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A BENCHMARK SIMULATION FOR TESTING MOIST NONHYDROSTATIC NUMERICAL MODEL FORMULATIONS

George H. Bryan and J. Michael Fritsch
Pennsylvania State University, University Park, PA

1. INTRODUCTION

Many methods can be used to evaluate a numerical modeling system. The most powerful methods are ones in which model output can be compared to known results, e.g., solutions that can be derived through dynamical analysis, or solutions that converge under certain conditions. Examples of commonly used analytic and/or benchmark cases include certain mountain wave solutions, inertia-gravity waves, a nonlinearly evolving cold pool and density current, and a rising warm thermal. Tests such as these are important for a number of reasons, e.g., for establishing the fidelity of a new numerical modeling system, or for testing the accuracy, efficiency, and efficacy of a new numerical technique.

Unfortunately, none of these analytic/benchmark cases include moist processes. Moreover, despite the varying methods used to include moist processes in numerical models, there does not appear to be a commonly agreed-upon method to evaluate a moist model formulation.

This paper presents a new simulation that can be used as a benchmark for testing numerical models with moisture. The design of the simulation is analogous to the nonlinear warm thermal benchmark case used by Tripoli (1992) and Wicker and Skamarock (1998), but includes phase changes of water vapor and cloud water.

2. THE NUMERICAL MODEL

The numerical model used for this study is described in detail in Bryan and Fritsch (2002). The governing equations for this model were carefully formulated so that, when combined, the equation set conserves mass and energy. In particular, the potential temperature (θ) and nondimensional pressure (π) equations are significantly different from those used in other numerical models (see Table 1). Most notably, these prognostic equations retain two commonly neglected effects: 1) the heat capacity of liquid water, and; 2) the diabatic contribution to π. However, since the model equations are not written in a conservative form, the numerical model integrations do not exactly conserve mass or energy, even though the equation set technically does.

For this study, the following processes are ignored: hydrometeor fallout, ice-phase microphysics, the Coriolis force, and subgrid-scale turbulence. Computational mixing is also neglected, despite the fact that a constant background mixing coefficient is normally applied in warm bubble tests such as this. Background mixing was avoided here in order to focus on the formulation of the governing equations, and to highlight the importance of mass and energy conservation in numerical models.

Corresponding author address: George H. Bryan, 503 Walker Bldg., University Park, PA 16802; email: bryan@essc.psu.edu

3. THE DRY COUNTERPART SIMULATION

A simulation presented by Wicker and Skamarock (1998) was chosen as the dry reference case. The simulation is two-dimensional, with a domain height of 10 km and width of 20 km. Rigid wall boundary conditions are specified on all four sides of the domain. The initial unperturbed environment is calm (zero initial winds everywhere), hydrostatic, and neutrally stable, defined by a constant potential temperature of 300 K. A warm perturbation, with a maximum amplitude of 2 K, is placed at the center of the domain. Results of a simulation with 100 m grid spacing after 1000 s of integration are presented in Fig. 1. Similar to the results of Wicker and Skamarock (1998), the thermal rises and expands over time. Two "rotors" develop on the sides of the thermal. Large θ gradients develop in the middle of the thermal (i.e., within the "arch" spanning between the two rotors).

4. THE MOIST SIMULATION

Seeking to obtain a similar result for a moist atmosphere, we again specify the initial environment to be hydrostatic and characterized by exactly neutral stability. In the dry case, it is possible to define neutral stability based on only one thermodynamic variable: potential temperature. However, a moist atmosphere is not as simple. To simplify the specification of the moist base state, two assumptions are made: 1) the total water mixing ratio (r_t) is constant at all levels; and 2) phase changes are exactly reversible. Under these two assumptions, a neutrally stable environment can be obtained using one conservative thermodynamic variable. We use the wet equivalent potential temperature (θ_e, Emanuel 1994 pg. 120), which is conserved for a reversible moist adiabatic atmosphere with constant total water mixing ratio. Using the hydrostatic equation for a moist atmosphere and the definition of θ_e, the vertical profiles of pressure, temperature, and mixing ratios of moisture can be obtained if values for θ_e and r_t are specified (see Bryan and Fritsch 2002 for further details).

