# 15.1 LOCAL POP FORECAST EQUATIONS FOR PHILADELPHIA, PA

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## 1. INTRODUCTION

The National Weather Service (NWS) has for a number of years used model-based statistical guidance as an aid to forecasting weather elements such as probability of precipitation (PoP). Such guidance is usually generated at a central location for many forecast points (see, e.g., Jacks et.al. 1990). However, PoP equations have also been derived at local NWS offices. Gerapetritis (1999) used logistic regression to derive PoP equations for Columbia, SC, from a limited number of variables extracted from the Forecast Output Statistics (FOUS) messages FRH63 and FRHT63. His motivation was to have PoP equations that were derived, at least in part, from Eta model forecasts available to the office. At that time, no centrally-generated Eta-based PoP guidance was available to the field. Despite using a very limited number of predictor variables, he reported modest improvement over PoP forecasts from Model Output Statistics based on the Nested Grid Model (FWC MOS).

The purpose of our study also was to develop Eta-based PoP equations for operational use until equations from the Meteorological Development Laboratory (MDL) became available. When this study was undertaken, centrally-generated Eta-based PoP guidance still was not available to the field, although it is anticipated that operational Eta MOS guidance will be made available to field offices in June, 2002. Besides having the equations for operational use, we were interested in the two additional questions: (1) Would our approach result in equations that remained skillful despite frequent changes to the Eta model, including changes to model resolution? And, (2) Would our approach result in equations that were useful despite the lack of independence of the data records?

This rest of this paper will describe the development of predictor data for the PoP forecast equations (Section 2), the use of logistic regression to derive the equations (Section 3) and the verification of the equations against MDL guidance forecasts (Section 4). Finally, we will discuss the significance and limitations of our results (Section 5).

## 2. PoP PREDICTOR DATA

Our approach was to manually extract independent predictor variables from as many Eta model runs as practical within operational constraints, without regard to the independence of the individual records and with only limited regard to model changes. We also accumulated values for the dependent variable, a binary representation (0 for no; 1 for yes) of whether at least 0.01 inch of precipitation fell in a twelve hour period at Philadelphia International Airport (PHL) as measured by the Automated Surface Observing System (ASOS).

Data collection began on 30 March 1998, and remains ongoing. Data were obtained from as many Eta runs as possible. However, without automatic data collection some runs were missed. In the warm seasons, data from 52% of the runs were collected. In the cold season, data from 62% of the runs were collected. All data that were collected were used.

Data from each model run covered three time periods: (1) 12 to 24 hours, (2) 24 to 36 hours, and (3) 36 to 48 hours, from time of model run. Data from the 0000 UTC and 1200 UTC Eta model runs were combined. However, the data were stratified by season, with the cold season running from 1 October to 31 March and the warm season from 1 April to 30 September. This would eventually result in six separate Eta-based PoP equations for PHL, i.e., three forecast periods times two seasons.

There have been changes to the Eta model over the course of this study, e.g., from a 32 km grid with 45 layers to a 22 km grid with 50 layers on 26 September 2000, and then to a 12 km grid with 60 layers on 27 November 2001. Grid resolution changes can affect the magnitude of many forecast variables; however for this study, the only action taken in response was to remove the cold season data from 1998 to 1999 and 1999 to 2000 before re-computing the cold season equation in December, 2001. This means that after the 26 September 2000 change, data used to derive all equations spanned one change.

The independent variables which made up the pool of potential predictors are listed in Table 1. These variables were chosen subjectively, but are all related directly or indirectly to forecast moisture and/or upward motion. They were extracted either from the FOUS message FRH61, or by digital read-out from a graphical image display of Eta model gridded data on the Advanced Weather Information Processing System (AWIPS). Values for each of the independent variables were available every six hours, so for most of the variables during each 12-hour forecast period there were three separate values from which to choose. One exception was model precipitation, for which two 6-hour forecasts were combined into one 12-hour forecast.

