30 minutes. The uncertainty of each half-hour sensible heat flux derived from the sonic is taken as about 25 W m^{-2}, which is approximately 15% of the maximum flux values.

2.3 Lapse Rate

The lapse rate γ is the change in potential temperature per unit height. For the calculations, the rawinsonde sounding of the National Weather Service at 5:00 AM CST is used. The sounding site is the Weather Service Forecast Office at Old Hickory, on the Cumberland river northeast of Nashville. The sounding is used to find a profile of the potential temperature in the height range where the boundary layer develops. The assumption is made that the profile stays the same during the day except as it is changed by boundary layer mixing, thus neglecting advection and subsidence.

Our estimate for the uncertainty in the measurements of γ is 50%. This range is rather large. It is based on the fact that γ had to be estimated using soundings made at a different place than Dickson, and that the values are rather variable in the soundings themselves. The measurements in the lower layers of the atmosphere should be treated with special care, because local effects could play a significant role. This uncertainty estimate also takes into account the assumptions that advection and subsidence are negligible.

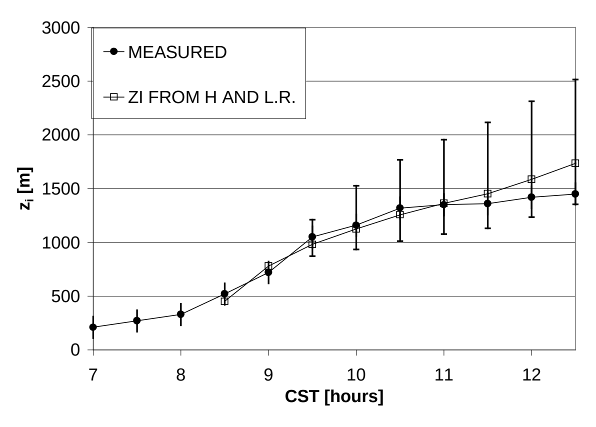

Figure 1: BL height at Dickson for 4 July 1999 calculated from (1) using the lapse rate (section 2.3) and sensible heat flux (section 2.2) - squares. BL height measured by the wind profiler (circles).

3. RESULTS

3.1 Boundary-Layer Heights

BL heights diagnosed from equation (1) are compared to measured z_i in Figure 1 for 4 July 1999. For this day, the equation seems to work quite well. The error bars in Figure 1 are calculated from the assumed uncertainties in the measurements. The difference between the measured and the calculated values are within 20% (after comparisons for the whole study period).

None of the methods account for subsidence or horizontal advection. Although subsidence is probably important, the error bars in Figure 1 show that the uncertainty of the calculation is large enough that we cannot conclude unambiguously that it is important.

3.2 Lapse Rate

The validity of using the Old Hickory soundings to estimate the lapse rate for the Dickson site can be tested, since z_i and H were both measured. Plugging the measured values into (1), we find for the sample day (4 July) that there is a significant difference between the estimated and measured values in the lower layers of the atmosphere. Below 700 m the measured lapse rates are much larger than the calculated ones. Above 700 m both curves come together. The most likely reason for this difference is the difference in location (see Measurements). Because local effects (the presence of a lake rather than forest, for example) have more influence on the lower atmosphere, the lapse rate can differ significantly. Above 700 m, the atmosphere over Dickson and the sounding site is the same. Due to this difference in location, the calculations in the lower layers of the atmosphere should be treated with care.

3.3 Yi Method

Yi et al. (2001) used a large dataset from March - November 1998 to derive the constants a and b of (2) by fitting profiler-measured z_i and flux measurements from a sonic anemometer in a forest clearing in Wisconsin. We can use the same procedure for our smaller dataset of only 10 days. Combining (1) with (2) and ignoring the constant b gives γ. Results are shown in table 1.

Table 1: Comparison of constants in equation (2) for the Yi-method and this study. R^2 is the square of the linear correlation coefficient between the result of (2) using the fitted a and b and the measured z_i .

	a	b	R^2	γ,K km^{-1}
Yi-method	0.78	86	0.98	4
This study	0.61	150	0.55	7

The values of a and b determined in this analysis should be approached with caution, given the limited size of our dataset and the low correlation we obtained.

The Yi method assumes that the lapse rate is constant during the day and constant for all days. In our measurements, the lapse rate is usually lower in the first half of the morning BL growth phase and higher in the second half. The difference between calculated and measured height (Figure 2) can be rather large (up to more than 100%) when the lapse rate is far from its average. The error-bars in the calculated values in Figure 2 are found from the assumed uncertainties in the measured heat flux and the standard deviation of a and b from standard formulas for the uncertainty of least-squares fits (Taylor 1997).

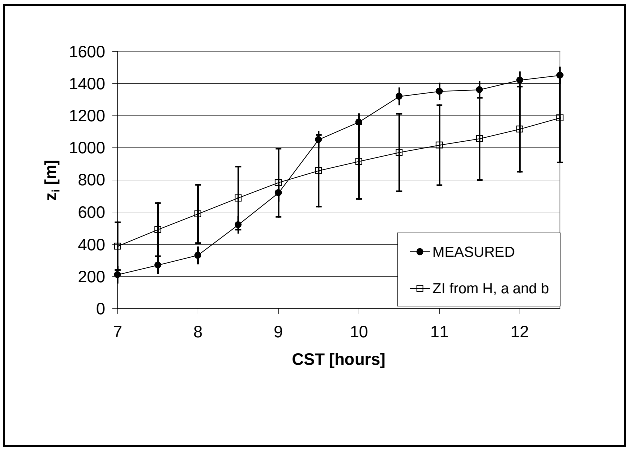

Figure 2: BL height at Dickson derived from (2) (squares) and measured (circles) for 4 July 1999. Error bars are derived from the extimated precision of the lapse rate and heat flux (see text).

3.4 Batchvarova and Gryning Method

The relationship (3) from Batchvarova and Gryning (1994) contains terms that are important when the atmosphere is nearly neutral (L large and negative) and with very low heights. In our data the momentum flux was so small and the convection so strong that the mechanical production and spin-up terms were negligible. In the study period, the mechanical production term was a maximum of less than 3% of the buoyancy term.

4. CONCLUSIONS

Given a known stationary stratification of the lower atmosphere, without advection and subsidence, the boundary-layer height can be calculated from heat flux and lapse rate measurements with uncertainty of about ± 20%. The largest source of uncertainty is the lapse rate, here taken from a balloon-borne sounding. The Yi-method should only be used when the additional assumption that the lapse rate does not vary in height or time is reasonable or required, as for example when the lapse rate is unknown. The extra complexity of the Batchvarova and Gryning (1994) method is only needed in near neutral lower atmospheres (small heat fluxes). In other words, for our data in continental summertime conditions, mechanical turbulence production and 'spin-up' are negligible. These terms may still be important in the early morning and under other types of conditions.

None of the relationships we tested explicitly accounts for vertical (subsidence) or horizontal advection. Subsidence is probably important, but it cannot be shown in this data set because the difference subsidence would make falls within the error-bars of the calculation.

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