All other parameters are the same as for the dry case. Results after 1000 s for a case in which θ_e=320 K and r_t=0.020 are presented in Fig. 2. The results of this moist case are very similar to the results of the dry case, especially with regards to the structural details such as the two "rotors" that form on the sides of the thermal and the "arch" that connects them. The moist thermal rises slightly faster than the dry thermal, and after 1000 s the

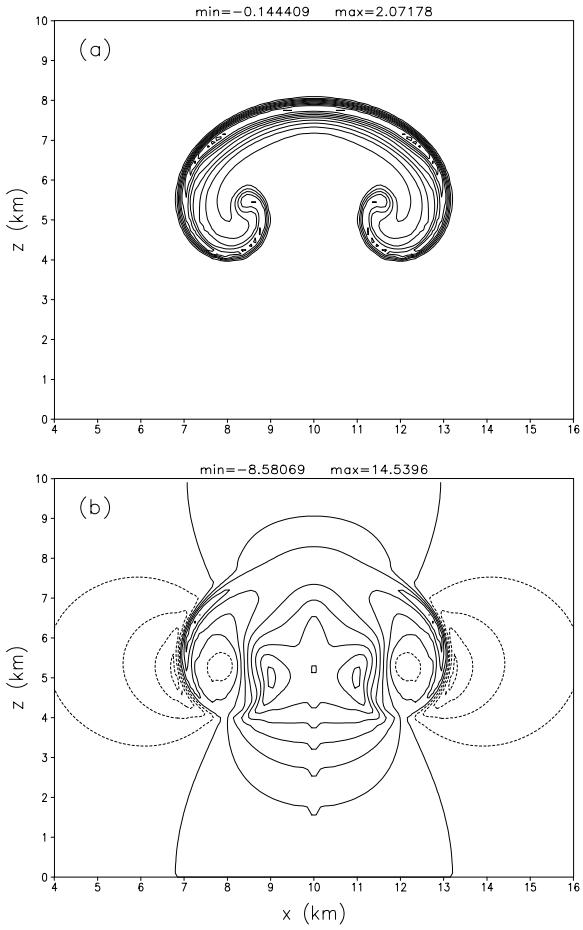

Fig. 1 Results of the dry thermal simulation. (a) Perturbation potential temperature, contour interval 0.2 K, zero contour omitted. (b) Vertical velocity, contour interval 2 m s^{-1}, negative contours dashed.

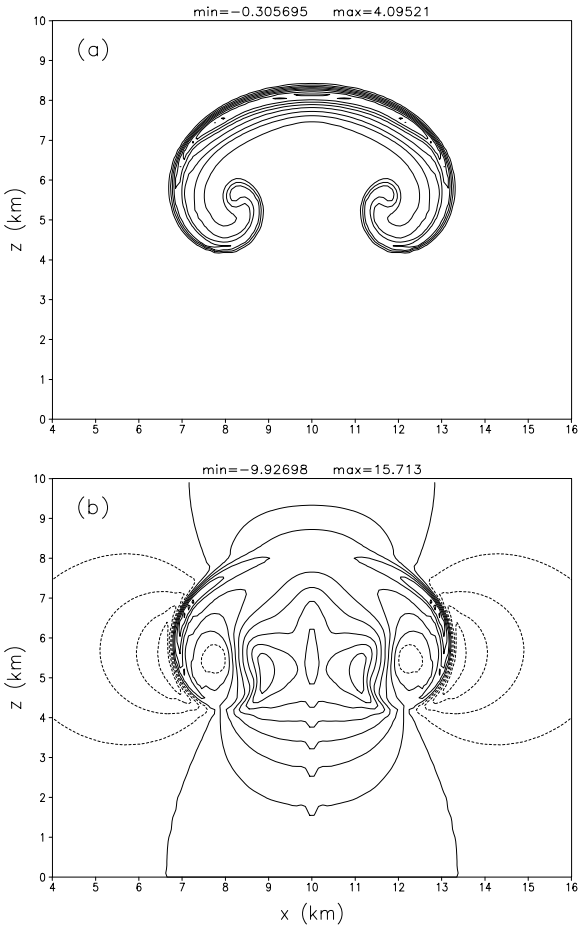

Fig. 2 Results of the moist simulation. (a) Perturbation wet equivalent potential temperature, contour interval 0.5 K, zero contour omitted. (b) Vertical velocity, contour interval 2 m s^{-1}, negative contours dashed.

vertical velocity field has higher maximum and minimum values. Nevertheless, the structural details are remarkably similar.