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Table 1: Eta model forecast variables used as potential PoP predictors.

| Variable | Units | Source | Abbreviation |
|---|---|---|---|
| 12-hour Model pcpn | inches | FRH61 | PCPN |
| 850 mb vertical velocity | µbar/sec | AWIPS Graphic | VV850 |
| 700 mb vertical velocity | µbar/sec | AWIPS Graphic | VV700 |
| 500 mb vertical velocity | µbar/sec | AWIPS Graphic | VV500 |
| 925 mb relative humidity | percent | AWIPS Graphic | RH925 |
| 850 mb relative humidity | percent | AWIPS Graphic | RH850 |
| 700 mb relative humidity | percent | AWIPS Graphic | RH700 |
| Previous period MSLP change | mb | FRH61 | DELP1 |
| Current period MSLP change | mb | FRH61 | DELP2 |
| Wind Direction | degrees | FRH61 | WDIR |
| Lifted Index | deg C | FRH61 | LI |
| 1000 mb Moisture Flux Div | g $kg^{-1}$ $12hrs^{-1}$ | AWIPS Graphic | MFD1000 |
| 925 mb Moisture Flux Div | g $kg^{-1}$ $12hrs^{-1}$ | AWIPS Graphic | MFD925 |
| 850 mb Moisture Flux Div | g $kg^{-1}$ $12hrs^{-1}$ | AWIPS Graphic | MFD850 |

Generally, the values assigned for each 12-hour time period were the values considered most conducive to precipitation. For example, if the 12-hour forecast value for 850 mb vertical velocity from a given run was -1.2 µbar/s (negative = upward), the 18-hour forecast value was -2.3 µbar/s and the 24-hour forecast value was -3.8 µbar/s, the period one (12 to 24 hours) value retained for this study was -3.8 µbar/s. Moreover, if that 24-hour forecast vertical velocity value was more conducive to precipitation than either the 30-hour or 36-hour value, it was also retained for period two (24 to 36 hours).

Regarding surface pressure change, the greater six-hour pressure fall during the 12-hour period was retained. If the pressure rose steadily over the 12-hour period, then the smaller six-hour pressure rise was retained. If the pressure rose or remained steady over one six-hour period and remained steady over the other six-hour period in a twelve hour period, then zero was retained.

Wind direction was treated in the following manner. First, the mean wind direction for each 12-hour period was determined. Then, the cosine of the difference between that mean wind direction and each of eighteen compass points between 10 degrees and 180 degrees inclusive were computed. This yielded, for each mean wind direction for each period, eighteen compass point values that ranged from -1 to +1. If the mean wind direction was, for example, 160 degrees, then of the eighteen values the value with the greatest absolute magnitude was 1 and it corresponded to 160 degrees. If the mean wind direction was 280 degrees, then the value with the greatest absolute magnitude was -1 and it corresponded to 100 degrees. The compass point whose values correlated best with the occurrence of measurable precipitation corresponded to the wind direction most correlated with precipitation, and it stood the best chance of being retained.

## 3. DERIVATION OF EQUATIONS

The statistical technique chosen for this study was logistic regression analysis. The logistic regression approach results in an equation that expresses the probability of some categorical event (in this case, the occurrence of measurable precipitation) as the sum of weighted values of quantitative variables. A complete explanation of logistic regression can be found in Freeman (1987). Computer routines to perform this type of analysis are included in various commercially-available statistical software packages.

In this technique, the statistic used to select predictors from the pool of independent variables is the likelihood ratio chi-square statistic ($G^2$). A model with a high residual $G^2$ value has little predictive value, so the more the residual $G^2$ is reduced by inclusion of a given independent variable, the better the predictive value of that variable. A p-value is associated with the reduction in residual $G^2$ by a given variable, and low p-values are associated with variables that have high predictive value.

The study first ran each independent variable in a one-predictor variable equation, and the variable that resulted in the greatest reduction in residual $G^2$ was kept, provided the associated p-value was less than or equal to 0.10 and the sign of the coefficient associated with the variable made sense meteorologically. For example, if 1000 mb moisture flux divergence resulted in the greatest reduction in residual $G^2$, and the associated p-value was less than 0.10 but the sign of the associated coefficient was positive (meaning that greater moisture flux divergence resulted in higher PoPs), then 1000 mb moisture flux divergence was not retained.

After one independent variable was retained, the next step was to run each remaining independent variable, together with the one retained in the first step, in a two-predictor variable equation. The one that resulted in the greatest additional reduction in residual $G^2$, subject to the constraints above, was kept. This process continued until there were no remaining variables that result in a reduction in residual $G^2$ corresponding to a p-value of less than 0.10 and whose coefficients made meteorological

Table 2. Example set of PoP forecast equations. See Table 1 for predictor abbreviations.

