8.1 THE SHAPES OF THE PRODUCTION REGIONS OF VELOCITY SPECTRA AND COSPECTRA IN THE NEUTRAL ATMOSPHERIC SURFACE LAYER

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1. INTRODUCTION

For many years the turbulence spectra obtained during the Kansas experiment (Kaimal et al., 1972) have served as the standard results for the atmospheric surface layer (ASL) over land (e.g. Kaimal & Finnigan, 1994) and at sea, both in the ASL above the water surface (Nicholls & Readings, 1981) and in the benthic surface layer over the sea bed (Lien & Sanford, 2000). Given this wide acceptance, it is surprising that only their inertial sub-ranges have been adequately explained: by Kolmogorov theory. The 'production' regions of these spectra—at the peak wavenumbers and smaller—await a full explanation.

Uncertain foundations invite challenges. Thus Hunt and co-workers (Hunt & Morrison, 2000; Hunt & Carlotti, 2001; Carlotti et al., 2001) have proposed that the spectra of streamwise (***u***) and transverse (***v***) velocity follow a $\boldsymbol{k}^{-1}$ power law at the spectral maximum, which is to say the spectra plotted as $\boldsymbol{k}\,\boldsymbol{E}_{ij}(\boldsymbol{k})$ really have broad, flat peaks rather than rounded peaks. They also propose a model that is consistent with this feature. Implicit in their case is that the Kansas results are unreliable.

Here the Kansas results are accepted at face value and the paper investigates what kind of turbulence structure might underlie them. Townsend's ideas of attached and detached structures, and of active and inactive components of turbulence are used throughout (Townsend 1976). To simplify matters, discussion centres on cases where inactive motions contribute little to the spectra, so we deal principally with spectra from neutral or near-neutral conditions.

2. COHERENT STRUCTURES

The first proposition is that the active parts of spectra can be explained in terms of the properties of a set of coherent structures. A coherent structure is a simple, recognizable and recurring pattern of motion that is localized in both physical and wavenumber space. The word coherent implies that it has a single dynamic, so all of its parts share the same length, velocity and time scales. Examples of coherent structures are quasi-streamwise vortices and hairpin vortices, but not wall streaks that seem to have quasi-regular (harmonic) spacing in the cross-wind direction and indefinite length. Here the term 'eddy' is used as a synonym for 'coherent structure'. Figure 1 shows a schematic coherent structure with the required characteristics

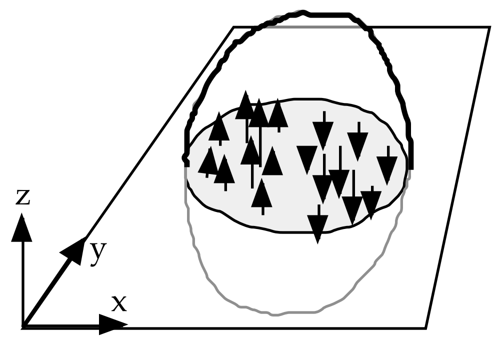

Figure 1. A schematic eddy. Main features are that an outline can be defined and its internal structure is simple. This is illustrated here by showing the vertical velocity not random but organized into well-defined areas of updraft and downdraft.

Eddies may be attached or detached. The distinction is that attached eddies extend down to the ground while detached eddies do not. This means that an attached eddy 'knows' where the ground is, and its dynamic is influenced by pressure reflection at the ground. Detached eddies do not contact the ground and so respond only to local conditions in the flow: Eddies of the inertial subrange are detached.

This distinction is illustrated in Fig. 2 using, as exemplar, the distribution of vertical velocity squared ($\boldsymbol{w}^2$) integrated over a horizontal plane at each height, $\boldsymbol{z}$. The integrated variance approaches zero at the ground for the attached eddy, but somewhat above it for the detached eddy. It also approaches zero at an upper height, ***h***, which defines the length scale of an attached eddy. Notice that it is the assumption of small complexity that ensures that ***w*** approaches zero at the ground.

