4.6 Countergradient fluxes in the clear convective boundary layer. The role of the entrainment flux *

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1. Introduction

In a first-order K-diffusion closure model it is assumed that the turbulent vertical flux $\langle w'\chi'\rangle$ of a generic quantity χ can be parameterized as an eddy-mixing coefficient K_χ multiplied by the local mean vertical gradient

$$\langle w'\chi'\rangle = -K_\chi \frac{\partial\langle\chi\rangle}{\partial z}, \qquad (1)$$

where the operator $\langle\rangle$ represents the horizontal slab-mean value. However, since a few decades it is known from aircraft observations that in the interior of the clear convective boundary layer (CBL) the vertical flux of the (virtual) potential temperature flows counter the mean vertical gradient (Lenschow, 1970; Warner, 1971). In order to allow for a countergradient heat flux, $\langle w'\theta'\rangle$, a correction term (γ_θ) to (1) has been proposed (Deardorff, 1972; Holtslag and Moeng, 1991),

$$\langle w'\theta'\rangle = -K_\theta \left(\frac{\partial\langle\theta\rangle}{\partial z} - \gamma_\theta\right). \qquad (2)$$

This type of parameterization has been discussed in detail by Stevens (2000). Other general expressions for the scalar flux have been suggested by, for example, Wyngaard and Weil (1991) and Cuijpers and Holtslag (1998). These authors proposed a correction term $\langle w'\chi'\rangle_{NL}$ that depends on the skewness of the vertical velocity field,

$$\langle w'\chi'\rangle = -K_\chi \frac{\partial\langle\chi\rangle}{\partial z} + \langle w'\chi'\rangle_{NL}. \qquad (3)$$

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In this investigation we systematically explore for which ratio between the entrainment flux and the surface flux the vertical scalar flux is not down the mean vertical gradient. For this study results from a large-eddy simulation (LES) of a convective boundary layer have been used. The linearity of the transport equation for the passive scalars allows the use of the principle of superposition of variables, and the inclusion of a top-down and bottom-up scalar to the simulation therefore makes that a scalar with any flux ratio can be reconstructed.

2. Set-up of the experiment

a. The principle of the superposition of variables

To diagnose the fields of an arbitrary passive scalar χ we will make use of the linearity of its transport equation and apply the principle of superposition of variables (Wyngaard and Brost, 1984; Moeng and Wyngaard, 1984) . Let χ be given by a linear superposition of two variables ϕ and χ,

$$\chi = a\psi + b\phi + c. \qquad (4)$$

with a, b and c arbitrary constants. The variable ψ has a flux $\langle w'\psi'\rangle_0$ at the surface, and is therefore referred to as a 'bottom-up' scalar. In contrast, the 'top-down' scalar ϕ has no surface flux. However, a turbulent flux is generated by entrainment at the top of the boundary layer ($\langle w'\phi'\rangle_T$) if it is initialized with a jump $\Delta\langle\psi\rangle$ across the inversion. After applying Reynolds decomposition on (4) and multiplication by w' we can express the vertical flux $\langle w'\chi'\rangle$ as a function of the bottom-up and top-down fluxes,

$$\langle w'\chi'\rangle = a\langle w'\psi'\rangle + b\langle w'\phi'\rangle. \qquad (5)$$

By choosing appropriate values for a and b we can obtain any arbitrary flux ratio r_χ, which is defined as the ratio of the flux of χ at the top of the boundary layer and at the surface, indicated by the subscripts T and 0, respectively,

$$r_\chi = \frac{\langle w'\chi'\rangle_T}{\langle w'\chi'\rangle_0}. \tag{6}$$

b. Large-Eddy Simulation of the CBL

The large-eddy simulation has been performed with the IMAU/KNMI model, e.g. VanZanten (2000). The filtered prognostic equation for the resolved part of an arbitrary conserved variable χ reads

$$\frac{\partial \chi}{\partial t} = -\frac{\partial u_j \chi}{\partial x_j} - \frac{\partial \overline{u''_j \chi''}}{\partial x_j}, \tag{7}$$

where $\overline{u''_j \chi''}$ is the subgrid flux. In the LES model the subgrid flux is expressed as the product of an eddy diffusivity and the local gradient of the resolved variable. The eddy diffusivities of all the passive scalars are identical. The simulation has been done with 256 x 256 x 80 grid points. The horizontal and vertical grid spacings were 100 m and 20 m, respectively. The results presented in this paper represent a time average over the fourth hour of the simulation.