It is important to reiterate that the model formulation for this simulation does not neglect any term in the governing equations. In particular, the specific heat of liquid water is included, and the diabatic contribution to the pressure equation is included; it is a common assumption in numerical models to neglect these two effects. Furthermore, the error in total mass and energy conservation is quite small (about 10^{-4} percent), especially compared to model formulations that ignore certain terms in the governing equations (which will be presented in the next section). Given this high degree of accuracy in mass and energy conservation, and the similarity to the dry case, it seems reasonable that this case can be considered a moist benchmark to which moist numerical models can be compared. Additionally, as in the dry case, the simulation proposed here is remarkably insensitive to the values used to define the initial neutrally stable sounding. This result helps establish the fact that the moist simulation design is robust, i.e., the correct result is not dependent on a specific initial thermodynamic environment. This is an important point, since it provides further confidence that the results truly represent a benchmark solution.

5. SENSITIVITY TO GOVERNING EQUATIONS

The assumptions of reversible phase changes and the absence of hydrometeor fallout clearly make this test case a simplification of reality. However, we have found the case to be valuable for testing the formulation of numerical models. As an example, four different model formulations (Table 1) are tested and presented in this section. Equation Set A makes two approximations that are commonly used in cloud models: the diabatic contribution to the pressure equation is ignored, and the specific heat of water is neglected. This equation set is similar to that used in the Klemp-Wilhelmson model, MM5, and ARPS, as well as several other numerical models. For Equation Set B, only the specific heat of water is neglected in the thermodynamic and pressure equations. Since the diabatic contribution to the pressure equation is included, this equation set conserves mass. This

Table 1 Summary of thermodynamic and pressure equations evaluated with the moist benchmark simulation, where "BM" refers to the benchmark equation set. See Bryan and Fritsch (2002) for definitions of symbols

Equation Set	Thermodynamic Equation	Pressure Equation
BM	$\frac{D\theta}{Dt} = -\theta\left(\frac{R_m}{c_{vml}} - \frac{Rc_{pml}}{c_p c_{vml}}\right)\frac{\partial u_i}{\partial x_i} + \left[\frac{c_v L_v}{c_{vml} c_p \pi} - \frac{R_v}{c_{vml}}\theta\left(1 - \frac{R}{c_p}\frac{c_{pml}}{R_m}\right)\right]\dot{r}_{cond}$	$\frac{D\pi}{Dt} = -\pi\frac{R}{c_p}\frac{c_{pml}}{c_{vml}}\frac{\partial u_i}{\partial x_i} + \frac{R}{c_p}\left(\frac{L_v}{c_{vml}\theta} - \pi\frac{R_v c_{pml}}{R_m c_{vml}}\right)\dot{r}_{cond}$
A	$\frac{D\theta}{Dt} = \frac{L_v}{c_p \pi}\dot{r}_{cond}$	$\frac{D\pi}{Dt} = -\pi\frac{R}{c_v}\frac{\partial u_i}{\partial x_i}$
B	$\frac{D\theta}{Dt} = \frac{L_v}{c_p \pi}\dot{r}_{cond}$	$\frac{D\pi}{Dt} = -\pi\frac{R}{c_v}\frac{\partial u_i}{\partial x_i} + \left(\frac{RL_v}{c_p c_v \theta} - \pi\frac{R_v}{c_v}\right)\dot{r}_{cond}$
C	$\frac{D\theta}{Dt} = \left[\frac{c_v L_v}{c_{vml} c_p \pi} - \frac{R_v}{c_{vml}}\theta\left(1 - \frac{R}{c_p}\frac{c_{pml}}{R_m}\right)\right]\dot{r}_{cond}$	$\frac{D\pi}{Dt} = -\pi\frac{R}{c_p}\frac{c_{pml}}{c_{vml}}\frac{\partial u_i}{\partial x_i} + \frac{R}{c_p}\left(\frac{L_v}{c_{vml}\theta} - \pi\frac{R_v c_{pml}}{R_m c_{vml}}\right)\dot{r}_{cond}$
D	$\frac{D\theta_{il}}{Dt} = 0$	$\frac{D\pi}{Dt} = -\pi\frac{R}{c_v}\frac{\partial u_i}{\partial x_i}$

