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A SIMPLE AND GENERAL SUBGRID MODEL SUITABLE BOTH FOR SURFACE LAYER AND FREE-STREAM TURBULENCE

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1. INTRODUCTION

The representation of the full depth of the atmospheric boundary layer with a unique parameterization is a difficulty shared by all numerical models from Large-eddy simulations (LES) to general circulation models (GCM). LES separate the scales of turbulence into two ranges, resolved and subgrid scales. The resolved scales are assumed to contain most of the energy of turbulent motion whilst on subgrid scales, motions are the less energetic. This approach works well far from regions of large gradients. In the surface region, the energy is always subgrid in all models from LES to GCMs. Past studies (e.g. Mason and Thomson 1992; Sullivan *et al.* 1994) have shown that the standard subgrid-scale eddy viscosity approach leads to overpredict the shear near the ground. To overcome this problem, the same authors have suggested modifications to subgrid turbulence schemes. It is worthwhile to notice that in all the approaches, the subgrid mixing length is assumed to be the Prandtl mixing length κz near the wall. Our goal (Redelsperger *et al.* 2001, hereafter RMC2001) was to provide a physical explanation of this problem and a solution suitable for any models and reliable for inhomogeneous surface conditions, complex topography and any vertical stability conditions.

2. PROBLEM

2.1 Analytical view

To illustrate the problem, we will look at the behaviour of turbulence closures in considering the one-dimensional, neutrally stratified case. Using the equilibrium theory for an horizontally homogeneous turbulent flow, the turbulent kinetic energy equation reduces to a balance of dissipation ε with shear production :

$$\varepsilon = -\overline{u'w'}\frac{\partial \overline{u}}{\partial z} \qquad (1)$$

where u, w are the components of wind along x and z directions, respectively. In the common subgrid-scale eddy viscosity approach, the Reynolds stress is modelled as:

$$-\overline{u'w'} = K_m\frac{\partial \overline{u}}{\partial z} \quad ; \quad K_m = \mathcal{L}_K E^{1/2} \qquad (2)$$

where E denotes the subgrid turbulent kinetic energy and $\mathcal{L}_K$ has the dimension of a length.
Dimensional arguments show that the dissipation can be expressed as

$$\varepsilon = \frac{E^{3/2}}{\mathcal{L}_\varepsilon} \qquad (3)$$

where $\mathcal{L}_\varepsilon$ has the dimension of a length.
In mesoscale and large-scale models, the lengths are generally specified using empirical formulations or by using the properties of the boundary layer in the considered column of the atmosphere (e.g. Troen and Mahrt 1986; Cuxart *et al.* 2000). In the surface layer, the lengths are assumed to be the Prandtl mixing length κz as generally in the LES. In LES models, for the case of free turbulence (i.e. away from the ground), spectral arguments can be used to show that:

$$\mathcal{L}_K = C_K L \quad ; \quad \mathcal{L}_\varepsilon = \frac{L}{C_\varepsilon} \qquad (4)$$

where L is the mesh size and

$$C_\varepsilon = \pi\left(\frac{2}{3\alpha_3}\right)^{\frac{3}{2}} \quad ; \quad C_K = \frac{1}{\pi}\left(\frac{2}{3\alpha_3}\right)^{\frac{3}{2}} \qquad (5)$$

where α_3 is the "3D" Kolmogorov constant. Measurements give $\alpha_3 = 1.6 \pm 0.02$ (Andreas 1987).
Using the equilibrium hypothesis (Eq. 1) together with eqs 2 and 3, the subgrid turbulent kinetic energy and the Reynolds stress can be written as:

$$E = \mathcal{L}_\varepsilon \mathcal{L}_K\left(\frac{\partial \overline{u}}{\partial z}\right)^2 \qquad (6)$$

$$\overline{u'w'} = {\mathcal{L}_K}^{3/2}{\mathcal{L}_\varepsilon}^{1/2}\left(\frac{\partial \overline{u}}{\partial z}\right)^2 \qquad (7)$$

On the other hand, the similarity theory gives

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$$E = \alpha {u_*}^2 = \alpha \kappa^2 z^2 \left(\frac{\partial \overline{u}}{\partial z}\right)^2 \quad (8)$$

$$\overline{u'w'} = (\kappa z)^2 \left(\frac{\partial \overline{u}}{\partial z}\right)^2 \quad (9)$$

where ${u_*}^2 = \overline{u'w'}|_{sfc}$, Observational data suggests α ranges from 3.75 to 5.47.

