P3.10 DYNAMIC EVALUATION OF DRAG COEFFICIENTS FOR FLOW OVER SCALE-SIMILAR COMPLEX BOUNDARIES

Stuart Chester*, Charles Meneveau, and Marc Parlange
The Johns Hopkins University, Baltimore, Maryland

1 INTRODUCTION

In traditional simulations of flow over complex terrain with unresolved surface features, the effect of the small-scale geometry is often parameterized using a drag coefficient determined in an *ad hoc* fashion. In these simulations, there is usually a large separation of scales between the resolved roughness and the small-scale surface roughness. One example of this is turbulent flow over a "flat" surface (i.e. resolved geometry is a flat wall) with surface roughness at a much smaller scale, which can be parameterized with a roughness length or drag coefficient. Due to the separation of scales, the drag from the small scales is less likely to depend on the details of the flow in question and hence can be successfully parameterized in this manner. However in situations where the surface possesses a wide range of scales, with significant unresolved features, the drag from the unresolved scales may depend on the particular flow and shape of the geometry under consideration. As a result, simulations using *ad hoc* parameterizations of the large unresolved surface features (i.e. features just smaller than the minimum resolved scale) are less likely to give accurate results. This raises the question of whether information from the resolved surface scales may be used to obtain accurate values of the drag coefficient, especially in geometries with some degree of scale-similarity. This would ensure that the drag coefficient used in a simulation is matched to the geometry and flow under consideration. The problem is similar to that in large-eddy simulation (LES) where traditionally the values for model coefficients for a given flow are prescribed. The dynamic framework of Germano *et al.* (1991) allows LES model coefficients to be determined using information contained in the smallest resolved flow scales, and has been used successfully in applications of LES. In this presentation, an analogous formulation for dynamic determination of drag coefficients is outlined, and results from preliminary 2-D *a priori* tests are given.

2 FORMULATION

For the sake of brevity, only a brief description of the dynamic formulation is given here. Similar to the dynamic model of Germano *et al.* (1991), the present formulation relies on test filtering (i.e. explicitly filtering the resolved scales) to extract information from the smallest resolved surface scales. Hence, an important part of a dynamic formulation for drag coefficients involves defining filter operations that act on surface geometry. Another key component of the formulation is a Germano identity that relates the force that the fluid exerts on the complex surface when it is considered at different scales. The force $\boldsymbol{F}_\Delta$ that the fluid exerts on the surface at scale Δ is modeled as

$$\boldsymbol{F}_\Delta = \Delta^2 \left(\widetilde{\boldsymbol{\sigma}} \cdot \widetilde{\boldsymbol{n}} + \frac{1}{2} \rho \mathrm{C_D}(\mathrm{Re}) |\boldsymbol{U}_{t,\Delta}| \boldsymbol{U}_{t,\Delta} \right), \qquad (1)$$

where $\widetilde{\boldsymbol{\sigma}}$ is the resolved stress tensor, $\widetilde{\boldsymbol{n}}$ is the resolved surface normal, ρ is the fluid density, $\mathrm{C_D}(\mathrm{Re})$ is a drag coefficient (depending on Reynolds number), and $\boldsymbol{U}_{t,\Delta}$ is a tangential velocity vector at scale Δ. This force model contains a resolved-scale contribution and the effect of the unresolved geometry scales is taken into account through the drag coefficient. For simplicity, this particular model only includes a drag force contribution from the unresolved geometry and neglects any lift force contribution from the unresolved scales. The dynamic formulation allows either a low Reynolds number scaling ($\mathrm{C_D} \propto \mathrm{Re}^{-1}$) or a high Reynolds number (inertial) scaling ($\mathrm{C_D}$ is Re independent) for the drag coefficient, but the type of scaling must be specified. A requirement for the dynamic model is that the surface display some degree of self-similarity at the smallest resolved scales.

