

## 13.4 AN INTEGRAL MIXING LENGTH SCALE FORMULATION FOR A TKE-L TURBULENCE CLOSURE IN ATMOSPHERIC MODELS.

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### 1 Introduction

The TKE-1 scheme uses a prognostic equation of Turbulent Kinetic Energy (TKE or  $E$ ) combined with a diagnostic length scale  $l_{m,h}$  to compute the eddy diffusivities:

$$K_{m,h} = l_{m,h} \sqrt{E}.$$

There is much debate on the length scale  $l_{m,h}$ . In many schemes the length scale is composed from a basic length scale (mainly as a function of height  $z$ ) and a correction based on local stability [see e.g. the length scale used in ECHAM4 as discussed in Roeckner et al. (1996)]. For a convective boundary layer this seems to do no justice to the nonlocal nature of the turbulence. Moreover this approach also gives rise to numerical instabilities [Lenderink et al. (2000) and <http://www.knmi.nl/samenw/eurocs/> under diurnal cycle of Cu clouds].

A natural way to incorporate nonlocal stability into the length scale formulation is proposed by Bougeault and Lacarrère (1989). In this method, the length scale is computed from distance which an upward (and downward) adiabatic parcel can travel before being stopped at a level where it has lost all its kinetic energy by buoyancy effects. In this way, the stability of a whole layer is incorporated into the length scale. This method is attractive since it is based on the simple physical concept that the major part of transport is done by the largest eddies. The scheme works well for convective boundary layers. It however seems to have problems in performing a realistic transition from a neutral to a convective boundary layer. To illustrate this consider Fig. 1, where a convective boundary layer is shown, capped by a strong inversion. What happens if in this situation the surface heat flux slowly decreases to zero? Or, in other words, what happens when the convective BL slowly evolves into a neutral BL? The answer is that, as long as the profile is not stable, the upward parcels will keep on rising until they hit the inversion, and the downward parcel will go down to the ground. So, basically the Bougeault and Lacarrère (1989) length scale (hereafter B&L length scale) remains unchanged, irrespective of local stability. A

similar argument holds when, with the same buoyancy forcing, the wind increases. When applied to the operational limited area model HIRLAM problems to obtain a realistic wind profiles under neutral conditions arose (Lenderink and de Rooy, 2000). The apparent reason for that was that, when tuned to convective conditions, the scheme did too much mixing for neutral conditions.

Here we will present a length scale formulation which may serve as an in-between (between local and non-local); it uses a two-stream approach with two length scales (started from top and bottom of the mixing domain) defined by vertical integrals over stability.

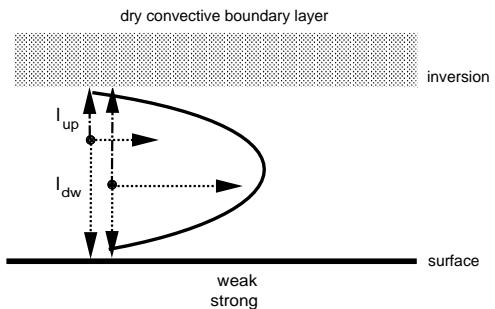


Figure 1: Illustration of the Bougeault and Lacarrère (1989) length scale. At each height a adiabatic parcel is released. Parcels rise (sink) until their kinetic energy is consumed by buoyancy.

### 2 Integral Length Scale

The basic idea of the new length scale formulation is shortly presented here; details can be found in Lenderink (2002). The length scale is denoted  $l_{int}$  which reminds us that it has been computed from integrals over stability. The formulation is designed to represent mixing in the range from close to neutral (and stable up to  $Ri < 0.1 - 0.2$ ) to unstable conditions.

The integral length scale  $l_{int}$  is computed from an “averaging” over two length scales  $l_{up}$  and  $l_{dw}$  by

$$\frac{1}{l_{int}} = \frac{1}{l_{up}} + \frac{1}{l_{dw}}$$

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These two length scale are defined as integrals over stability by:

$$l_{up}(z) = \int_{z_{bottom}}^z F(Ri) dz'$$

$$l_{dw}(z) = \int_z^{z_{top}} F(Ri) dz'$$

where  $F(Ri)$  is a function of the Richardson number that will be defined below, and  $z_{bottom}$  and  $z_{top}$  are the lower and upper boundary of the mixing domain.