So far, the equations for the subgrid scheme (6, 7) and for the similarity laws (8, 9) have been derived separately. Now imposing that both are valid in the surface layer, the two following equations are obtained:

$$\mathcal{L}_K = \frac{1}{\sqrt{\alpha}} \kappa z \;\; ; \; \mathcal{L}_\varepsilon = \alpha^{\frac{3}{2}} \kappa z \quad (10)$$

Assuming that $L = \kappa z$, it can be finally written:

$$C_\varepsilon|_{sfc} = \frac{1}{\alpha^{\frac{3}{2}}} \;\; ; \; C_K|_{sfc} = \frac{1}{\sqrt{\alpha}} \quad (11)$$

For $\alpha = 3.75$ (5.47), we should have for the neutral surface layer:

$$C_\varepsilon|_{sfc} = 0.137 \; (0.078) \quad (12)$$

$$C_K|_{sfc} = 0.516 \; (0.428) \quad (13)$$

In fact, the constants in the subgrid turbulence schemes are kept to their free-stream values (Eq. 5) as discussed above. Commonly used values of C_ε and C_K are thus 0.7 and 0.066, respectively. In the following, C_K and C_ε are used exclusively for these free-stream values.

For cases where the primary balance is between shear production and dissipation, these free-stream values will imply in the surface layer a value of the turbulent kinetic energy which is too small and a vertical wind shear which is too large. It is important to notice that the problem did not originate from a bad choice of constants but from the use in the surface layer of a subgrid scheme derived for free-stream turbulence. Two main hypotheses, used to derive the subgrid fluxes given by Eqs. 6 and 7 from the full second order moment equations, seem to fail in the surface layer: turbulence is isotropic and the mesh size lays inside the inertial range of the energy spectrum.

These remarks and the simple derivation above gives a first simple explanation as to why the constant C_K used in GCM and mesoscale models is generally set up empirically to values larger than 0.066 (value as large as 0.5). For LES models, two types of solutions to this problem have been proposed. Firstly, larger values of constants $C_\varepsilon|_{sfc}$ and $C_K|_{sfc}$ can be empirically specified. The drawback is that these values are no longer adapted to free-stream turbulence where they should be determined as described above. Secondly, we can make the subgrid turbulence scheme more complex by introducing an anisotropic term which will become larger near the surface. This last method has shown to be efficient in improving the simulation of the surface layer in LES. The present study seeks a more general method, simple to implement in any models and suitable for LES models as well as mesoscale models and GCMs.

2.2 Spectral view

Away from the ground, one usually considers the three-dimensional spectrum $E(k)$, which represents the turbulent kinetic energy, within the range of wave numbers $[k, k + dk]$. Close to the ground, one cannot consider the turbulence to be nearly isotropic, and therefore one must be more careful with the definitions. Writing u_i for the fluctuating velocity field, one can define $R_{ij}(r,z) =< u_i(x,y,z)u_j(x+r,y,z) >$ where $< . >$ is the ensemble average. Then, we denote by $E_{ij}(k_1, z)$ the Fourier transform of R_{ij} with respect to r and $E(k_1,z) = \frac{1}{2}(E_{11}(k_1,z) + E_{22}(k_1,z) + E_{33}(k_1,z))$. Obviously, the turbulent kinetic energy at the height z is given by $\int_{-\infty}^{+\infty} E(k_1,z)dk_1$.

Some recent measurements have shown that these velocity spectra change dramatically close to the ground (e.g. Kim & Adrian 1999, Fuehrer & Friehe 1999) in the following way : for $k_1 > 2\pi/z$, E_{11}, E_{22} and E_{33} behave in the same ways as in homogeneous turbulence, but for $2\pi/\Lambda < k_1 < 2\pi/z$, where Λ is a very large length scale (up to 12 times the boundary layer height or the pipe radius in pipe experiments), E_{11} and E_{22} have a self similar behaviour in k_1^{-1} while E_{33} is roughly flat (Fig. 1). This behaviour is analysed from a theoretical point of view in Hunt & Morrison (2000) and Hunt & Carlotti (2001).

