10.2 ON THE PREDICTABILITY OF THE STABLE BOUNDARY LAYER AND THE ROLE OF INITIAL CONDITIONS

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1. INTRODUCTION

The behavior of the stable boundary layer is critical to weather forecasts and air-quality modeling. In recent years there has been increased interest and concern about the proper formulation of stable boundary layer parameterizations in models of all scales (GWEX GABLS Working Group). Evidently, application of traditional Monin-Obukov local closure with stability functions having relatively sharp cut-offs at theoretical critical Richardson numbers (Blackadar, 1979) can produce decoupled solutions that are too cold, leading to deterioration of model performance (Derbyshire,1999; Viterbo et al., 1999). Intermitttancy of turbulence in the real atmosphere not incorporated in the model parameterizations may be a key factor in the performance problem (Van de Weil, 2002; Mahrt, 1999). Another potential factor may be that non-local effects are not handled in the application of the parameterizations (McNider and Pielke, 1981). Unlike the daytime boundary layer, which is dominated by the incoming solar heat flux, the stable boundary layer is affected by many physical parameters and processes, any of which can alter the outcome (Mahrt 1999). These parameters include surface roughness, surface heat capacity, geostrophic wind, low-level jets, latent heating in the soil, etc. The dynamic competition enters as nonlinearities in the diffusion terms. Thus, the system may support exotic behavior such as stable and unstable attracting solutions, limit cycles, hysteresis (loss of predictability).

It is the purpose of this paper to use the techniques of nonlinear analysis (Seydel, 1988; Doedel et al., 1986, 1991) to explore the influence of the parameters and the behavior of the equations used in forecast and air-quality models. Nonlinear analysis allows understanding of the equations as a function of imposed parameters such as surface roughness, pressure gradient, downward longwave radiation, etc. A previous paper (McNider et al., 1995) truncated the boundary layer equations to a two-layer system that allowed construction of the bifurcation diagrams, definition of bifurcation points (turning points), and computation of the eigenvalues which allows definition of the local stability of the steady solutions. That study (referred in this paper as MCN95) showed that the truncated system did indeed support hysteresis so that for a range of geostrophic winds and surface-roughness values the system had multi-valued solutions that would reduce the predictability of the system. This also makes the system sensitive to initial conditions.

While MCN95 showed that this exotic behavior existed in the truncated system, there was some question whether this might be an artifact of the two-layer system and might not be applicable to the full PDE system. The present study provides an analysis of the full ***n***-layer system to determine whether the essential characteristics of the two-layer equations are retained. In addition, we examine the role of initial conditions in the final solutions.

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2. STATEMENT OF THE PROBLEM

We consider a simple, one-dimensional (single column model) (see Stull,1988).

$$\frac{\partial u}{\partial t} = f_{co}(v - v_G) + \frac{\partial}{\partial z}\left(K_m(z, u_z, v_z, \theta_z)\frac{\partial u}{\partial z}\right),$$
$$\frac{\partial v}{\partial t} = f_{co}(u_G - u) + \frac{\partial}{\partial z}\left(K_m(z, u_z, v_z, \theta_z)\frac{\partial v}{\partial z}\right),$$

$$\frac{\partial\theta}{\partial t} = R_c(\theta) + \frac{\partial}{\partial z}\left(K_h(z,u_z,v_z,\theta_z)\frac{\partial\theta}{\partial z}\right),$$

with the boundary conditions (for t ≥ 0)

$$u(z_0,t) = 0, \qquad u(Z,t) = u_G,$$
$$v(z_0,t) = 0, \qquad v(Z,t) = v_G,$$
$$\theta(z_0,t) = \theta_g(t), \qquad \theta(Z,t) = \theta_Z,$$

coupled with the IVP for the ground potential temperature $\theta_g = \theta_g(t)$ (for z=0, t>0)

$$\frac{d\theta_g(t)}{dt} = \frac{1}{C_g}\left(I_1(\theta)|_{z=z_0} - \sigma\theta_g^4(t) - H_0(z,u_z,v_z,\theta_z)|_{z=z_0}\right)$$
$$-\kappa_m(\theta_g(t) - \theta_m),$$

and the initial conditions

$$u(z,0) = u_G, \quad v(z,0) = v_G, \quad \theta(z,0) = \theta_A,$$
$$\theta_g(0) = \theta_A.$$

Here,

$K_m = K_m(z,u_z,v_z,\theta_z)$ and
$K_h = K_h(z,u_z,v_z,\theta_z)$

are the exchange coefficients for momentum and heat, which are functions of the space variable ***z*** and the partial derivatives:

$$u_z = \partial u(z,t)/\partial z, \qquad v_z = \partial v(z,t)/\partial z, \qquad \theta_z = \partial\theta(z,t)/\partial z.$$

