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GENERATION OF LARGE-SCALE SEMI-ORGANIZED STRUCTURES IN TURBULENT CONVECTION

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1. INTRODUCTION

In the last decades it has been recognized that the very high Rayleigh number convective boundary layer (CBL) has more complex nature than might be reckoned. Besides the fully organized component naturally considered as the mean flow and the chaotic small-scale turbulent fluctuations, one more type of motion has been discovered, namely, long-lived large-scale structures, which are neither turbulent nor deterministic. These semi-organized structures considerably enhance the vertical transports and render them essentially non-local in nature. In the atmospheric shear-free convection, the structures represent three-dimensional Benard-type cells composed of narrow uprising plumes and wide downdraughts. They embrace the entire convective boundary layer ($\sim$ 2 km in height) and include pronounced large-scale ($\sim$ 5 km in diameter) convergence flow patterns close to the surface (see, e.g., Atkinson and Wu Zhang, 1996; Etling and Brown, 1993, and references therein). In sheared convection, the structures represent CBL-scale rolls stretched along the mean wind. Life-times of the semi-organized structures are much larger than the turbulent time scales. Thus, these structures can be treated as comparatively stable, quasi-stationary motions, playing the same role with respect to small-scale turbulence as the mean flow.

In a laboratory turbulent convection several organized features of motion, such as plumes, jets, and the large-scale circulation, are known to exist. The experimentally observed large-scale circulation in the closed box with a heated bottom wall (the Rayleigh-Benard apparatus) is often called the "wind" (see, e.g., Niemela et al., 2001, and references therein). There are several unsolved theoretical questions concerning these flows, e.g., how do they arise, and what are their characteristics and dynamics.

In spite of a number of studies, the nature of large-scale semi-organized structures is poorly understood. The Rayleigh numbers, Ra, based on the molecular transport coefficients are very large (of the order of $10^{11} - 10^{13}$). This corresponds to fully developed turbulent convection in atmospheric and laboratory flows. At the same time the effective Rayleigh numbers, $\mathrm{Ra}^{(\mathrm{eff})}$, based on the turbulent transport coefficients are not high, e.g., $\mathrm{Ra}^{(\mathrm{eff})} \sim \mathrm{Ra}/(\mathrm{RePe})$, where Re and Pe are the Reynolds and Peclet numbers, respectively. They are less than the critical Rayleigh numbers required for the excitation of large-scale convection. Hence the emergence of large-scale convective flows (which are observed in the atmospheric and laboratory flows) seems puzzling.

The main goal of this study is to suggest a mechanism for excitation of large-scale circulations (large-scale convection). We analyzed relevance of the obtained results to turbulent convection in the atmosphere and laboratory experiments.

2. THE CONCEPT OF CONVECTIVE WIND

The proposed "convective-wind theory" of turbulent sheared convection distinguishes between the "true turbulence", corresponding to the small-scale part of the spectrum, and the "convective wind" comprised of large-scale semi-organized motions caused by an inverse energy cascade through large-scale instabilities. The true turbulence in its turn consists of the two parts: the familiar "Kolmogorov-cascade turbulence" and an essentially non-isotropic "tangling turbulence" caused by tangling of the mean-velocity gradients with the Kolmogorov-type turbulence. These two types of turbulent motions overlap in the maximum-scale part of the spectrum. The tangling turbulence does not exhibit any direct energy cascade, and its spectrum is steeper than the Kolmogorov-turbulence spectrum.

In this study the convective-wind motions were investigated using perturbation analysis. It was demonstrated that their typical length and time scales are much larger than the true-turbulence scales. This justifies separate treatment of the above two types of motions.

It is proposed that the term turbulence (or true turbulence) be kept only for the Kolmogorov + tangling turbulence part of the spectrum. This concept implies that the convective wind (as well as semi-organized motions in other very high Reynolds number flows) should not be confused with the true turbulence.

In the light of this observation, further attempts to develop an overall turbulence closure covering the whole spectrum of non-regular motions do not look promising. Indeed, traditional math-statistical tools are adequate as applied to the true turbulence but become a Procrustean bad for semi-organized motions, such as convective wind. This factual inconsistency explains why modern convective-turbulence closures, despite their enormous complexity, are not sufficiently advanced to reproduce the transport properties of convective flows over a range of regimes.