This gives the following approximate behaviour:

$$E(k_1) = \begin{cases} \frac{3}{10}\alpha_3 \varepsilon^{2/3} k_1^{-5/3} \text{if } \; k_1 > 2\pi/z \\ \frac{6}{55}\alpha_3 \varepsilon^{2/3} \frac{z^{2/3}}{(2\pi)^{2/3}} [\frac{21}{12}\frac{1}{k_1} + \frac{z}{2\pi}] \\ \text{if } \; 2\pi/z > k_1 > 2\pi/\Lambda \end{cases} \quad (14)$$

with $\varepsilon(z) = u_*^3/(\kappa z)$. In homogeneous isotropic turbulence, one simply has $E_{hom} = \frac{3}{10}\alpha_3 \varepsilon^{2/3} k_1^{-5/3}$ for all k.

This form of the spectra produces a deficit of turbulent kinetic energy for a given dissipation, as indicated by the shaded area in Figure 2.

In the homogeneous isotropic turbulence case, on can compute E_{hom} the subgrid kinetic energy by:

$$E_{hom} = \int_{k_H}^{\infty} E_{hom}(k)dk = \frac{3\alpha_3}{2}\varepsilon^{2/3} k_H^{-2/3} \quad (15)$$

Considering as the cut off wave number of the filter is $k_H = \pi/L$, one get

$$\mathcal{L}_\varepsilon = \frac{L}{C_\varepsilon} \quad \text{with} \quad C_\varepsilon = \pi\Big(\frac{2}{3\alpha_3}\Big)^{3/2}. \qquad (16)$$

In the case of turbulence blocked by the ground, having a behaviour as shown in Fig. 1, a completely rigourous argument is not possible any more, because there is no clear definition of what the large eddies are. However, as it can be seen in Fig. 2, for a given dissipation, there is a deficit of turbulent kinetic energy in the case of wall-bounded turbulence. Define

$$\Xi_\varepsilon = \frac{\int_{k_H}^{\infty} E_{hom}(k_1)dk_1}{\int_{k_H}^{\infty} E_{bl}(k_1)dk_1}, \qquad (17)$$

where E_{bl} is given by Equation 14. We claim that Ξ_ε, which is defined with respect to the streamwise direction, is a good measure of the global deficit of turbulent kinetic energy : the turbulent kinetic energy of the actual flow is equal to E_{hom} (given by Eq. 15) divided by Ξ_ε.

$$E = \frac{E_{hom}}{\Xi_\varepsilon} \quad \Rightarrow \quad \varepsilon = C_\varepsilon \frac{E^{3/2}}{L/\Xi_\varepsilon^{3/2}} \qquad (18)$$

It is easy to calculate Ξ_ε and then using Eqs 18 to get :

$$\varepsilon = C_\varepsilon \frac{E^{3/2}}{A_\varepsilon z} \ ; \ A_\varepsilon = \frac{1}{2}\Big(\frac{7}{22}\ln Z_* + \frac{2}{11}(1 - Z_*) + 1\Big)^{3/2} \qquad (19)$$

where $Z_* = \frac{2L}{z}$ Evaluating $\mathcal{L}_K$ is even less easy to do rigourously. Computations (see details in RMC2001) lead to the following expressions:

$$\mathcal{L}_K = C_K A_K z \qquad (20)$$

$$A_K = \frac{7 + (8/3)Z_*}{33\sqrt{(7/22)\ln Z_* + (2/11)(1 - Z_*) + 1}}\Big(Z_*\Big)^{2/3} \qquad (21)$$

From these derivation, it can be viewed that A_K and A_ε tend to large values when $z/L \to 0$. Note that the models where it is assumed that $\mathcal{L}_K/C_K = C_\varepsilon \mathcal{L}_\varepsilon = Min(L, \kappa z)$, are discarded by the present analysis. This corresponds to assuming that $A_\varepsilon = A_K = 0.4$ close to the ground, which is far too small. Our new values do not violate any physical consideration, since the Karman constant arises from consideration of a single length scale close to the ground, which is not correct in choosing the subgrid model of LES, where the filter characteristic length is relevant in all the domain of computation. These calculations having been made using many approximations, especially for the blocked case, we can only be confident in the predicted values of A_ε and A_K, up to a multiplicative coefficient of order one. On a more theoretical point of view, the main result of this spectral analysis is to show how the energy deficit of blocked turbulence for a given dissipation (this can be called 'anomalous dissipation') has a dramatic effect on the coefficients to be used in models close to the ground, causing A_ε and A_K to become large ($A_K \propto z^{-2/3}$, $A_\varepsilon \propto (\ln 1/z)^{3/2}$).

