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I. INTRODUCTION

Cloud base height profiles (CBHP) are known to have highly fluctuating and irregular structure. The dynamics of CBHP evolution is determined by a variety of processes in the atmosphere (Garraff, 1992). This irregular structure is a benchmark for non-linear dynamical processes. The type of the correlations of the CBH values and their time dependence are studied applying the Detrended Fluctuation Analysis (DFA) statistical method (Peng, 1994). The scaling properties of the CBHP are studied by the multifractal approach.

In this report we present an analysis of the time dependence of the correlations and the multi-affine properties of a CBHP signal $y(t)$ (Fig.1.a) measured with a ground-based laser (ref to <http://www.arm.gov>) having a temporal resolution of 30 seconds, taken on December 7-12, 1997 at the Southern Great Plains (SGP) (Oklahoma, USA) site of the Atmospheric Radiation Measurement Program of the Department of Energy.

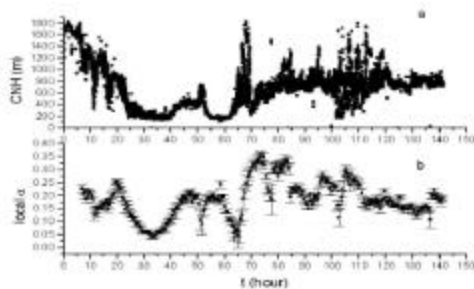


Fig.1 a) Cloud base height profile data measured at SGP site; b) local correlations study

II. DFA METHOD AND THE TIME DEPENDENCE OF THE CORRELATIONS

The DFA technique (Peng, 1994) consists of dividing a random variable sequence $y(n)$ of length N into N/t non-overlapping boxes, each containing t points. Then, the local trend (assumed to be linear in this investigation $z(n)=an+b$) in each box is computed using a linear least-square fit to the data points in that

box. The detrended fluctuation function $F(t)$ is then calculated following

$$F^2(t) = \frac{1}{t} \sum_{n=kt+1}^{(k+1)t} [y(n) - z(n)]^2, k = 0, 1, 2, \dots, \left(\frac{N}{t} - 1\right) \quad (1)$$

Averaging $F^2(t)$ over the intervals gives the fluctuations $\overline{F^2(t)}$ as a function of t . If the $y(n)$ data are random uncorrelated variables or short range correlated variables, the behaviour is expected to be a

power law $\left\langle F^2(t) \right\rangle^{1/2} \propto t^a$.

The DFA exponent a so obtained represents the correlation properties of the signal. For the studied case an a exponent equal to 0.20 ± 0.002 for time lags less than 16 hours are found. The time dependence of the correlations is next discussed. Results are plotted also in the Fig.1.b as a function of the cloud life-time for an observation box $w=7h$ moved along the profile by $\Delta w=30$ min till the end. The results are similar to these ones, found for the marine stratocumulus diurnal evolution (Kitova, 2002): a more stable structure of the boundary layer corresponds to a lower value of the a -indicator, i.e., larger anti-persistence, thus a set of fluctuations, tending to induce a greater stability of the clouds. In contrast, during the periods of higher instability in the planetary boundary, less anti-persistent (more persistent like) behavior of the system drags it out of equilibrium, corresponding to a larger a values.

III. MULTI-AFFINITY AND INTERMITTENCY

First, we tested the scaling properties of the power spectral density $S(f)$ of the CBHP signal and obtained (figure not shown) that $S(f) \sim f^\beta$ with $\beta=1.35 \pm 0.06$ for frequencies lower than $1/15 \text{ min}^{-1}$ and $\beta=0$ for higher frequencies. Similar values for the spectral exponents, e.g. $\beta=1.28 \pm 0.1$ and $\beta=1.49 \pm 0.08$, are obtained in the work of Kitova (2002) and Gospodinova (2001) for cloud base data measurements during the Atmospheric Stratocumulus Transition Experiment (ASTEX).

The multi-affine properties of $y(t)$ can be described by the so-called " q -th" order structure functions (Davis, 1994; Ivanova, 1999; Ivanova and Ausloos, 1999):

$$C_q = \left\{ \left[y(t_{i+r}) - y(t_i) \right]^q \right\} \quad i = 1, 2, \dots, N-r \quad (2)$$

where the averages are taken over all possible pairs of points that are $t = t_{i+r} - t_i$ apart from each other with $r > 0$. Assuming a power law dependence of the structure function, the $H(q)$ spectrum is defined (Davis, 1994) through:

$$C_q(t) \propto t^{qH(q)}, \quad q \geq 0 \quad (3)$$

The *intermittency* of the signal can be studied through the so-called singular measure analysis. The first step that this technique requires is defining a basic measure

$$de(l, l) = \frac{|\Delta y(l, l)|}{\langle \Delta y(l, l) \rangle}, \quad l = 0, 1, \dots, N-1 \quad (4)$$

where $\Delta y(l, l) = y(t_{l+1}) - y(t_l)$ is the small-scale

gradient field and $\langle \Delta y(l, l) \rangle = \frac{1}{N} \sum_{l=1}^{N-1} |\Delta y(l, l)|$

It should be noted that we use spatial/temporal averages rather than ensemble averages, thus making an ergodicity assumption (Holley, 1993) as our only recourse in such an empirical data analysis. Next we define a series of ever more coarse-grained and ever shorter fields $de(r, l)$ where $0 < l < N-r$ and $r=1, 2, 4, \dots, N=2^m$. The average measure in the interval $[l, l+r]$ is

$$de(r, l) = \frac{1}{r} \sum_{l'=l}^{l+r-1} de(l, l'), \quad l = 0, \dots, N-r \quad (5)$$