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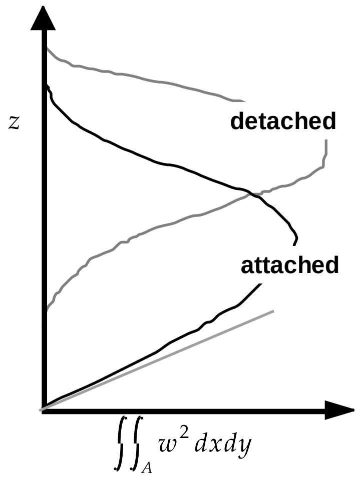

Figure 2. Schematic of the height distribution of the variance of vertical velocity in attached and detached eddies. Attached and detached eddies are distinguished by how their variances approach zero at the ground.

3. SMALL-WAVENUMBER ASYMPTOTES

Here we derive the asymptotic behaviour of velocity spectra and cospectra in the ASL in neutral conditions. We assume that the important eddies are attached and statistically self-similar when scaled on ***h*** and ***u***$_*$ The derivation employs wavelet transforms, as described by Perrier et al. (1995) and Farge et al. (1996).

Consider a continuous wavelet transform of the vertical velocity signal $\boldsymbol{w}_{hi}(\boldsymbol{x}, \boldsymbol{z})$ as measured along a horizontal transect at height ***z*** through a single eddy of height ***h***. Here ***x*** is distance along the transect and all quantities are non-dimensionalized using ***u***$_*$ and ***h***. The wavelet transform of this signal is

$$\tilde{f}_{hzi}(x,r) = \frac{1}{r}\int_{-\infty}^{\infty} w_{hzi}(x')\,\psi\!\left(\frac{x'-x}{r}\right) dx'$$

where $\psi(..)$ is a wavelet function, compact in both physical and wavenumber space. The wavelet spectrum of this signal is

$$\tilde{E}_{hzi}(x,k) = \frac{1}{2c_\psi k_0}\left|\tilde{f}_{hzi}\!\left(x, \frac{k_0}{k}\right)\right|^2$$

where $\boldsymbol{k}_0$ is the peak wavenumber for the analysing wavelet,

$$c_\psi = \int_0^\infty |\hat{\psi}(k)|^2 \frac{dk}{k}$$

and $\hat{\psi}$ is the Fourier transform of ψ. Wavenumber, ***k***, is then related to wavelet dilation, ***r***, by $\boldsymbol{k} = 2\pi/\boldsymbol{r}$. If the transect passes through ***n*** such eddies, the wavelet transform of the set of intersected eddies is

$$\tilde{f}_{hz}(x,r) = \frac{1}{r}\int_{-\infty}^{\infty}\left[\sum_{i=1}^{n} w_{hi}(x', z)\right]\psi\!\left(\frac{x'-x}{r}\right) dx'$$

which is the superposition of the wavelet transforms of the individual signals

$$\tilde{f}_{hz}(x,r) = \sum_{i=1}^{n} \tilde{f}_{hzi}(x,r).$$

The wavelet spectrum for the sum of the signals is

$$\tilde{E}_{hzi}(x,k) = \frac{1}{2c_\psi k_0}\left|\tilde{f}_{hz}\!\left(x, \frac{k_0}{k}\right)\right|^2$$

Expansion shows that this can be expressed as the superposition of the individual spectra plus product terms $2\tilde{f}_i\hat{f}_j$, i ≠ j whose effect is to redistribute energy away from the positions and scales of the individual eddies and towards the positions and scales of eddy clusters. The product terms are significant wherever the individual wavelet transforms overlap with significant amplitude.

The average spectrum for all eddies of (normalized) height 1 is

$$\tilde{E}_{hz}(k) = \frac{1}{2c_\psi k_0 L}\int_{-\infty}^{\infty}\left|\sum_{i=1}^{n}\tilde{f}_{hzi}(x,r)\right|^2 dx$$

where ***L*** is the length of the transect.

This equation is written for eddies of just one size, but the summation could be extended over any set of intersected eddies. If all intersected eddies were to be included, it would describe the complete wavelet spectrum, $\tilde{E}_z(k)$. Alternatively, summation could be over all eddies whose individual wavelet transforms are non-zero at $\boldsymbol{r} = 1$. This set of eddies is sufficient to evaluate $\tilde{E}_z(k)$ at $\boldsymbol{k} = 2\pi$, so

$$\tilde{E}_z(2\pi) = \frac{1}{2c_\psi k_0 L}\int_{-\infty}^{\infty}\left|\sum_{i=1}^{n}\tilde{f}_{hzi}(x,1)\right|^2 dx$$

The contributing eddies would have heights spread about 1. Eddies much taller than 1 would contribute to the extent that they have internal structure with scale ~1, and eddies shorter than 1 would contribute if they occur in groups of scale ~1. Eddies shorter than ***z*** do not contribute. If eddies are neither very complex nor occur in very large clusters then the main contributors to the spectrum at $\boldsymbol{k} = 2\pi$ will be eddies with heights near 1. The range may be a decade or more wide but that is not important to the asymptotic behaviour of the spectrum. All that is necessary is that there exists a height ***z*** that is small compared to the finest structural scales within eddies of heights ≥1.

