P1.13

SELF-SIMILARITY CONSTRAINTS FOR CONVECTIVE BOUNDARY LAYERS

Jianjun Duan* Bjorn Stevens

University of California Los Angeles, Los Angeles, California

1. INTRODUCTION

The study is inspired by the idea that the convective boundary layer (CBL) driven by the surface heat flux Q_0 should under certain conditions, grow self-similarly. The CBL with depth h can be interpreted with a two-layer-model: the lower layer is the relatively mixed layer ranging from the surface to the height h_m, the upper one is the interfacial layer with the depth of $\Delta h = h - h_m$.

At the initialization, a uniform lapse rate $\Gamma = \frac{\partial\theta}{\partial z}$ is specified throughout the flow. With the constant surface heat flux Q_0, a CBL develops and grows with time into the capping inversion, while it is always topped by the stable stratification Γ. Our LES experiment shows such a growth of CBL is self-similar, as in Fig. 1. By using "self-similar", we mean that the shape of the θ profile does not depend on the time provided Γ and Q_0 are constants. That is, the profiles at different moments are non-dimensionally equivalent.

Horizontal homogeneity is assumed. The molecular diffusion is always insignificant and is neglected. The first law becomes

$$\frac{\partial\theta}{\partial t} = -\frac{\partial Q}{\partial z}. \tag{1}$$

2. ANALYTIC INTERPRETATION WITH MIXED LAYER ASSUMPTION

In the limit of strong capping inversion,

$$\Delta h \ll h. \tag{2}$$

So the inversion can be approximately treated as a jump of $\Delta\theta$ across h. With mixed layer assumption,

$$\Delta\theta = \theta_t - \theta_m. \tag{3}$$

where θ_m is θ in the mixed layer and θ_t is θ at the top of CBL. In our problem, $\theta_t = \theta_0 + \Gamma h$, θ_0 is the initial surface value of θ.

With the initial conditions

$$\Delta\theta = 0, \tag{4}$$

$$h = 0, \tag{5}$$

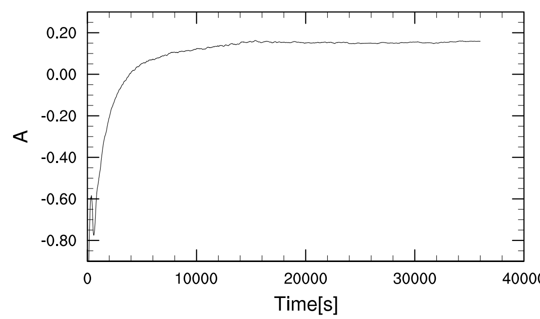

Figure 1: Time series of $A = \frac{\Delta\theta}{h\Gamma}$ in LES experiment.

we have the solution as

$$\Delta\theta = \sqrt{\frac{2\Gamma Q_0 t}{b^2 - 1}}, \tag{6}$$

$$h = \frac{1+b}{\Gamma}\Delta\theta, \tag{7}$$

where $b = 1 - \frac{1}{\alpha}$. $1 - \alpha$ is the nondimensional average heating rate in the mixed layer. Because $\frac{\Delta\theta}{h} = \frac{\Gamma}{1+b}$, the growth is self-similar.

3. SELF-SIMILARITY SCALING WITH FINITE-DEPTH INTERFACIAL LAYER

In the previous section we had an analytic solution for self-similarly growing CBL without resolving the structure of the interfacial layer. Here it is assumed that there exist self-similar profiles for θ and Q in the CBL:

$$Q = Q_0 f(\sigma), \tag{8}$$

$$\theta = \theta_H + A\Gamma H\phi(\sigma), \tag{9}$$

where

$$\sigma = \frac{z}{H} \tag{10}$$

and $\phi(\sigma)$ and $f(\sigma)$ are the self-similar nondimensional profiles of θ and Q respectively. Note we have had self-similarity built in when scaling θ. $A = \frac{\Delta\theta}{h\Gamma}$ is a nondimensional constant. H is a length scale, which for reasons we elaborate on further below, is chosen such that $H \geq h$. θ_H is θ at $z = H$. The coordinate transformation

$$(t, z) \longrightarrow (H, \sigma) \tag{11}$$

* *Corresponding author address:* Jianjun Duan, Dept. of Atm. Sciences, 405 Hilgard Ave., Box 951565, Los Angeles CA 90095-1565; e-mail: jjduan@atmos.ucla.edu

allows us to take advantage of the above self-similar nondimensional profiles. We have

$$\frac{\partial}{\partial t} = \frac{dH}{dt}\frac{\partial}{\partial H} - \frac{\sigma}{H}\frac{dH}{dt}\frac{\partial}{\partial \sigma}, \tag{12}$$

$$\frac{\partial}{\partial z} = \frac{1}{H}\frac{\partial}{\partial \sigma}. \tag{13}$$