3 SIMULATIONS AND REPRESENTATION OF COMPLEX GEOMETRY

Due to the above requirement that the surface display self-similarity, an ideal surface to test the above formulation is a fractal surface. In this study, two-dimensional simulations of flow over a Koch-curve geometry are performed on a regular Cartesian grid. The flow is periodic in the horizontal direction, and stress-free boundary conditions are applied at the top of the domain. In the *a priori* tests, no-slip boundary conditions are applied on the Koch-curve. Current simulations are limited to $\mathrm{Re} \approx 500$, based on the domain height and maximum velocity, and are performed without a turbulence model. Higher Reynolds number simulations using a simple turbulence model are planned. The domain with streamlines from one of the simulations used in the *a priori* tests is shown in figure 1. The geometry is represented on the Cartesian grid through the use of body forces, in a similar approach to that used by Mohd-Yusof (1997). To accurately represent the complex, no-slip surface, different interpolation schemes are tried ranging from

*e-mail: chester@titan.me.jhu.edu

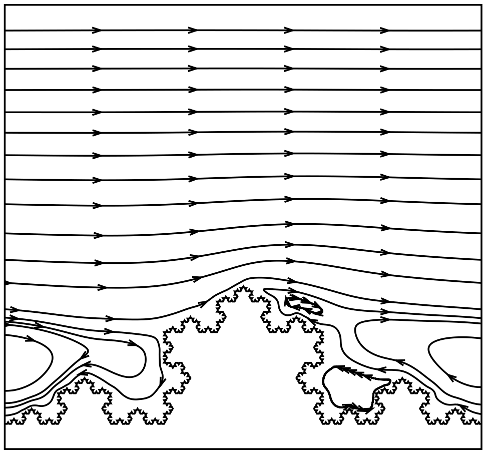

Figure 1: Geometry and streamlines from a simulation used in an *a priori* test ($\mathrm{Re} \approx 500$).

no interpolation to a combination of linear and bilinear interpolation (Kim *et al.*, 2001).

4 PRELIMINARY RESULTS: *A PRIORI* TESTS

In *a priori* tests, simulations are performed over detailed Koch-curve surfaces and can be thought of as direct numerical simulations, in which all relevant surface scales are resolved. In these cases, the Koch-curve shape consists of 1024 surfaces each of length ℓ_{Koch} = 1/243 (normalized by the height of the domain). To perform the *a priori* tests, the relevant quantities from the unfiltered surface are filtered and then used to calculate the dynamic drag coefficient. For comparison, an "exact" value of the drag coefficient can be found by minimizing the squared error in the model expression for the surface force (equation 1).

In the low Reynolds number tests ($\mathrm{Re} \approx 500$) with an inertial scaling of the drag coefficient, it is found that there is high variability in the accuracy of the dynamic formulation, depending on the resolved scale Δ. When $\Delta = 9\ell_{\mathsf{Koch}}$, the dynamic value of the drag coefficient differs from the exact value by about 10 %. However, when $\Delta = 3\ell_{\mathsf{Koch}}$ there is a large discrepancy of 90 %, probably due to the fact that at this scale, the local Reynolds number is $O(1)$ so that the assumption that the drag coefficient follows an inertial scaling is not justified. When a low Reynolds number scaling is used ($\mathrm{C_D} \propto \mathrm{Re}^{-1}$), the error in the drag coefficient drops to 14 % at scale $\Delta = 3\ell_{\mathsf{Koch}}$. We note that the higher Reynolds number inertial scaling is of primary interest and here, the low Reynolds number scaling is used only to check the dynamic drag coefficient formulation. In the same low Reynolds number tests with $\Delta = 27\ell_{\mathsf{Koch}}$ (where the local Reynolds number is $O(100)$), large errors are obtained (about 140 %) and the dynamic coefficient becomes negative when an inertial scaling for the drag coefficient is used. The cause for this problem lies in the fact that the force model used assumes that the velocity vector near the surface points down the pressure gradient. However, in this complex flow it is possible that this velocity actually points up the local pressure gradient, causing a negative contribution to dynamic drag coefficient. This can occur in high shear situations where viscous stress can cause the flow to move up the pressure gradient, and also in the neighborhood of highly curved streamlines. These results show that higher Reynolds number *a priori* tests are desirable, to ensure an inertial scaling of the drag coefficient and to ensure that viscous stresses in the flow are less important (reducing the likelihood of negative contributions to the coefficient for the present model). In addition to these preliminary *priori* tests, *a posteriori* tests performed with an imposed dynamic drag coefficient are needed to quantify the accuracy of this dynamic model in applications.

5 CONCLUSION

A dynamic drag law for scale-similar surfaces that takes unresolved geometry into account in the calculation of the drag coefficient has been formulated. The formulation gives reasonable results (10 % error) at certain scales in low Reynolds number *a priori* tests, although at different scales larger errors are obtained. Higher Reynolds number tests required to observe the dynamic drag coefficient behavior in an inertial scaling regime need to be undertaken.

References

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