Warm Season, First Period:

*PoP = 1/[1 + exp(-11.15 + 1.548*PCPN - 0.3506*VV500 + 0.06330*RH925 + 0.06487*RH700 - 0.07724*LI)]*

Warm Season, Second Period :

*PoP = 1/[1 + exp(-9.588 + 2.466*PCPN - 0.1463*VV500 + 0.04871*RH925 + 0.06244*RH700 - 0.1402*DELP1 - 0.05298*LI)]*

Warm Season, Third Period :

*PoP = 1/[1 + exp(-7.668 + 3.182*PCPN - 0.1134*VV500 + 0.04745*RH925 + 0.03540*RH700 - 0.1284*DELP1 - 0.06658*LI)]*

Cold Season, First Period :

*PoP = 1/[1 + exp(-11.03 - 0.3234*VV700 - 0.3604*VV500 + 0.08094*RH925 + 0.03781*RH700 - 0.1312*LI)]*

Cold Season, Second Period :

*PoP = 1/[1 + exp(-13.34 - 0.2902*VV700 - 0.2015*VV500 + 0.1070*RH925 + 0.02831*RH700 + 0.8183*cosine(220 - mean wind direction))]*

Cold Season, Third Period :

*PoP = 1/[1 + exp(-10.32 + 5.426*PCPN - 0.1868*VV500 + 0.06731*RH925 + 0.03770*RH700)]*

sense. The resulting logistic regression equation is of the form:

$$PoP = 1/[1 + \exp(B_0 + B_1X_1 + ... + B_nX_n)] \qquad (1)$$

where:

PoP = probability of precipitation
$B_0$ = constant
$B_1$ through $B_n$ = coefficients
$X_1$ through $X_n$ = predictor variables.

The first warm season PoP equation was generated after the 1998 warm season. The first cold season PoP equation was generated after the 1998 to 1999 cold season. We employed an iterative process, so that during the course of a given season, after additional data had been accumulated, the equation was re-derived.

The most recent versions of the equations are given in Table 2. The most commonly selected predictors for the warm season equations were model precipitation, 500 mb vertical velocity, relative humidity at 925 mb and 700 mb, and the lifted index. For the cold season, the most frequently selected predictors were vertical velocity at 700 mb and 500mb, and relative humidity at 925 mb and 700 mb.

## 4. VERIFICATION

The measurement used to verify our locally-derived PoP equations was the Brier Score. The Brier Score is defined as

$$BrierScore = \frac{1}{n}\sum_{i=1}^{n}(Fi - Oi)^2 \qquad (2)$$

where $F_i$ are the PoP forecasts with values ranging from 0 to 1, and $O_i$ is either 1 if measurable precipitation occurs or 0 if it does not. Thus, *lower* Brier Scores indicate better forecasts. More information on the Brier Score can be found in Wilks (1995).

Verification was conducted on independent data collected after equations were derived, although that data would subsequently be included in the larger dependent data set for later derivations. Brier scores were computed for the local-equation PoPs, and also for PoPs from the corresponding FWC MOS.

The verification results for the warm season equations are shown in Table 3, and for the cold season equations in Table 4. Verifications scores for combined forecast periods only are graphed in Figure 1. Overall the scores for the local PoPs show a modest improvement over the FWC scores, with somewhat more improvement during the cold season.

More recently, PoP guidance based on the NCEP AVN model (MAV bulletins) has become available to local NWS offices. The study verified the local PoP equations against the MAV PoPs for the most recent warm and cold seasons. The results are shown in Table 5. Again, the local equations show modest improvement over the MAV forecasts.

