9.2 ON REASONS FOR THE OBSERVED VARIATION OF THE VON KARMAN CONSTANT IN THE ATMOSPHERIC SURFACE LAYER

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Evidence obtained in recent years by experimental studies of the atmospheric surface layer indicates that the von Karman scaling factor k probably does decrease with the increasing intensity of turbulence, at least over the range of Reynolds numbers observed. Our brief review (Frenzen and Vogel, 1995) surveys some of these results. Further, Oncley et al. (1990; 1996) have presented a semi-log plot of a large number of k values obtained by two other field experiments in addition to their own, a plot which both showed the declining trend in k and suggested that the variation could be represented as an inverse function of the roughness Reynolds number, $Re_0 = u_* z_0/\nu$. A particular function of this form was then determined by Frenzen and Vogel who based it on the maximum of $k = 0.41$ observed at very small Re_0 and an average of $k = 0.39$ determined by 29 normally-distributed values measured at moderate Re_0 in two of their field experiments. The resulting relation was shown to suggest that k varies in the surface layer by only ±5% around a central value of 0.39. To date no plausible explanation for this variation has been given.

One of us (Frenzen, 2002) has recently shown that the observed variation of k in the surface layer can be traced to changes with increasing Reynolds number that are made by two features of the turbulence field: 1) a decrease in the rate at which the momentum transfer increases with increasing turbulence which is caused by the increasing range of isotropy in the developing turbulence spectrum, and 2) changes in the magnitude of the dissipation deficit in the TKE budget. To illustrate how these developments affect the magnitude of k, consider the influence of changing Reynolds number on an expanded version of the definition of the von Karman factor implied by the log profile.

Given the usual form obtained by rearranging the log profile relation,

$$k\,z = u_*/\left(\frac{\partial \overline{U}}{\partial z}\right) \quad , \tag{1}$$

multiplying the numerator and denominator on the right with u_*^2 produces a modified definition expressed in terms of the mechanisms that affect k:

$$k\,z = u_*^3/\left(u_*^2\frac{\partial \overline{U}}{\partial z}\right) = (\tau/\rho)^{3/2}/\left[(\tau/\rho)\,\frac{\partial \overline{U}}{\partial z}\right] \tag{2}$$

The function in the numerator on the far right is a measure of the momentum flux at height z, while the terms in the denominator represent the rate at which TKE is produced by wind shear at the same level; that is, they are equivalent to π_τ in the abbreviated TKE budget equation,

$$\varepsilon = \pi_\tau + \pi_B - D \ . \tag{3}$$

Here ε is the turbulence dissipation rate, π_τ and π_B represent the rates of TKE production by wind shear and bouyancy, and D is the net rate at which TKE is removed from the budget by the sum of the divergent transport terms, the turbulent transport of turbulence and the pressure-velocity correlation. Since the discussion can be limited to the neutral case, $\pi_B = 0$ and (3) can be rewritten as $\pi_\tau = (\varepsilon + D)$. Substitution in (2) then gives

$$k\,z = \frac{(\tau/\rho)^{3/2}}{\pi_\tau} \simeq \frac{(\tau/\rho)^{3/2}}{(\varepsilon + D)} \tag{4}$$

which can be reduced to

$$k\,z = \frac{(\tau/\rho)^{3/2}}{A\,\varepsilon} \tag{5}$$

by introducing the dissipation multiplier $A \equiv (\varepsilon + D)/\varepsilon$ in order to write the total TKE removal as a single term. Since it can be shown that $(A-1)/A = D/(\varepsilon + D)$, the relative amount by which A exceeds unity represents a relative measure of the amount of TKE removed by the divergent transport terms. In other words, $(A-1)/A$ provides a relative measure of the "dissipation deficit", the amount by which the dissipation alone fails to balance the TKE budget.

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Effects of the first mechanism proposed can be isolated from those of the second by assuming that A remains constant. Any change in k must then be due to a change in ratio of (a) the value of the momentum flux function in the numerator of (5), to (b) the measure of the intensity of turbulence in the intertial subrange given, through the Kolmogorov -5/3 relation, by the dissipation rate in the denominator. Although (a) and (b) both increase with Re_0, the fact that the growing intertial subrange is accompanied by a steady increase in the range over which higher frequency fluctuations become isotropic causes (a) to increase more slowly than (b). Because the onset of isotropy causes the horizontal and vertical components of the fluctuations affected to become uncorrelated, the rate at which the integrated covariance (i.e., the momentum flux) increases with increasing Re_0 is reduced. On the other hand, since fluctuation variance is unaffected by isotropy, the rate at which the integrated variance (i.e., the turbulence intensity and hence the dissipation rate) increases with Re_0 remains much the same. Thus, the ratio of (a) to (b) decreases with increasing Re_0, and therefore the magnitude of k decreases as well. Although the effects of the changing momentum flux are raised to the 3/2 power in the numerator of (5), this effect of increasing isotropy is probably small since the fraction of fluctuation frequencies affected remains a relatively small portion of the complete cospectrum.

The principle cause of the observed variation of k is therefore probably the changing magnitude of the dissipation deficit in the TKE budget. Because of the comparitively high rate at which TKE is produced by stronger shear in the layers near a boundary, not all of the turbulence produced in the surface layer dissipates locally. Instead, the budget balance is maintained by the divergent transport terms which transfer the excess TKE, down the TKE gradient, to regions of smaller production somewhere above the surface layer. The existence of a dissipation deficit in the surface layer ensures $A > 1$. Any additional change in the deficit will increase A in (5) and thereby cause a change of the opposite sign in the magnitude of k.

Turbulence data obtained in the 1991 Glencoe experiment (Frenzen and Vogel, 2001), conducted in the Hay plains of central New South Wales, Australia, and in a 1989 field experiment in southeastern Wyoming (Frenzen and Vogel, 1992), were investigated in search of evidence of either effect. However, these data are limited due to the relatively narrow range of Reynolds numbers observed. Further studies are needed over a much wider range of Reynolds numbers in order to detect the relatively subtle changes in the mechanisms that affect k. Ideally, these studies should use the same instrumentation to minimize error. Further, they should be conducted in conditions of fully developed turbulence over a wide variety of roughnesses ranging from the open sea, to extensive forested or urbanized areas.

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