The scaling properties of the generating function are then searched for through

$$\langle de(r, l) \rangle^q \propto r^{-K(q)}, \quad q \geq 0 \quad (6)$$

Thus the multi-fractal properties of the CBHP signal are expressed by two sets of scaling functions, $H(q)$ describing the roughness of the signal and $K(q)$ describing its intermittency. The value obtained for $H(q)$ function at $q=1$, $H(q=1)=H_1=0.27 \pm 0.03$ is close to the a exponent of DFA method, $a=0.20 \pm 0.002$ of this study and is similar to the results for the cloud base height data series measured during ASTEX, for which $a=0.24 \pm 0.002$ for June 18, 1992 and $a=0.21 \pm 0.005$ for June 15, 1992 (Kitova, 2002) and for which $H(q=1)=H_1=0.23 \pm 0.04$ for June 14, 1992 and $H(q=1)=H_1=0.21 \pm 0.03$ for June 15, 1992 (Gospodinova, 2001).

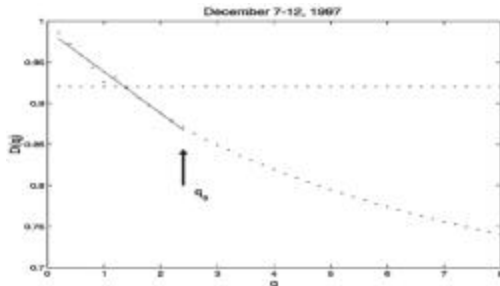


Fig.2 Hierarchy of generalized dimensions $D(q)$ for the CBHP data in Fig.1.a. The straight line is drawn to enhance the value q_s at which $D(q)$ function starts to deviate from a linear dependence. This deviation is related to sampling problems due to a single realization. The dashed line defines the monofractal case $D_1=0.92$

The intermittency of the signal can be also expressed through the generalized dimensions $D(q)$ as introduced by Grassberger, 1983 and Hentschel and Procaccia, 1983

$$D(q) = 1 - \frac{K(q)}{q-1} \quad (7)$$

The multi-affinity of $y(t)$ means that one should use different scaling factors $H(q)=H_q$ in order to rescale such a signal. This also implies that *local* roughness exponents exist (Vandewalle, 1998) at different scales. The density of the points $N_\epsilon(t)$ that have the same roughness exponent is assumed (Halsey, 1986) to scale over the time span t as

$$N_g(t) \propto t^{-h(g)} \quad (8)$$

From Ref. Barabasi, 1991 the following relations are found:

$$g(q) = \frac{d(qH(q))}{dq} \quad (9)$$

and

$$h(g_q) = 1 + qg(q) - qH(q) \quad (10)$$

In Fig. 3 the CBHP multi-affine properties are presented via the $h(?)$ -curve.

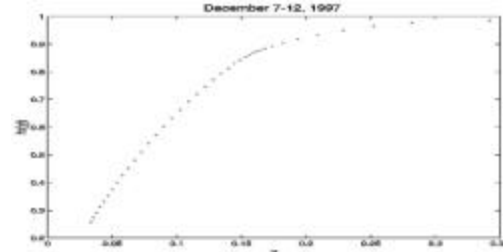


Fig.3 The $h(?)$ -curve for the CBHP data in Fig.1.a

IV. CONCLUSIONS

We have demonstrated the diurnal evolution of the cloud dynamics and the multi-affine structure of cloud base height profiles. Further work will be directed toward relating these statistical parameters to the dynamical properties of the clouds, an important step toward understanding, modelling and predicting their dynamical behavior.

Acknowledgements

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References

- Barabasi, A-L, P. Szepefalussy and T. Vicsek, *Physica A* **178**, 17 (1991)
- Davis, A., A. Marshak, W. Wiscombe and R. Cahalan, *J. Geophys. Research* **99**, 8055 (1994)
- Garratt, J.R., *The Atmospheric Boundary Layer*, (Cambridge University Press, 1992)
- Gospodinova, N., K. Ivanova, E.E. Clothiaux and T. Ackerman, XI-th International School on Quantum

Electronics, 18-22 September 2000, Varna, Bulgaria,
In Proc. SPIE, vol. **4397**, pp. 476-480, (2001)
 Grassberger, P., *Phys. Rev. Lett.* **97**, 227 (1983)
 Halsey, T.C., M.H. Jensen, L.P. Kadanoff, I. Procaccia,
 and B.I. Shraiman, *Phys. Rev. A* **33**, 1141 (1986)
 Hentschel, H.G.E. and I. Procaccia, *Physica D* **8**, 435
 (1983)
 Holley, R. and E.C. Waymire, *Ann. J. Appl. Prob.* **2**,
 819 (1993)
 Ivanova, K. and T. Ackerman, *Phys. Rev. E* **59**, 2778
 (1999)
 Ivanova, K. and M. Ausloos, *Eur. Phys. J. B* **8**, 665
 (1999)
 Kitova, N., K. Ivanova, M. Ausloos, T.P. Ackerman,
 and M.A. Mikhalev, *Int. J. Mod. Phys. C*, **13**, No. 2
 (2002)
 Peng, C.-K., S.V. Buldyrev, S. Havlin, M. Simons,
 H.E. Stanley and A. Goldberger, *Phys. Rev. E* **49**,
 1685 (1994)