3. SOLUTION

The previous spectral analysis based on the existence of a k_1^{-1} range in the energy spectrum indicates that the subgrid scale lengths should be thus taken larger than the commonly used Prandtl length κ z. An approach is here proposed to match the usual subgrid turbulence scheme and the similarity laws through the use of adequate expressions of subgrid scale lengths near the surface. This approach is in agreement with the physical analysis given above and contrasts with the empirical modification of Smagorinsky constant as suggested by other authors.

The spectral calculations of Section 2.2 showed the following behaviour of the length scales :

$$L_\varepsilon = A_\varepsilon z \ ; \ L_K = A_K z, \qquad (22)$$

where $\mathcal{L}_K = C_K L_K$ and $\mathcal{L}_\varepsilon = \frac{L_\varepsilon}{C_\varepsilon}$, C_K and C_ε being kept equal to their free-stream values (Eq. 5).

Using the results of Section 2.1, the similarity laws (Eqs. 8 and 9), are exactly derived from the subgrid scheme by using the same value of constants C_ε and C_K both in the surface layer and the free-stress layer. The constants A_ε and A_K are then given by:

$$A_\varepsilon = \alpha^{3/2}\kappa C_\varepsilon \ ; \ A_K = \frac{1}{\alpha^{1/2}}\frac{\kappa}{C_K} \qquad (23)$$

With $\alpha = 3.75$, A_ε and A_K are equal to 2.03 and 3.13 respectively. For $\alpha = 4.63$, we have $A_\varepsilon = A_K = 2.79$ (which is of the same order as the estimates given from the spectral considerations in Section 2.2). This value of α is close to the evaluation of various observational estimates and is in particular close to the value of 4.75 given in Stull (1988). Thus it seems reasonable to use this value as the proposed solution leading to a single value of subgrid scale length $L = A_K z = A_\varepsilon z$ near the surface (this greatly simplifies the adaptation of preexisting models to our new formulation).

This solution can be easily generalized (RMC2001) in considering stability functions. This solution is very simple to implement in a model, as opposed to other solutions previously proposed in the literature.

Stability functions for momentum, temperature and turbulent kinetic energy as deduced from this new scheme can be compared to the Monin-Obukov profiles as well as to the standard subgrid scheme with subgrid scale lengths equal to κz or the mesh size Δ as usual in LES models. Fig. 3 clearly show the improvements brought by the new scheme over the usual methods. The use of κz for subgrid scale lengths leads to an overestimate of the values of the vertical gradients of momentum and potential temperature and the dissipation. The use of the mesh size leads to better results for unstable conditions, though still overestimating the momentum gradient and the dissipation by a factor of around 2.

4. CONCLUSION

The present work provides a physical explanation and a general solution suitable for any atmospheric models, including inhomogeneous surface conditions, complex topography and any vertical stability. The energy deficit of blocked turbulence for a given dissipation, has been shown to have a dramatic effect on the mixing and dissipation lengths to be used in subgrid models close to the ground. To take into account this 'anomalous dissipation', modifications in the standard subgrid schemes derived for free-stream turbulence have been proposed. In particular, it is argued that subgrid scale lengths equal to κz should not be used. These modifications are simple to implement in models and are physically justified by recent measurements of spectra close to the ground. This method is also easily applicable to mesoscale and large-scale models.