In accordance with the convective-wind concept, the three-fold approach is proposed instead of the traditional, overall closures. It includes: (i). Application of the Kolmogorov closure to the true turbulence and the Orszag (relaxation) closure to the tangling turbulence; (ii). Using these closures, analytical investigation of the basic features and scales of convective-wind structures. (iii). Numerical modelling of complex flows with due regard to appropriate resolution (or parameterization) of the above structures.

This clearly reminds large-eddy simulation (LES) techniques, in which sub-filter scale closures are of the Kolmogorov type, whereas semi-organized motions are resolved. The proposed approach offers scope for providing LES with better grounded turbulent transport coefficients and additional tools, such as a priori knowledge about the basic features of semi-organized structures, and physical grounds for the optimal choice of the filter scale.

An interesting prospect is to attempt to parameterize the transport properties of the structures. This would lead to flux-calculation schemes consisted of advanced closures for the true turbulence and appropriate parameterizations for the dominant type of structures.

First results from the proposed theory demonstrate a qualitative agreement with the known unexplained observations. Examples are: (a) the turbulent heat conductivity increases with the decrease of shear and the turbulent Prandtl number decreases by a factor four with decrease of shear; (b) in the presence of the mean shear, the horizontal heat flux is oriented against the regular mean flow.

The developed theory predicts the convective wind instability in a shear-free turbulent convection. This instability causes formation of large-scale semi-organized fluid motions (convective wind) in the form of cells. Spatial characteristics of these motions, such as the minimum size of the growing perturbations and the size of perturbations with the maximum growth rate, are determined.

This study predicts also the existence of the convective-shear instability in the sheared turbulent convection. This instability causes formation of large-scale semi-organized fluid motions in the form of rolls (sometimes visualized as the boundary-layer cloud streets). These motions can exist in the form of generated convective-shear waves, which have a nonzero hydrodynamic helicity. Increase of shear promotes excitation of the convective-shear instability.

3. TURBULENT FLUX OF ENTROPY

Now we discuss in a more detail the mechanism of formation of semi-organized structures. Traditional theoretical models of the boundary-layer turbulence, such as the Kolmogorov-type closures and similarity theories (e.g., the Monin-Obukhov surface-layer similarity theory) imply two assumptions: (i) Turbulent flows can be decomposed into two components of principally different nature: fully organized (mean-flow) and fully turbulent flows. (ii) Turbulent fluxes are uniquely determined by the local mean gradients. For example, the turbulent flux of entropy is given by

$$\langle s\mathbf{u}\rangle = -\kappa_T \boldsymbol{\nabla} \bar{S} \;, \qquad (1)$$

where κ_T is the turbulent thermal conductivity, $\bar{S}$ is the mean entropy, $\mathbf{u}$ and s are fluctuations of the velocity and entropy.

However, the mean velocity gradients can affect the turbulent flux of entropy. The reason is that additional essentially non-isotropic velocity fluctuations can be generated by tangling of the mean-velocity gradients with the Kolmogorov-type turbulence. The source of energy of this "tangling turbulence" is the energy of the Kolmogorov turbulence. We showed that the tangling turbulence can cause formation of semi-organized structures due to excitation of large-scale instability. This process is nothing but the inverse energy cascade.

The tangling turbulence was introduced by Wheelon and Batchelor for a passive scalar and by Golitsin and Moffatt for a passive vector (magnetic field). Anisotropic fluctuations of a passive scalar (e.g., the number density of particles or temperature) are generated by tangling of gradients of the mean passive scalar field with random velocity field. Similarly, anisotropic magnetic fluctuations are excited by tangling of the mean magnetic field with the velocity fluctuations. The Reynolds stresses in a shear flow is another example of a tangling turbulence. Indeed, they are strongly anisotropic in the presence of shear and have a steeper spectrum ($\propto k^{-7/3}$) than a Kolmogorov turbulence. The tangling turbulence contributes to the turbulent flux of entropy. Calculations based on the Navier-Stokes equations, the entropy evolution equation formulated in the Boussinesq approximation and employing the Orszag (relaxation) closure hypothesis yield the following expression for the turbulent flux of entropy $\boldsymbol{\Phi} \equiv \langle s\mathbf{u}\rangle$:

$$\begin{aligned}\boldsymbol{\Phi} &= \boldsymbol{\Phi}^* + (\tau_0/5)[-5\alpha(\boldsymbol{\nabla}\cdot\bar{\mathbf{U}}_\perp)\boldsymbol{\Phi}^*_\parallel \\ &\quad +(\alpha+3/2)(\bar{\boldsymbol{\omega}}\times\boldsymbol{\Phi}^*_\parallel) + 3(\bar{\boldsymbol{\omega}}_\parallel\times\boldsymbol{\Phi}^*)] \;, \qquad (2)\end{aligned}$$

Here, τ_0 is the correlation time of the Kolmogorov turbulence corresponding to the maximum scale of turbulent motions, $\bar{\boldsymbol{\omega}} = \boldsymbol{\nabla}\times\bar{\mathbf{U}}$ is the mean vorticity, the mean velocity vector is presented as $\bar{\mathbf{U}} = \bar{\mathbf{U}}_\perp + \bar{U}_z\mathbf{e}$, and $\boldsymbol{\Phi}^*_\parallel = \Phi^*_z\mathbf{e}$, $\bar{\boldsymbol{\omega}}_\parallel = \bar{\omega}_z\mathbf{e}$, $\boldsymbol{\Phi}^* = -\kappa_T\boldsymbol{\nabla}\bar{S} - \tau_0\Phi^*_z(d\bar{\mathbf{U}}^{(0)}(z)/dz)$, $\bar{\mathbf{U}}^{(0)}(z)$ is the imposed horizontal large-scale flow velocity (e.g., a wind velocity), $\mathbf{e}$ is the vertical unit vector and α is the degree of thermal anisotropy of the background turbulent convection (without mean-velocity gradients). In the isotropic case, $\alpha = 1$. When $-9/2 < \alpha < 1$, thermal structures have the form of columns or thermal jets, and when $1 < \alpha < 3$, they have the "pancake" form. The last three terms on the right hand side of Eq. (2), depending on the mean-velocity gradients and caused by the tangling turbulence, can result in the excitation of large-scale instability and formation of semi-organized structures (convective wind).

The turbulent flux of entropy can be also obtained from simple symmetry reasoning. Indeed, $\langle s\mathbf{u}\rangle = \Phi^*_i + \beta_{ijk}\nabla_j\bar{U}_k$, where β_{ijk} is an arbitrary true tensor. Then using the identity $\nabla_j\bar{U}_i = (\delta\bar{\mathbf{U}})_{ij} - (1/2)\varepsilon_{ijk}\,\bar{\omega}_k$, the turbulent flux of entropy becomes

$$\langle su_i\rangle = \Phi^*_i + \eta_{ij}\bar{\omega}_j + (\bar{\boldsymbol{\omega}}\times\boldsymbol{\sigma})_i + \mu_{ijk}(\delta\bar{\mathbf{U}})_{jk} \;, \qquad (3)$$

where $(\delta\bar{\mathbf{U}})_{ij} = (\nabla_i\bar{U}_j + \nabla_j\bar{U}_i)/2$ and ε_{ijk} is the fully antisymmetric Levi-Civita tensor. Clearly, $\boldsymbol{\Phi}^*$ determines the contribution from the Kolmogorov turbulence, whereas the last three terms describe the contribution of the tangling turbulence. In Eq. (3), η_{ij} is a symmetric pseudo-tensor, $\boldsymbol{\sigma}$ is a true vector, μ_{ijk} is a true tensor symmetric in the last two indexes, $\boldsymbol{\Phi} \equiv \langle s\mathbf{u}\rangle$ and $\boldsymbol{\Phi}^*$ are true vectors. These tensors and vectors can be constructed using two vectors: $\boldsymbol{\Phi}^*$ and the vertical unit vector $\mathbf{e}$. Therefore $\eta_{ij} = 0$, $\boldsymbol{\sigma} = A_1\boldsymbol{\Phi}^* + A_2\Phi^*_z\mathbf{e}$, and $\mu_{ijk} = A_3\Phi^*_z e_{ijk} + A_4\Phi^*_i e_{jk}$, where A_k are the unknown coefficients and $e_{ijk} = e_i e_j e_k$. This yields the following expression of the turbulent flux of entropy in a divergence-free mean velocity field:

$$\begin{aligned}\mathbf{\Phi} &= \mathbf{\Phi}^* - (A_3 + A_4)(\boldsymbol{\nabla}\cdot\bar{\mathbf{U}}_\perp)\mathbf{\Phi}^*_\parallel \\ &+(A_1 + A_2)(\bar{\boldsymbol{\omega}}\times\mathbf{\Phi}^*_\parallel) + A_1(\bar{\boldsymbol{\omega}}_\parallel\times\mathbf{\Phi}^*) \\ &-A_4(\boldsymbol{\nabla}\cdot\bar{\mathbf{U}}_\perp)\mathbf{\Phi}^*_\perp] \, . \end{aligned} \quad (4)$$

Equations (2) and (4) coincide if one sets $A_1 = 3\tau_0/5$, $A_2 = (\tau_0/5)(\alpha - 3/2)$, $A_3 = \tau_0\alpha$, and $A_4 = 0$.

4. MECHANISMS OF THE LARGE-SCALE INSTABILITY

The mechanism of the convective wind instability, associated with the second term in the expression for the turbulent flux of entropy [see Eq. (2)], in the shear-free turbulent convection at $\alpha > 0$ is as follows. Perturbations of the vertical velocity $\bar{U}_z$ with $\partial\bar{U}_z/\partial z > 0$ have negative divergence of the horizontal velocity , i.e., $\mathrm{div}\,\bar{\mathbf{U}}_\perp < 0$ (provided that $\mathrm{div}\,\bar{\mathbf{U}} \approx 0$). This results in the vertical turbulent flux of entropy and causes an increase of the mean entropy. On the other hand, the increase of the the mean entropy increases the buoyancy force and results in the increase of the vertical velocity $\bar{U}_z$ and excitation of the large-scale instability. Similar phenomenon occurs in the regions with $\partial\bar{U}_z/\partial z < 0$ whereby $\mathrm{div}\,\bar{\mathbf{U}}_\perp > 0$. This causes a downward flux of the entropy and the decrease of the mean entropy. The latter enhances the downward flow and results in the instability which also causes formation of a large-scale semi-organized convective wind structure. Thus, nonzero $\mathrm{div}\,\bar{\mathbf{U}}_\perp$ causes redistribution of the vertical turbulent flux of entropy and formation of regions with large vertical fluxes of entropy. Thereby the regions with $\mathrm{div}\,\bar{\mathbf{U}}_\perp < 0$ are separated by the regions with low vertical flux of entropy with $\mathrm{div}\,\bar{\mathbf{U}}_\perp > 0$. This results in a formation of a large-scale circulation of the velocity field.

Another mechanism of the convective wind instability is associated with the third term in the expression (2) for the turbulent flux of entropy when $\alpha < -3/2$. This term describes the horizontal flux of the mean entropy. The latter results in increase (decrease) of the mean entropy in the regions with upward (downward) fluid flows. On the other hand, the increase of the mean entropy results in the increase of the buoyancy force, the mean vertical velocity $\bar{U}_z$ and the mean vorticity $\bar{\boldsymbol{\omega}}$. This causes the large-scale convective wind instability. The second term in the turbulent flux of entropy at $\alpha < -3/2$ causes a decrease of the growth rate of the instability because, when $\alpha < -3/2$, it implies a downward turbulent flux of entropy in the upward flow. This decreases both, the mean entropy and the buoyancy force. Note that, when $\alpha < -3/2$, the thermal structure of the background turbulence has the form of a thermal column.

The mechanism of the convective-shear instability associated with the last term in the expression (2) for the turbulent flux of entropy is as follows. The vorticity perturbations generate perturbations of entropy. Indeed, for two vortices with opposite directions of the vorticity $\bar{\boldsymbol{\omega}}_\parallel$, the turbulent flux of entropy is directed towards the boundary between the vortices. The latter increases the mean entropy between the vortices. Such redistribution of the mean entropy causes increase of the buoyancy force and formation of upward flows between the vortices. Finally, the vertical flows generate vorticity, etc. This results in excitation of the instability and generation of the convective-shear waves.

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