The initial height of the boundary layer was set to 810 m, and the jumps across the inversion were $\Delta\langle\theta\rangle$ = 5.0 K, $\Delta\langle q_t\rangle$ = -2.5 $g \cdot kg^{-1}$, $\Delta\langle\psi\rangle$ = 0.0, and $\Delta\langle\phi\rangle$ = 0.1. Above the inversion these quantities were set at a constant value. The surface fluxes were constant during the entire simulation, $\langle w'\theta'\rangle_0$ = 0.05 mKs^{-1}, $\langle w'q'_t\rangle_0$ = 1.5 $\cdot 10^{-2}$ $(g \cdot kg^{-1})ms^{-1}$, $\langle w'\psi'\rangle_0$ = 0.001 ms^{-1}, $\langle w'\phi'\rangle_0$ = 0.0 ms^{-1} and u_* = 0.01 ms^{-1}. There were no large-scale forcings like subsidence or radiation prescribed .

3. Results

The vertical fluxes for the bottom-up and top-down scalar are shown in Fig. 1. In the same figure we show the eddy diffusivities K_ψ and K_ϕ, which we have computed from,

$$\langle w'\psi'\rangle = -K_\psi \frac{\partial\langle\psi\rangle}{\partial z}, \tag{8}$$

and similarly for K_ϕ. The fact that K_ψ and K_ϕ have a positive value at all levels in the boundary layer indicate that the bottom-up and top-down scalar fluxes are down the mean gradient. Note that due to the development of a small inversion with time the bottom-up flux has a small flux due to entrainment at the top of the boundary layer. If the superposition principle is applied to construct the field of scalar which has no entrainment flux at the top of the boundary layer, negative eddy-diffusivities K_ψ are found at the top of the boundary layer (Moeng and Wyngaard, 1984).

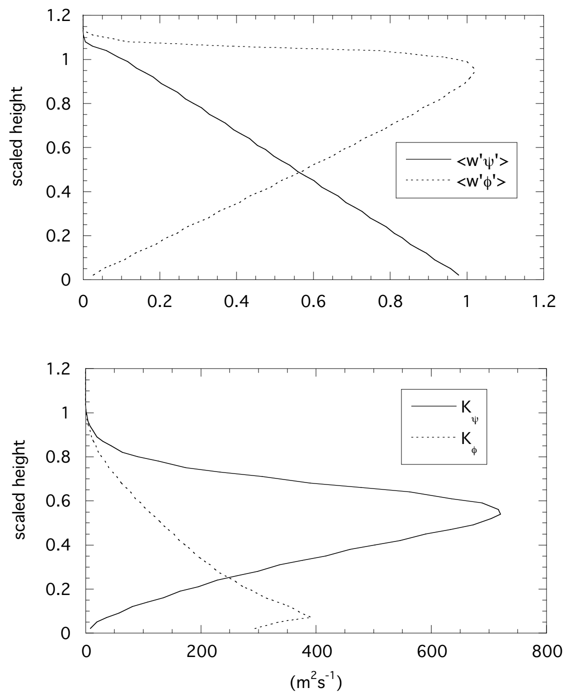

Figure 1: The a) vertical flux and b) the eddy diffusivity for the bottom-up and the top-down scalars. The fluxes have been scaled by their maximum value.