First let us inspect how the length scale is designed to behave. We illustrated the behavior of the length scale in Fig. 2 in the (simplest) case of a convective boundary layer capped by a strong inversion. In this case  $z_{top}$  is the inversion height,  $z_{bottom}$  the surface. If we assume that  $F$  is a constant,  $l_{up}$  and  $l_{dw}$  are proportional to the distance to the inversion and surface, respectively. If it is a strong inversion, the length scale is now very similar to the B&L length scale because in that case all upward parcels will just above the base of the inversion and the downward parcels will hit the surface. In this respect our method can be considered as a “poor man’s” parcel method, obtaining rather similar results for convective situations to B&L, though at a much lower computational cost.

Next, we take  $F$  as a function of the Richardson number  $Ri$ . In Fig. 2 two situations are shown, one strongly convective representative for a midday situation (thick lines), one only weakly convective (thin lines) representative for the late afternoon. In these two situations, the behavior of  $l_{up}$ ,  $l_{dw}$  and  $l_{int}$  is shown. So roughly,  $F$  is chosen as an increasing function of instability; for the near neutral conditions at late afternoon  $F$  should represent neutral scaling, whereas for more convective conditions  $F$  is chosen larger.

We assume the follow form of  $F$ :

$$\begin{aligned} F_m(Ri) &= \alpha_n - \frac{2}{\pi}(\alpha_c - \alpha_n)(\alpha_r Ri), & Ri > 0 \\ &= \alpha_n - \frac{2}{\pi}(\alpha_c - \alpha_n) \arctan(\alpha_r Ri), & Ri \leq 0 \end{aligned}$$

In this equation  $\alpha_n$  determines the **neutral** scaling,  $\frac{2}{\pi}(\alpha_c - \alpha_n)\alpha_r$  the stability dependency near neutral and  $F_m$  is limited by  $\alpha_c$  for **convective** conditions.

The constants  $\alpha_n$  and  $\alpha_r$  are found by a matching argument of  $l_{up}$  (which determines the behavior of  $l_{int}$  close to surface) to a linearized form of the flux profile relations close to neutral conditions. This is not a straightforward matching [like e.g., ECHAM4 turbulence is matched to Louis (1979)] since an integral over  $Ri$  occurs in the definition in  $l_{up}$ , but it can be done (approximately) assuming (based on flux profile relations) a linear dependency of  $Ri$  with  $z$  (again

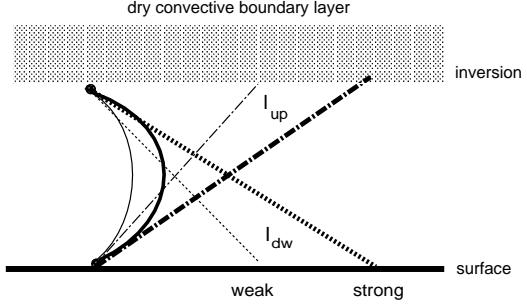


Figure 2: Illustration of the computation of the new length scale. Two “parcel” are released: one starts at the inversion, the other at the surface. The length scale increases as a function of the stability. Thick lines corresponds to a convective, thin lines to a more neutral situation.

for close to neutral conditions). (Figure 6 shows that, despite linearization near neutral, results are favorable over a rather large stability range.)

Finally, for very stable conditions and away from the surface the length scale is matched with a local buoyancy based length scale  $c\sqrt{E}/N$  with  $N^2$  the Brunt-Vaisala frequency.

### 3 Single Column Model results

The model was run for a dry convective boundary layer with shear. The model was started from a stable boundary layer at night, and integrated with prescribed surface fluxes of heat and moisture and with a prescribed surface roughness. The case was based on the GCSS WG-1 ARM case (Brown and co authors, 2002), but with clouds removed by lowering the specific humidity profile.