The discussion above relates to the wavelet spectrum while most published spectral results for the ASL have been presented as Fourier spectra. The two are related by

$$\tilde{E}(k) = \frac{1}{c_\psi k}\int_0^\infty E(\omega)\left|\hat{\psi}\left(\frac{k_0\omega}{k}\right)\right|^2 d\omega$$

Importantly, wavelet and Fourier spectra have the same slope when the Fourier spectrum follows a power law $E_z(k) \propto k^{-\beta}$, provided that the analysing wavelet has at least (β - 1)/2 vanishing moments. This is true of any wavelet for the k^0 power law considered here.

We are now ready to introduce some physical assumptions and so deduce the asymptotic behaviour of spectra at small wavenumbers. Let the turbulence be statistically homogeneous in the direction of the transect so that $E_z(2\pi)$ approaches a stable mean value for large enough L. Let the turbulence also be statistically self-similar under inner scaling so that $E_z(2\pi)$ is independent of the choice of h. Then we can write

$$\tilde{E}_z(2\pi) = g(z)$$

where $g(z)$ is a universal function of height of the kind shown in Fig. 2. This function may be expanded into a Taylor series about z = 0, so for z << 1 and with $g(z)$ = 0 at the ground we have

$$\tilde{E}_z(2\pi) = a\,z + O(z^2)$$

where a is a constant. Taking the limit gives

$$\tilde{E}_z(2\pi) = a\,z \qquad : z << 1$$

The parameter h does not appear explicitly, so this holds for any h. Detached eddies do not contribute to the spectrum in this limit so this equation defines the asymptotic behaviour of the whole spectrum at kz << 1. In dimensional form this asymptote is

$$\tilde{E}(k) = a\,u_*^2\,z \qquad \text{for} \quad kz << 1.$$

Before declaring that similar reasoning leads to k^0 asymptotes to the u, v spectra and the uw cospectrum we must show that the fluctuations in these variables go to zero at the ground, just as for w (Fig. 2). For inviscid eddies we can not invoke a non-slip boundary condition to give this directly, but without viscosity changes in horizontal velocity can only be transported to the ground by vertical displacements of air, so $w \to 0$ at the surface ensures that u, v, $uw \to 0$ also. The u, v spectra and the uw cospectrum therefore also approach k^0 asymptotes as $kz \to 0$. This agrees with the neutral spectra observed at Kansas.

4. POSITIONS OF SPECTRAL PEAKS

A notable feature of the neutral Kansas spectra is that the positions of their various peaks range over almost a decade in wavenumber. If the same eddies generate all these spectra then eddy characteristics apart from size distribution, which is common to all, must help determine peak positions. The wavenumber formalism presented above invites us to suppose that aggregation properties of eddies as well as their individual form, number and complexity will influence spectral shapes. To prepare for this we must take another look at the properties of active, attached eddies.

The first point to note is that these eddies are attached, so they have no bulk vertical motion. Therefore within each eddy the updrafts must balance the downdrafts at each level. That is to say, there will be a strong tendency for the w signal along any horizontal transect through an eddy to have a zero mean. This means that the amplitude of the wavelet transform will decrease rapidly with increasing dilation. This will reduce the effect of any aggregation on the w spectrum by reducing any contribution to the spectrum from product terms, $\tilde{f}_i\hat{f}_j$. Also, we do not expect whole eddies to move laterally in the flow so we have similar expectations for the v spectrum.