formulation is similar to that used in COAMPS, and in some respects is similar to models that integrate a density equation rather than a pressure equation (such as the WRF model). In Equation Set C, the specific heat of water is included, as is the diabatic contribution to the pressure equation, but the term involving divergence in the thermodynamic equation is neglected; scale analysis suggests that this term is small and, presumably, negligible. To our knowledge, this equation set has not been used in the literature, but is included here as an example of how approximate forms of the governing equations can be tested numerically. Equation Set D uses the ice-liquid water potential temperature (θ_{il}) of Tripoli and Cotton (1981). Equation set D also neglects the diabatic contribution to the pressure equation, and the specific heat of water in the pressure equation. This equation set is similar to that used in RAMS and in the University of Wisconsin Nonhydrostatic Modeling System.

Results (Fig. 3) clearly show the dramatic impact of neglecting terms from the complete thermodynamic and pressure equations – none of the simulations using approximate equations compare well with the benchmark solution (Fig. 2a). In all of these cases, the thermal rises much slower than the thermal in the benchmark run. In the θ_e fields, large undershoots (i.e., anomalously low values, depicted by dashed contours) develop in all cases.

It is interesting to note that the output from runs A and B are very similar. Both thermals rise to ~6.9 km, and the vertical motion patterns are nearly identical. This result suggests that the extra effort required to conserve mass in a numerical model (by including the diabatic contribution to the pressure equation) may not lead to significant improvements in results unless total energy is also conserved (as in the benchmark).

The results from run C were surprising. Among all the simulations, this run least resembles the benchmark case. The thermal only reached 5.8 km, and the θ_e pattern is quite different from the other runs. This test highlights the danger of neglecting terms that may seem unimportant under a scale analysis.

The simulation that used θ_{il} as the governing variable (run D) produced results that most closely match the benchmark. The thermal reaches ~7.6 km, and becomes only slightly distorted in shape. On the other hand, this formulation has the largest total mass and total energy errors out of all runs presented here.

The value chosen for r_t in these tests is abnormally high for the imposed temperature sounding. One might wonder whether these results only come about due to this unphysical initial environment. A comparison of simulations with different values for θ_e and r_t (not shown) reveals that the differences presented here are accentuated over those one would expect to find in more "normal" environments. Nevertheless, *it is clear that the mass-conserving and energy-conserving form of the thermodynamic and pressure equations can produce the desired results in all environments, and that these equations should be preferred over approximate equation sets*.

We have conducted additional simulations using realistic initial environments to address whether the conclusions drawn from this paper hold for more typical uses of numerical models. The simulations with the benchmark equation set tend to have the strongest updrafts, the highest cloud tops, and the most rainfall. Based on these results, we have concluded that the form of the governing equations used in a numerical model does have an impact on the results, although perhaps a small impact for most uses.