Using (8), (9), (12) and (13), (1) can be rewritten as

$$\left\{\frac{1}{A} + \phi(\sigma) - \sigma\frac{d\phi}{d\sigma}\right\} = -C\frac{df}{d\sigma}. \tag{14}$$

where $C = \frac{Q_0}{A\Gamma H \frac{dH}{dt}}$ is proportional to the heat capacity of the whole PBL and its value is decided by the heat budget. Because for $\frac{d\phi}{d\sigma} = 0$, (14) implies that f is linear with σ, (14) is consistent with mixed layer model. Equation (14) relates the profile of θ with that of Q in the case of self-similar growth. An integration yields

$$\begin{aligned} f(\sigma) =& \frac{1}{C}\left\{\sigma\left(\phi(\sigma) - \frac{1}{A}\right) - 2\int_{\sigma_0}^{\sigma}\phi(\sigma')d\sigma'\right\} \\ & - \frac{\sigma_0}{C}\left(\phi(\sigma_0) - \frac{1}{A}\right) + f(\sigma_0). \end{aligned} \tag{15}$$

where σ_0 is a reference value.

The parameters in (14) can be derived with the assumption of finite depth interfacial layer. Define

$$s = \int_m^1 \phi d\sigma \tag{16}$$

where $m = \frac{h_m}{H}$. We have

$$A = \frac{\Delta\theta}{\Gamma H} = \frac{\alpha}{2(1-\alpha)(s-m)+1}, \tag{17}$$

$$C = \frac{1}{A} + 2(s-m) = \frac{2(s-m)+1}{\alpha}. \tag{18}$$

We have done two kinds of numerical experiments which support our scaling. One is to prescribe a nondimensional flux profile (based on (14)) and use it to force the CBL. It grows self-similarly. The other is to calculate the flux (or the heating rate) using (14) at real time locally and use it to force the CBL. This approach is convergent and the result seems decent.

It needs to point out that within the second approach, we even don't need an accurate boundary layer depth estimation, provided that the value is bigger than the actual one. Suppose H is the estimated boundary layer depth and $H > h$. θ is linear function of z around $z = H$, so $\theta(z) = \theta(H) + \Gamma(z - H)$. Nondimensionlizaing it yields $\phi(\sigma) = \frac{1}{A}(\sigma - 1)$ (note that $\phi(1) = 0$). With this relation, using (14) we see $\frac{df}{d\sigma} = 0$. So even with estimated depth greater than the actual one, we still get no flux above the actual CBL top. This property implies a smooth matching across the CBL top for the flux profile.

4. KPP ISSUES

K Profile Parametrization (KPP)(Large et al. (1994)) method is widely used. Here we want to discuss KPP within this self-similarity framework. Substitute the flux with $-K\left(\frac{d\phi}{d\sigma} - \gamma\right)$ (Deardorff (1966)) in (14), we have

$$\left\{\frac{1}{A} + \phi(\sigma) - \sigma\frac{d\phi}{d\sigma}\right\} = C\frac{d}{d\sigma}\left[K\left(\frac{d\phi}{d\sigma} - \gamma\right)\right]. \tag{19}$$

where K is the turbulent diffusivity and γ is the nolocal term, which is assumed to be constant for the moment.

The usual practice(Large et al. (1994); Stevens (2000)) is

$$K = k\sigma(1-\sigma)^2 \tag{20}$$

where k is a constant. With such a K, $\sigma = 0$ and $\sigma = 1$ are singular points of (19). Stevens (2000) argued that the index should be smaller than 2.

Here we assume that

$$K = k\sigma(1-\sigma)^a \tag{21}$$

to discuss the issues of a.

Instead of solving (19), we study the behavior of this equation at neutral points (Stevens (2000)). By definition, at neutral points, $\frac{d\phi}{d\sigma} = 0$. Here we focus on the neutral point where ϕ achieve its minimum, so $\phi = -1$. We have

$$1 - \frac{1}{A} = kC(1-\sigma)^{a-1}[(1-\sigma-a\sigma)\gamma - \sigma(1-\sigma)\frac{d^2\phi}{d\sigma^2}]. \tag{22}$$

For a realistic ϕ profile, the term $\sigma(1-\sigma)\frac{d^2\phi}{d\sigma^2}$ is assumed to be negligible. So we finally get

$$1 - \frac{1}{A} = k\gamma C(1-\sigma)^{a-1}(1-\sigma-a\sigma). \tag{23}$$

Equation (23) relates the neutral height σ, the index a and nonlocal term γ together. we also see k and γ play the role together in the form of $k\gamma$. This is consistent with Stevens (2000).

Since (23) may have more than one solution for σ, while the realistic case only permits one solution, it is assumed here that the desired σ leads to a peak value of the right-hand-side of (23), which implies $\sigma = \frac{2}{a+1}$.

References

Deardorff, J. W., 1966: The counter-gradient heat flux in the lower atmosphere and in the laboratory. *J. Atmos. Sci.*, **23**, 503–506.

Large, W. G., McWilliams, and S. C. Doney, 1994: Oceanic vertical mixing: A review and a model with a nonlocal boundary layer parameterization. *Rev. Geophys.*, **32**, 363–403.

Stevens, B., 2000: Quasi-steady analysis of a PBL model with an eddy-diffusivity profile and non-local fluxes. *Mon. Wea. Rev.*, **128**, 824–836.