The time evolution of the potential temperature and the corresponding evolution of the eddy diffusivity  $K_h$  is plotted in Fig. 3, showing the growth of a convective layer from the surface (at 12 UTC = 6 local time) to about 1500 m at late afternoon. It shows that the  $E$ - $l$  model behaves very continuous, both in time and in space. The results, profiles and entrainment rates, are rather close to the LES results for this case [comparison shown in Lenderink (2002)].

### 4 Comparison to field data

The turbulence scheme is incorporated a high resolution (horizontally 18 km, vertical levels at 10, 35,

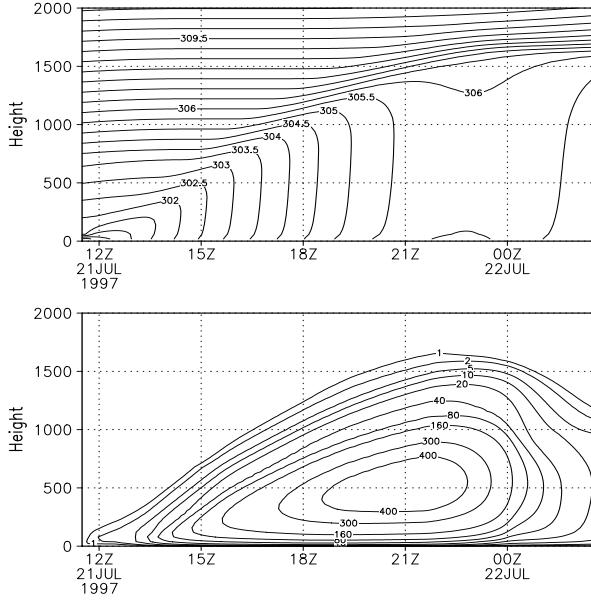


Figure 3: Time evolution of  $\theta$  (upper panel) and  $K_h$  (lower panel) for diurnal cycle of dry convective boundary layer with shear.

70, 125, 190 m) version of the KNMI limited area model RACMO. Results are compared to measurement at the 200m Cabauw tower (the Netherlands). Results are presented for februari 2001.

Results of the 10 m wind (in Fig. 4) show reasonable agreement between model predictions and measurements. At high wind speeds, however, there seems to be a level off in the model predictions. For reference, results obtained with the ECHAM4 turbulence scheme (similar to Louis (1979) close to surface) show very similar behavior.

As a more demanding measure of the model behavior we show in Fig. 5 the ratio between the 10 m wind and the 200 m wind ( $f_{10}/f_{200}$ ) as a function of the bulk Richardson number. Under neutral conditions and with a surface roughness of 0.05-0.30 m, this ratio is 0.55-0.60. This ratio is plotted for the measurements as well as the model output. Results of the model are reasonable, though the variations as a function stability in this ratio are somewhat moderate compared to the observations.

Finally, we plotted the flux profile relations  $\phi_m = \kappa z / u^* \partial U / \partial z$  computed from the MODEL output at every 10 min. It shows that the model surprisingly well captures the behavior for weakly stable conditions as predicted by the usual flux profile relation  $1 + 5z/L$ . Note also that a large spread is inevitable because at a 10 min level there is no (quasi-) stationary state. Further analysis showed that the behavior