The levels in the boundary layer where the mean vertical flux is counter the mean gradient are found from the following criterion,

$$\langle w'\chi'\rangle \frac{\partial\langle\chi\rangle}{\partial z} > 0. \tag{9}$$

Fig. (2) shows that countergradient fluxes are manifest for negative flux ratios ratios, in particular for the range $-2 \leq r_\chi \leq 0$. Obviously, there is no symmetry at a flux ratio $r_\chi = 0$. For quantities that have a very small flux ratio close to zero, the flux is countergradient in the upper part of the boundary layer. Hence, for quantities that have a positive vertical flux throughout the boundary layer ($r_\chi > 0.04$) a correction term like

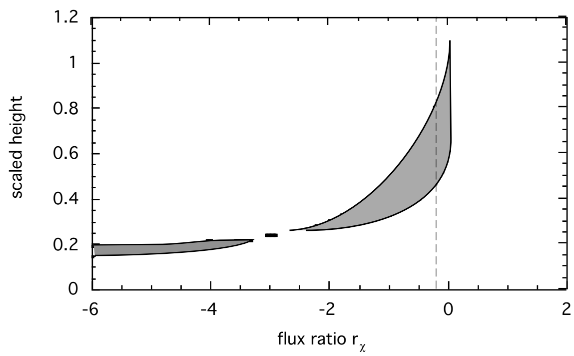

Figure 2: The shaded area indicates the levels at which the vertical turbulent flux is counter the mean gradient. The maximum flux ratio for which countergradient fluxes are observed is $r_\chi = 0.04$. Not shown is a very shallow layer just above the surface layer with countergradient fluxes for flux ratios $r_\chi \leq -6$. The height is scaled with the boundary layer depth (850 m). The vertical dashed line indicates the buoyancy flux ratio.

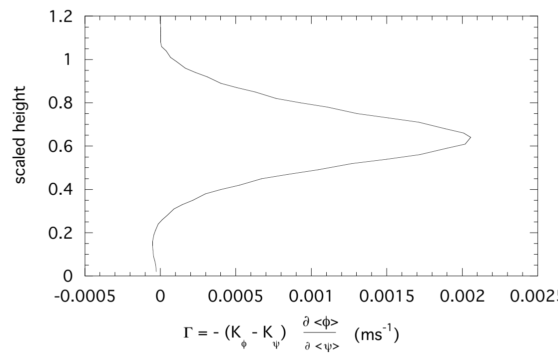

Figure 3: The factor Γ in Eq. (11).

γ_θ in (2) that accounts for countergradient behavior is unnessecary.

4. Flux parameterization

In this section we will present a parameterization for the vertical flux $\langle w'\chi'\rangle$ on the basis of the superposition principle. We aim to derive a 'classical' downgradient formulation and a 'correction' term which depends only on the flux ratio of the quantity under consideration. To this end, let us combine (5) and (8) to express the vertical flux $\langle w'\chi'\rangle$ as a function of the parameterized bottom-up and top-down fluxes,

$$\langle w'\chi'\rangle = -aK_\psi \frac{\partial\langle\psi\rangle}{\partial z} - bK_\phi \frac{\partial\langle\phi\rangle}{\partial z}. \qquad (10)$$

The vertical gradient of $\langle\chi\rangle$ can be incorporated by substituting the vertical derivative of the horizontal slab mean of (4) into (10),

$$\langle w'\chi'\rangle = -K_\psi \frac{\partial\langle\chi\rangle}{\partial z} - b(K_\phi - K_\psi)\frac{\partial\langle\phi\rangle}{\partial z}. \qquad (11)$$

The factor b is constrained by (5),

$$b = \frac{\langle w'\chi'\rangle_T}{\langle w'\phi'\rangle_T} - a\frac{\langle w'\psi'\rangle_T}{\langle w'\phi'\rangle_T}. \qquad (12)$$

where the factor a is given by

$$a = \frac{\langle w'\chi'\rangle_0}{\langle w'\psi'\rangle_0}, \qquad (13)$$

For a bottom-up flux which has no flux at the top of the BL the second term on the rhs of (12) cancels. However, since the bottom-up scalar in our case has a small entrainment flux due to the very weak inversion jump that has developed with time, its effect on the factor b is taken into account by (12).

