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USE OF ADJOINT-DERIVED FORECAST SENSITIVITIES TO CHARACTERIZE FORECAST UNCERTAINTY

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1. INTRODUCTION

A goal of ensemble forecasting is the assessment of the predictability of a given flow. The generation of the initial perturbations which describe an ensemble is governed by two principles: 1) the initial perturbations should be constructed using some knowledge of the statistics of analysis errors and 2) the initial ensemble perturbations should share the structure of those perturbations which amplify rapidly over the forecast interval of interest (i.e., in those regions in which the sensitivity of the forecast to initial condition uncertainty is largest).

A means of constructing ensembles is through the use of singular vectors (SVs, also referred to as "optimal perturbations"), which are those perturbations which amplify linearly most rapidly for a given norm, for a given basic state, over a prescribed time interval. In the construction of ensembles using SVs, the choice of the analysis error covariance metric as a measure of initial amplitude ensures that the SVs are constructed using knowledge of the characteristics of the analysis error. The rapidly growing property of SVs in addition to the fact that they are orthogonal at the initial and final (optimization) times provides that the ensemble generated from the SVs has maximum spread at the end of the optimization interval.

While there is evidence that there is some utility in the use of SVs for ensemble prediction, there are limitations to their efficacy. These limitations include computational cost, the validity of the assumption of linear dynamics, and the number of members needed to construct a reasonable ensemble. Calculation of SVs for ensemble prediction is costly, as several runs of both the linearized version of a numerical weather prediction (NWP) model and its adjoint are required in the iterative schemes used to solve the eigenvalue problem that defines the SVs. The concern over whether linear dynamics is appropriate arises when the initial perturbation is of large amplitude, whether the perturbation grows so rapidly that its amplitude is comparable to that of the basic state, or whether processes within the model that are described by a conditional (e.g., an "if-then" statement) change the sense of the conditional - resulting in effects that are not described by the linearization. Finally, there is no *a priori* means of identifying the number of ensemble members needed in an ensemble for a particular forecast.

In this presentation, we present an alternative means of constructing an ensemble forecast using adjoint derived forecast sensitivities. A forecast sensitivity is defined as the gradient of a response function (*R*, any differentiable function of output of an NWP model) with respect to that model's initial condition, (e.g., the sensitivity is $\nabla_{\mathbf{x}_0} R$). The adjoint of a NWP model serves as a tool for the efficient calculation of these sensitivities (Errico 1997). Forecast sensitivities allow for the identification of those regions in which a small change to the initial condition, $\mathbf{x}_0$, of an NWP model will have largest effect on a particular aspect of that model's forecast. The sensitivity may be used to estimate the change in the response function, δR, for a specified change in the initial condition, $\delta \mathbf{x}_0$,

$$\delta R = \langle \nabla_{\mathbf{x}_0} R, \delta \mathbf{x}_0 \rangle , \qquad (1)$$

where $\langle \mathbf{x}, \mathbf{y} \rangle$ denotes the inner product of two vectors **x** and **y**.

We suggest that given measures of analysis errors, and a forecast sensitivity gradient for a specific response function derived from a single integration of an adjoint model, we may be able to estimate the likely ranges of values the response function may take for a site or regionally specific forecast. The approach to be outlined below, does not suffer from the computation burden of the SV calculation, but concerns about the validity of linear dynamics and the requisite number of ensemble members remain. In this presentation, we explore the use of adjoint-based forecast sensitivities and differences between operational analyses to construct an ensemble of forecasts, for specific forecast aspects. The goal of this research to determine whether the technique can provide forecasters with a practical objective method for predicting the skill of a single deterministic forecast assuming that the primary source of forecast error is imperfect specification of initial conditions rather than model error. As part of this work, we also seek to

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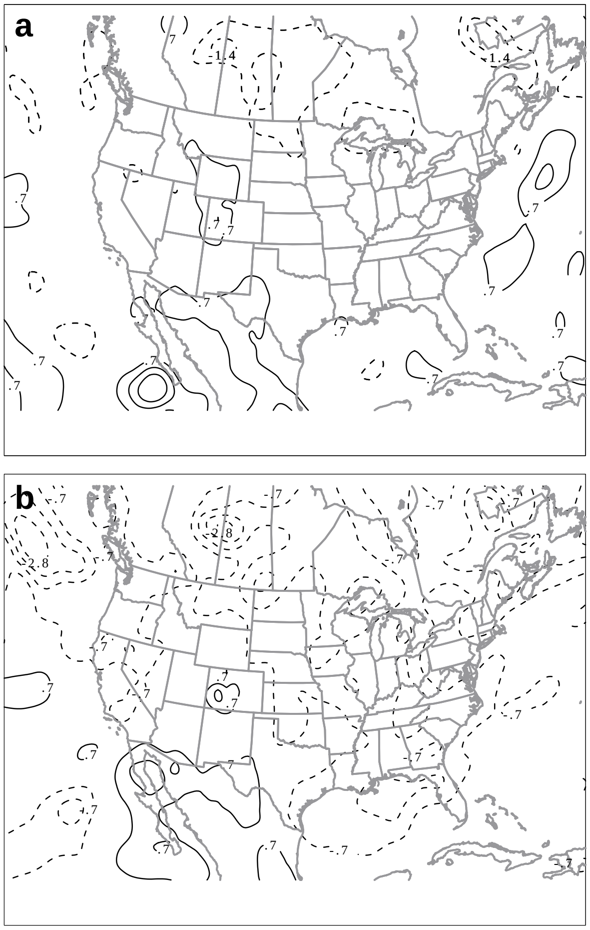

Figure 1. Differences between analyses of 650 hPa temperature in (a) NCEP's Eta and Aviation models and (b) NCEP Eta and UKMET models for 1200 UTC 29 September 2001. Contour interval 0.7^{o}C with negative values dashed.

determine objective measures of the validity of the tangent linear assumption, to evaluate the dependence of the forecast skill on characteristics of the larger scale flow, and to determine how large an ensemble is require to obtain a reliable estimate of bounds on the forecast.