## 5. DISCUSSION AND CONCLUSIONS

The verification statistics indicate the local equations manage minor overall improvement over the FWC and MAV during the warm season, and they manage somewhat greater improvement over the FWC and MAV during the cold season. This occurs despite the limited number of independent variables used, the modest accommodation for changes to the Eta model, and the likelihood of dependence between data records. The local PoP equations do have the advantage that they

Table 3. Brier-score verification statistics for warm-season Pop equations at PHL., vs. FWC guidance.

| Year | Forecast Period | Number Of Cases | Brier Score FWC | Brier Score Local |
|---|---|---|---|---|
| 1999 | 12-24 hrs | 159 | .0751 | .0664 |
| | 24-36 hrs | 159 | .0716 | .0815 |
| | 36-48 hrs | 159 | .0852 | .0796 |
| | Combined | 477 | .0773 | .0758 |
| 2000 | 12-24 hrs | 180 | .1137 | .1169 |
| | 24-36 hrs | 180 | .1321 | .1326 |
| | 36-48 hrs | 180 | .1519 | .1441 |
| | Combined | 540 | .1326 | .1312 |
| 2001 | 12-24 hrs | 193 | .0775 | .0764 |
| | 24-36 hrs | 193 | .0859 | .0879 |
| | 36-48 hrs | 193 | .1060 | .0946 |
| | Combined | 579 | .0898 | .0863 |

Table 4. Same as Table 3, except for the cold season.

| Year | Forecast Period | Number Of Cases | Brier Score FWC | Brier Score Local |
|---|---|---|---|---|
| 1999- | 12-24 hrs | 231 | .0718 | .0681 |
| 2000 | 24-36 hrs | 231 | .0788 | .0821 |
| | 36-48 hrs | 231 | .1021 | .0876 |
| | Combined | 693 | .0842 | .0793 |
| 2000- | 12-24 hrs | 193 | .0639 | .0643 |
| 2001 | 24-36 hrs | 193 | .0862 | .0894 |
| | 36-48 hrs | 193 | .1126 | .0908 |
| | Combined | 579 | .0875 | .0815 |
| 2001- | 12-24 hrs | 241 | .0599 | .0480 |
| 2002 | 24-36hrs | 241 | .0685 | .0571 |
| | 36-48 hrs | 241 | .0620 | .0710 |
| | Combined | 723 | .0635 | .0587 |

apply to a single forecast site (PHL), whereas the FWC and MAV equations for PHL were derived with data from a number of sites in the mid-Atlantic region.

Based on the comparison of the local equations to the FWCs, it appears that the technique of logistic regression may be robust enough to generate a PoP equation that outperforms a more rigorously derived equation from a larger pool of potential predictor variables, from a model not subject to modification (the NGM). Or, perhaps the Eta model is so superior to the NGM that it overwhelms the weaknesses of our approach, or that some combination of these factors is at work. The authors are aware of no study that has established overall ETA model superiority in predicting measurable precipitation when compared to the AVN model. In the absence of such a study, it seems unlikely that the marginal improvement of the local equations over the MAVs is due to ETA model superiority over the AVN.

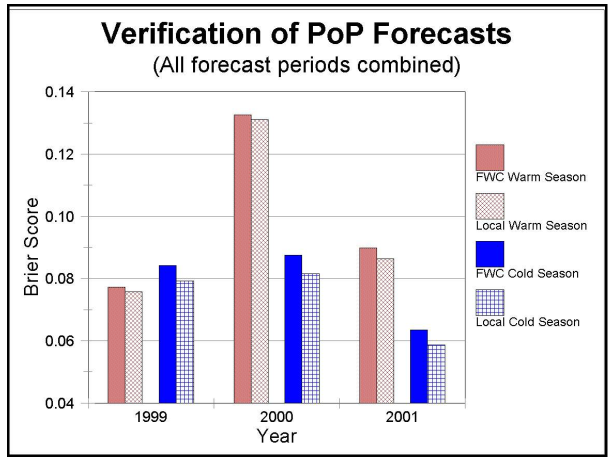


Fig 1. Brier scores for cold and warm season verification, all forecast periods combined (from Tables 3 and 4).

Table 5. Verification of local PoP equation vs. MAV forecast guidance.

| Season | Forecast Period | Number Of Cases | Brier Score MAV | Brier Score Local |
|---|---|---|---|---|
| Warm | 12-24 hrs | 146 | .0685 | .0747 |
| Season | 24-36 hrs | 146 | .1030 | .0981 |
| 2001 | 36-48 hrs | 146 | .0989 | .0892 |
| | Combined | 438 | .0901 | .0873 |
| Cold | 12-24 hrs | 231 | .0589 | .0492 |
| Season | 24-36 hrs | 231 | .0733 | .0574 |
| 2001- | 36-48 hrs | 231 | .0722 | .0736 |
| 2002 | Combined | 693 | .0681 | .0601 |

## 6. REFERENCES


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