The nonlinearity in the equations enters through these terms, originally defined as

$$K_m = f_m(Ri)\kappa^2 z^2 s, \qquad K_h = f_h(Ri)\kappa^2 z^2 s,$$

with the stability functions f_m and f_h (namely, the Richardson number formulations) given by England & McNider, 1995.

$$f_m(Ri) = f_h(Ri) = \begin{cases} (1 - Ri/Ri_c)^2, & if\ 0 \le Ri \le Ri_c, \\ 0, & if\ Ri \ge Ri_c, \end{cases}$$

and the vertical wind shear s defined by

$$s \equiv s(u_z,v_z) = \sqrt{u_z^2 + v_z^2},$$

R_c(_) is the clear-air radiative cooling rate for the air temperature and is taken to be zero in this dissertation. I_1 is the longwave back radiation from the atmosphere, given by Staley and Jurica [22, pp.349-356]

$$I_1 \equiv I_1(\theta)|_{z=z_0} = 0.67\sigma(1670Q_a)^{0.08}\theta^4|_{z=z_0},$$

and H_0 is the heat flux carried away from the surface by turbulence, given by Blackdar (1979)

$$H_0 \equiv H_0(z,u_z,v_z,\theta_z)|_{z=z_0} = -\rho C_p(K_h(z,u_z,v_z,\theta_z)_z|_{z=z_0}.$$

3.0 Bifurcation Analysis

3.1 Background

The set of equations listed above can be considered a nonlinear dynamical system with the nonlinearities entering through the turbulent diffusion terms. Such nonlinearities effectively prevent analytical solutions; however, the behavior of the solutions can be quantitatively examined as a function of external parameters using the methods of numerical bifurcation theory. Thus, despite the lack of a closed-form solution, parameter-dependence and other characteristics of the solution can be developed.

Techniques of nonlinear analysis have expanded in recent years to include numerical continuation methods. Sophisticated software packages are now readily available to carry out the analysis. For those not familiar with numerical continuation we provide the following example as an introduction. Seydel (1988) and Doedel et al., (1986) provide an excellent overview of practical techniques of nonlinear analysis and continuation.

Using numerical techniques, the steady solution of the system can then be traced out, thus providing an understanding of the dependence of the steady solution on the bifurcation parameter λ. In our case λ is one of the imposed parameters such as geostrophic wind or surface roughness. More importantly, techniques of linear stability analysis can be used to determine the characteristics of the perturbed solution from the steady solution. Thus, by numerically calculating the eigenvalues, a portion of the curves can be characterized as stable or unstable to perturbations or whether the perturbed solution will exhibit oscillatory behavior.

With this introduction we next use numerical continuation to analyze the system and characterize the behavior of the system as a function of external parameters. In the analysis we choose the external parameters to be geostrophic wind, u_g, roughness length, z_0 layer heights, z_1 and z_2, and the Coriolis parameter, ***f***.

3.2 Analysis

MCN95 truncated the system to a two-layer system reducing the PDE system to a set of ODE's. We now use the analysis software AUTO (Doedel and Kernevez, 1986) to carry out the analysis. Figure 1 is an example from that study showing that the system of equations supports hysteresis. This is indicated by the "S" shaped bifurcation diagram. Because of hysteresis, multiple solutions exist in some parameter regimes. This is a classical case of the lack of predictability in the system with potential sensitivity to initial conditions. For light geostrophic winds the system goes to a light-wind, cold solution, and at large geostrophic winds the system goes to a warm, windy solution. In the intervening region sharp variation occurs; for larger roughness lengths the system supports three solutions in the fold-back region.

We have now carried out a bifurcation analysis of the full PDE system (Shi, 1997). Figure 2 shows the bifurcation diagrams. The full PDE system also supports hysteresis and multi-valued solutions. The ***u***, ***v*** and θ in the diagrams are now represented as a vector norm (the magnitude of the ***z*** vector) so that actual values of the behavior of the PDE and the ODE systems cannot be compared. However, the transition between the cold, calm solution and the warm, windy solution occurs at similar values of geostrophic winds. The surface temperatures as a function of geostrophic wind are virtually identical. Thus, we conclude that the ODE system and its stability characteristics are a good surrogate for the full PDE system.