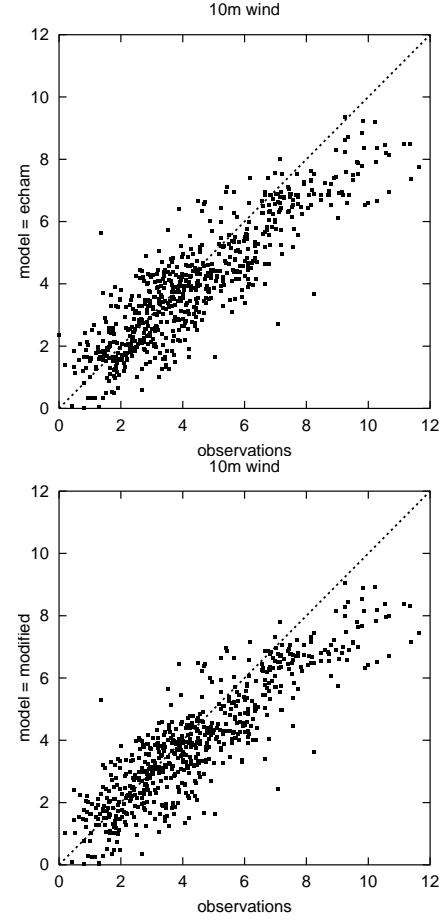


Figure 4: Comparison between observations of the 10 m wind at the Cabauw tower and model predictions. Upper panel, results of ECHAM4 turbulence scheme, lower panel results of the new turbulence scheme.

for weakly stable conditions is indeed governed by the integral length scale. The leveling off for higher stability is due to the stable, buoyancy based, length scale formulation.

## 5 Conclusion

We presented a new length scale formulation to be used in operational atmospheric models. In contrast to most length scale formulations, stability enters the length scale formulation in a non-local, vertically integrated sense, instead of only/mainly local [as e.g. in the Louis (1979) scheme]. This improves the stability characteristics of the scheme, mainly for convective situations.

Results in a single column model show good (numerically stable and physically reasonable) behavior for a convective boundary layer. In this case, the

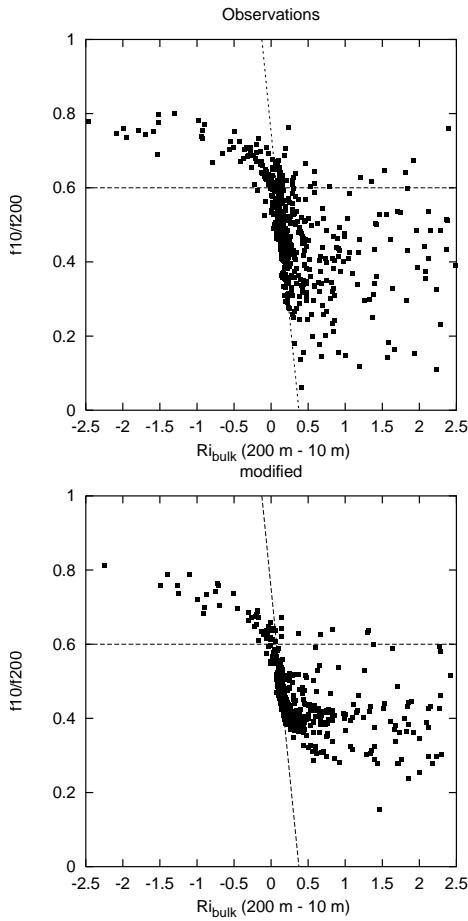


Figure 5: Ratio  $f_{10}/f_{200}$  as a function of the bulk  $R_i$  (200 m - 10 m) in the observations (upper panel) and the mode output (lower panel).

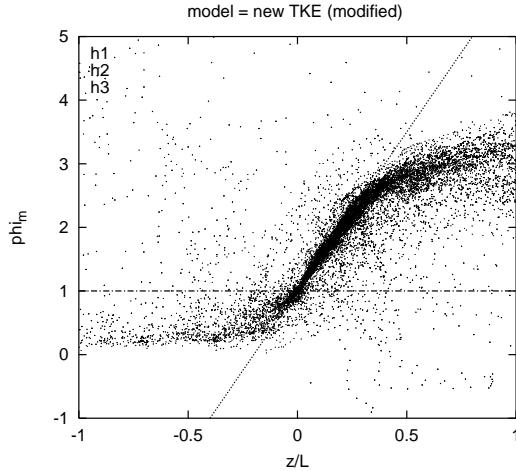


Figure 6: Flux profile relations  $\phi_m$  computed from the model output (at the 4 lowest model levels). The line corresponds to  $1 + 5 z/L$ .

length scale mimics the behavior of the Bougeault and Lacarrère (1989) length scale, though at a lower computational cost. Results in a limited area model showed a realistic behavior (that is, e.g., satisfying flux profile relations) near the surface. The scheme can also be well extended to moist, cloudy conditions [some results already shown in Lenderink and Siebesma (2000)].

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