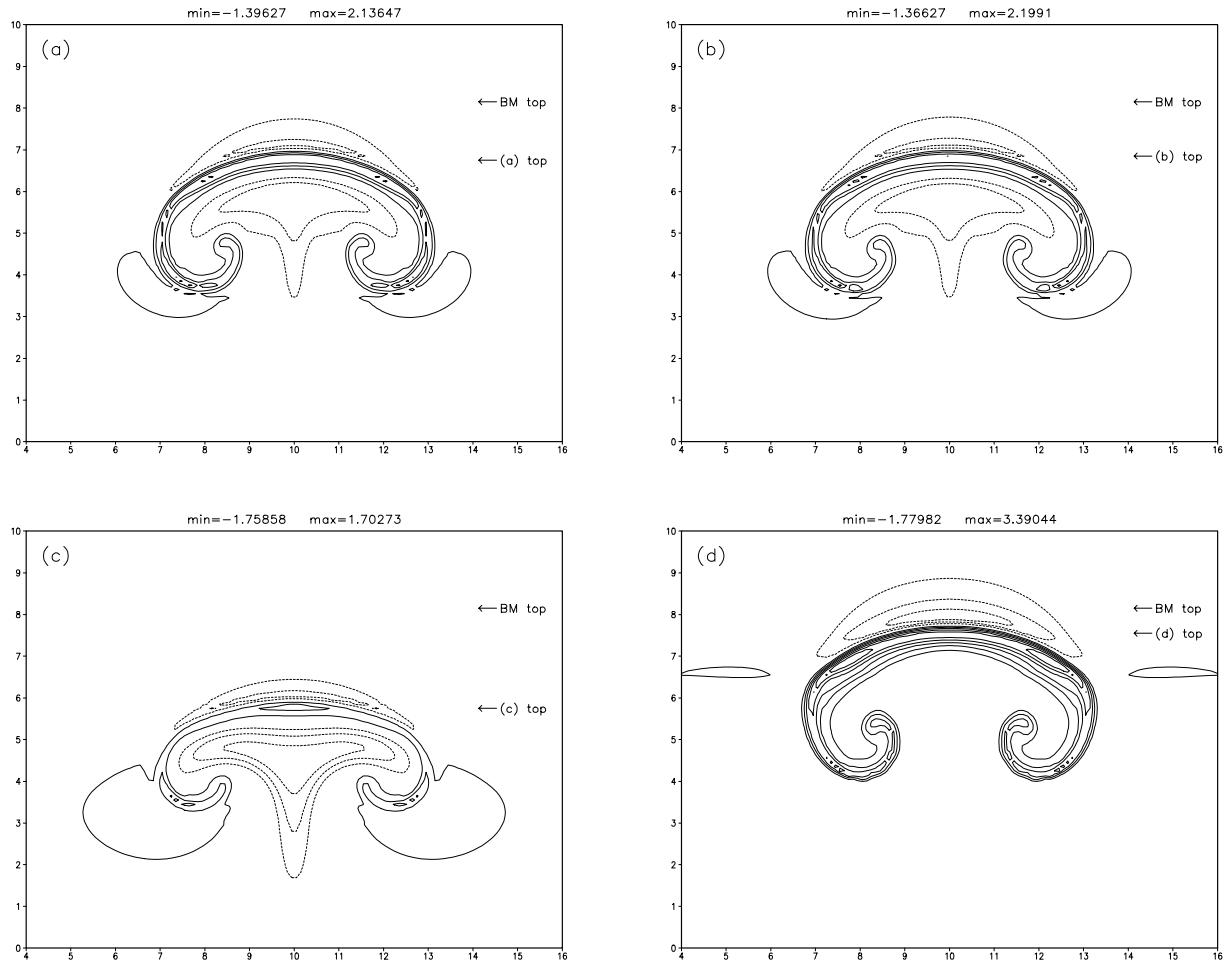

Fig. 3 Perturbation wet equivalent potential temperature from simulations using various model formulations: (a) equation set A, (b) equation set B, (c) equation set C, and (d) equation set D. Contour interval is 0.5 K. Negative contours are dashed. The zero contour is omitted. Near the right-hand side of each panel, the height of the top of the thermal is indicated, along with the height of the top of the thermal from the benchmark ("BM") simulation.

6. CONCLUDING REMARKS

Although the model used in this study does not exactly conserve mass, momentum, or energy, the results strongly suggest that conservation of these basic variables can be necessary to obtain accurate results in some instances. This result supports the need to construct numerical models around conservation principles, which is the driving principle behind several recent model development efforts (e.g., Ooyama 2001, Skamarock et al. 2001, and Satoh 2002).

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