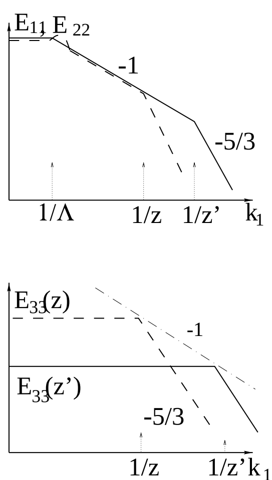

Figure 1: Sketch of the measured one-dimensional spectra in atmosphere and pipe very close to the boundary (cf. Hunt & Carlotti (2001))

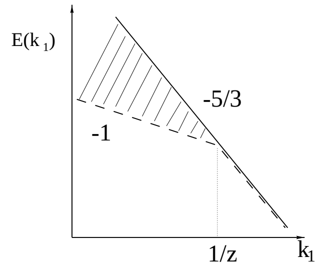

Figure 2: Illustration of the anomalous dissipation close to a surface : solid line (——) : 1D kinetic energy spectrum for isotropic turbulence for a given dissipation ; dashed line (– – –) : one-dimensional kinetic energy for wall blocked turbulence and the same dissipation.

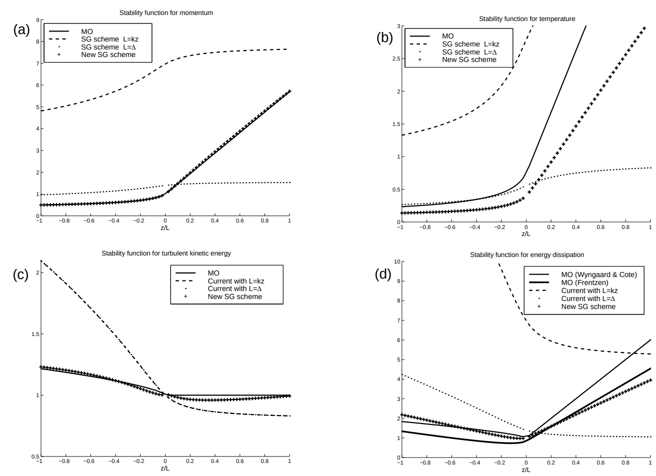

Figure 3: Theoritical profiles of stability function as function of z/L_{MO} for u (a), θ (b), E (c) and ε/E(d), respectively: Monin-Obukov similarity law (solid line), standard subgrid turbulence scheme at the first half-level of model with L= κz (dashed line) and L=Δ_z (dotted line) and new scheme (plus). L_{MO} stands for the Monin-Obukov length

References

Andreas, E.L., 1987: On the Kolmogorov constants for the temperature-humidity cospectrum and the refractive index spectrum, *J. Atmos. Sci.*,**44**, 2399-2406.

Cuxart, J., Bougeault Ph., and Redelsperger, J.L., 2000: A multiscale turbulence scheme apt for LES and mesoscale modelling, *Quart. J. Roy. Meteorol. Soc.*, **126**, 1-30.

Fuerher, P.L. and Friehe, C.A., 1999: A physically based turbulent velocity time series decomposition, *Boundary Layer Meteorology*, **90**, 241-295.

Hunt, J.C.R. and Carlotti, P., 2001: Statistical structure of the high Reynolds number boundary layer, *Flow, Turbulence, Combustion*, 66, 453-475.

Hunt, J.C.R. and Morrison, J.F., 2000: Eddy structure in turbulent boundary layers, *European Journal of Mechanics*, B Fluids, **19**, 673-694.

Kim, K.C. and Adrian, R.J., 1999: Very large scale motion in the outer layer, *Physics of Fluids*, **11(2)**, 417422.

Mason, P. J. and Thomson, D. J., 1992: Stochastic backscatter in large-eddy simulations of boundary layers. *J. Fluid Mech.*, **242**, 51–78.

Redelsperger, J.L., F. Mahé, and P. Carlotti, 2001: A simple and general subgrid model suitable both for surface layer and free-stream turbulence. Bound. Layer Meteor., 375-408.

Stull, R., 1988: *An Introduction to Boundary Layer Meteorology.* Kluwer Academic Publishers, Dordrecht, 637 pp.

Sullivan, P., McWilliams, J. & Moeng, C.H., 1994: A subgrid-scale model for large-eddy simulation of planetary boundary-layer flows, *Boundary-Layer Meteorol.*, **71**, 247-276.

Troen, I. and L. Mahrt, L., 1986: A simple model of the atmospheric boundary-layer: Sensitivity to surface evaporation, *Bound. Layer Meteor.*, **37**, 129–148.