Note that with a top-down scalar and any other quantity that has an arbitrary flux ratio the superposition principle enables to reconstruct all fields for χ. For example, it is possible to replace the bottom-up scalar by the humidity which will yield a similar expression as (11). If the humidity jump across the inversion is significant the second term in (12) cannot be neglected in that case.

The factor $\Gamma = -(K_\phi - K_\psi)\frac{\partial\langle\phi\rangle}{\partial z}$ from Eq. (11) is shown in Fig. (3). Γ has a maximum just above the middle of the boundary layer, and it has a small negative value near the surface. Note that for the CBL Γ has an uniform shape since the eddy diffusivities, and the vertical gradient of the bottom-down scalar have an universal vertical profile. In fact, the formula (11) has a similar structure as (3) and could therefore serve as a parametrization for vertical fluxes in the CBL.

However, we would like to express (11) in a form like (2),

$$\langle w'\chi'\rangle = -K_\psi \left(\frac{\partial\langle\chi\rangle}{\partial z} - b\gamma(z)\right), \qquad (14)$$

with

$$\gamma = -\left(\frac{K_\phi}{K_\psi} - 1\right)\frac{\partial\langle\phi\rangle}{\partial z}. \qquad (15)$$

Figure (4) shows that the factor γ is large and maximum in the inversion layer layer. The shape of the

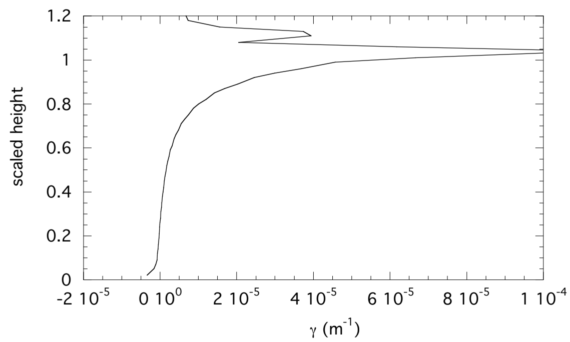

Figure 4: The correction factor γ according to (14).

vertical profile for Γ can be understood from the fact that the eddy diffusivity K_ψ is based on the bottom-up scalar, which has only a very weak entrainment flux. Thus, for any other quantity that has a significant jump across the inversion the correction term γ must account for the representation of the entrainment flux. Clearly, the complicated vertical structure for γ illustrates that it is hard to find an average value which can serve as a representative value for the entire boundary layer.

In summary, from the fields of an arbitrary scalar and a top-down passive scalar two eddy diffusivities can be computed. Both eddy diffusivities are included in the correction factor γ (or Γ). Then only one free parameter remains, namely b which depends on the surface and top (entrainment) fluxes of χ.

5. Conclusions

First, we have shown that for the CBL the existence of countergradient fluxes are predominantly limited to a range of negative flux ratios. For positive flux ratios ($r_\chi > 0.04$) we did not find evidence of countergradient fluxes.

Second, on the basis of the principle of superposition of variables we have derived a parameterization for the vertical flux in the CBL. It is an exact parameterization since no assumptions have been made in its derivation. It can be applied to conserved quantities with arbitrary flux ratios. The correction factor γ allows for countergradient fluxes. The form of the suggested parameterization is based on expressions suggested in literature and can therefore be used to validate current K-diffusion parameterizations.

Third, by the simplicity and elegance of the superposition of variables principle we like to advocate this approach for the study of other types of convective boundary layers, like stratocumulus.

Acknowledgement The investigations were supported by the Netherlands Organization for Scientific Research (NWO). This work was sponsored by the National Computing Facilities Foundation (NCF) for the use of supercomputer facilities.

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