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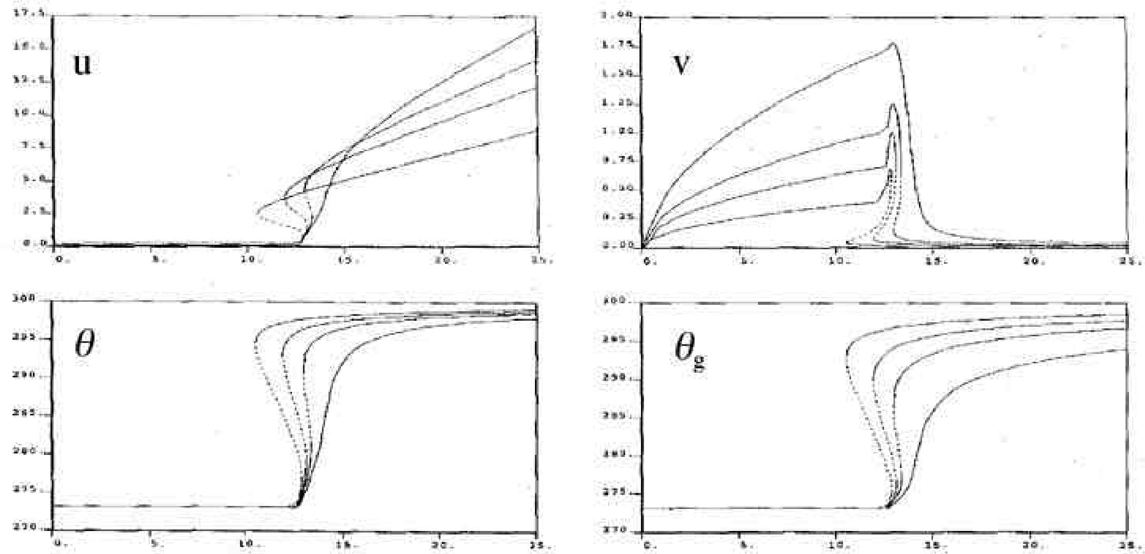

Figure 1. Bifurcation diagram for ODE systems. Bifurcation parameter on the x –axis is geostrophic wind. Four curves are given for roughness lengths –0.01,0.5,1.0 and 2.0m.

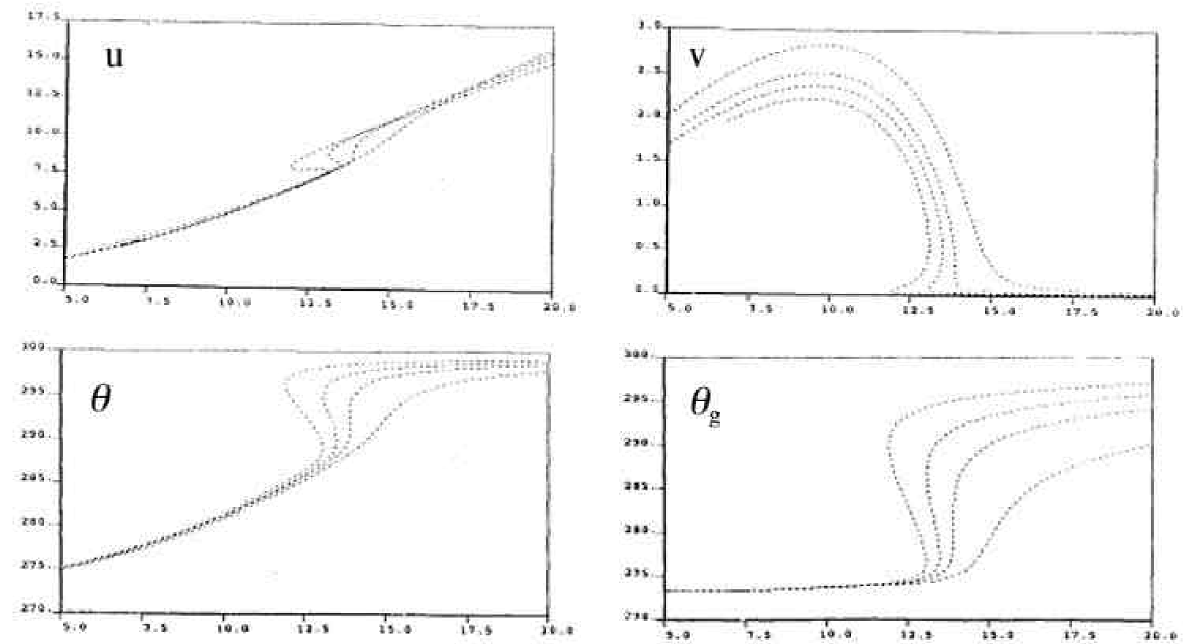

Figure 2. Same as figure 1 except for the PDE system. Values for u, v and _ are the magnitude norm for the z vector.