The second defining characteristics of these eddies is that they are active: they transfer momentum. That is to say, their net effect is to transfer faster air downwards towards the surface and slower air upwards away from it. By their nature, aggregates of similar-sized eddies will be zones in the flow where streamwise velocity nearer the ground (say $z < h/2$) is increased to above the average at that level, and streamwise velocity away from it (say $z > h/2$) decreased. We expect transform product terms $\tilde{f}_i\hat{f}_j$ to be larger and to shift spectral energy towards the larger scales of the aggregates. The same is true of uw because momentum is transported downwards in all eddies. If aggregation is a factor, we expect to find u and uw spectral peaks that are broader and at smaller wavenumbers than those of the w and v spectra.

The neutral spectra from Kansas do have characteristics that can be attributed to eddy aggregation. Unfortunately these spectra are not unambiguous because Taylor's frozen turbulence hypothesis must be used to convert them from frequency to wavenumber representations. Also, and at best, they provide along-wind transects through the turbulent field, so they give no information on cross-wind structure. The frozen turbulence hypothesis is problematic because eddy evolutionary and translational time scales are about equal in surface layers. The evolutionary time scale of an attached eddy is determined by the shear that drives it, $\partial\bar{u}/\partial z$, evaluated at $z \sim h/2$ where its energy is greatest. Thus the evolutionary time scale is ~ $0.2h/u_*$ for a logarithmic wind profile. The translational time scale of the same eddy at z is $\sim h/\bar{u}$ at the same height. The ratio of the two time scales is $O(1)$, so we expect an eddy to change form substantially while passing a fixed observer.

Data that circumvents both limitations can be got by aircraft. Nicholls & Readings (1981) report data from 11 aircraft runs made in the surface layer ($z < z_i$) and in near-neutral conditions (- 0.15 < $-z/L$ < 0) over the sea. Their spectra are not so well defined as those from Kansas and there is evidence of some inactive turbulence, but the positions of the active spectral peaks can be read fairly reliably. These data can be put beside the Kansas results as a cross-check on the use of Taylor's hypothesis in the Kansas data and to provide cross-wind transects. Peak positions are shown in Table 1, recorded as peak wavelength normalised by observation height.

	Kansas Along wind $\bar{u}/f_m z$	Aircraft Along wind $2\pi/k_1 z$	Aircraft Cross wind $2\pi/k_2 z$
u	17	15	2.5
v	4	2.5	2.5
w	1.8	2.5	2.5
uw	12	15	2.5

Table 1. Positions of spectral peaks as $\lambda_m/\boldsymbol{z}$ from Kaimal et al. (1972); column 2, and from Nicholls & Readings (1981); column 3, 4. Nicholls & Readings gave their results to only one significant figure, so the recalculated values here have been rounded.

The along wind data from tower and aircraft shown in Table 1 may be compared directly. They show similar peak wavelengths: evidence that use of Taylor's frozen turbulence has not distorted the Kansas results significantly, and that the conditions over the sea are 'normal'. This is necessary in the light of evidence that turbulence processes in the marine surface layer can be qualitatively affected by a decaying sea where momentum transfer is reversed.

An interesting feature of Table 1 is that the peak positions are different in the $\boldsymbol{k}_1$ (along-wind) and $\boldsymbol{k}_2$ (cross-wind) directions. All of the cross-wind peaks are at the same wavelength, equal to that of the along-wind peak in the ***w*** spectrum. The along-wind ***v*** peak is at a similar or slightly larger wavelength while the peak wavelengths of the along-wind ***u*** and ***uw*** spectra are at notably longer. This is consistent with the turbulent field consisting of groups of eddies that are aligned in the streamwise direction.

Direct support for this interpretation comes from a wavelet analysis of momentum flux using data from an instrument at 4 m over land (Mayer et al. 1994). Conditions were unstable and not well described in the paper, but the wavelet transform clearly shows a pattern of regularly decreasing smaller-scale coherent structures trailing behind and in clear association with the larger-scale ones.

A final observation is that the peak wavelenghs show that the peak variance or covariance is produced by eddies that are about twice the observation height, assuming horizontal and vertical eddy dimensions are about equal. This is as expected.

5. CONCLUSIONS

This paper has presented a qualitative account of some characteristics of the production regions of velocity spectra and cospectra in the ASL. It is based on the assumption that the turbulence giving rise to these spectra is dominated by active and attached coherent structures. It has been argued that the spectra can be interpreted as reflecting eddies that occur in groups aligned with the wind. A structural model with this characteristic is presented in a second paper at this meeting.

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