In section 2 we present the methodology and motivation for this work. A summary and plan for further study is outlined in section 3.

2. MOTIVATION AND METHODOLOGY

2.1 Motivation

Provided that the model being used to calculate the forecast sensitivities may be viewed as "perfect," motivation for our approach comes from the results of a number of studies that suggest that cases of major forecast errors may be explained by defects in the initial analyses (e.g., Rabier et al. 1996). Because the "true state" of the atmosphere is not known, and because the analyses used to initialize operational NWP models may be viewed as best estimates for the state of the atmosphere at a given time, then differences between the analyses may be viewed as plausible errors in the analyses.

At any given analysis time, comparison of analyses within and between operational centers reveals that there may be considerable discrepancies between the analyses. An example of such a comparison is found in Fig. 1 which shows differences between the analyses of 650 hPa temperature from the National Center for Environmental Prediction's (NCEP's) Eta and Aviation models (Fig. 1a), and the Eta and United Kingdom Meteorological Office (UKMET) global model (Fig. 1b). There are clearly regions on these difference maps where discrepancies in the analyses exceed 1K. Furthermore, the differences in 650 hPa temperature between the Eta and Aviation models, are smaller than the differences between the Eta and UKMET analyses at this analysis time.

To the extent that we may use the differences between the analyses as representing analysis uncertainty, knowledge of this initial uncertainty together with knowledge of the forecast sensitivity may be used to estimate changes in the response function.

2.2 Methodology

While in principle, any differentiable function of the model forecast state could be used for this study, for simplicity in interpretation, and because of its potential operational interest, the response function chosen is the temperature averaged over the upper Midwestern United States on the sigma surface $\sigma = 0.85$.

The procedure for constructing the ensemble forecasts follows:

- 36 hr forecast sensitivities are calculated using the MM5 Adjoint Modeling System (Zou et al. 1997) in a horizontal domain (identical to that shown in Fig. 1 with 48x70 gridpoints) with 10 evenly spaced sigma levels. The adjoint model is run "dry" about a moist basic state calculated from a forward run of the nonlinear model initialized using the Eta model analysis interpolated to the MM5 grid. The response function R is also calculated for the forward model run.
- The analysis differences, $\delta\mathbf{x}_0$ are determined from the differences between the Eta, Aviation, UKMET, and Navy NOGAPS model analyses interpolated to the MM5 grid. From these four different analyses, we may construct 12 initial perturbations (6 positive, 6 negative).
- An *estimate* for the change in the response function, δR, is calculated using (1) for each of the 12 initial analysis perturbations. Because the calculation is linear, only 6 independent (positive) perturbations are necessary, as the change for the negative perturbations is determined by multiplying the result by -1. From this calculation, bounds on the response function may be determined from the largest δR calculated.
- As a check of the linearity, the change in R, ΔR, is evaluated from differences in non-linear model runs using the positive and negative perturbations. As an example, one may compute

$$\Delta R^{\pm} = R(\mathbf{x}_0^{eta} \pm \delta\mathbf{x}_0^{eta-avn}) - R(\mathbf{x}_0^{eta}) \qquad (2)$$

using the perturbation derived from the Eta and Aviation model analyses. This $\Delta R^{\pm}$ may then be compared with

δR .

For sufficiently large ensemble size, and for a 'realistic' estimate of the initial condition uncertainty, we may determine those periods for which the chosen response function has enhanced or decreased predictability.

Output and verification statistics from this study are archived and available in near-real-time at the URL:

http://helios.aos.wisc.edu.

3. SUMMARY AND OUTLINE OF FUTURE WORK

The use of adjoint based sensitivities in the construction of site or regionally specific forecasts has been proposed and the methodology of the approach has been outlined. The approach takes advantage of uncertainties in operational analyses to derive a set of initial condition perturbations that may be used in conjunction with adjoint-derived forecast sensitivity gradients to calculate bounds on the value of a particular forecast aspect. Compared with the cost of SV generated ensembles, which require a forward run of the NWP model, followed by multiple integrations of the adjoint and tangent-linear versions of the NWP model, the computational cost of generating a ***single*** adjoint-derived sensitivity ensemble is the cost of one forward nonlinear model integration followed by one adjoint model integration. Furthermore, we note that the choice of response function allows for the forecast to be *tailored to specific forecast needs* (e.g., forecasts for a particular site or region, forecasts of severe weather indices, wind speed, average temperature, precipitation) as long as the forecast problem can be expressed as a differentiable function of the model output.

In order for this approach to be practical, several questions must first be addressed. These questions include:

1) What is the minimum number of initial perturbations necessary to generate a useful forecast?

2) For N different analyses, there are $N(N\text{-}1)$ initial perturbations (including both positive and negative) that may be generated from simply taking analysis differences. At present we have a relatively small number of different analyses. Are there means of increasing the ensemble size using a limit?

3) What is the maximum length of time for which the assumption of linear perturbation evolution is valid?

4) Is there any relationship between the size of the forecast bounds compared with the skill of the forecast?

In addition to these questions, we will explore the relationship between the size of the forecast bounds to both the amplitudes of the sensitivity gradient and magnitude of the initial analysis differences.

4. REFERENCES

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