12.2 ANISOTROPIC BACKGROUND ERROR CORRELATIONS IN A 3D-VAR SYSTEM

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1 Introduction

Often, the assimilation of new radar and satellite data has little or even negative impact on the model analysis from current 3D- (and 4D-)Var systems. These shortcomings can be traced, to a large extent, to the over-simplified and unrealistic background and observation error covariances that are in use. One knows, in particular, that the background error covariance matrix provides the main vehicle by which information from the observation increments is propagated to those grid points and model variables that are not directly used to formulate the observation operator. Owing to the lack of computationally feasible approaches, current 3D-Var systems assume background error statistics that hardly account for the geographical variability of data quality and quantity. However, recent developments in the use of recursive filters (Purser *et al.*, 2002a,b) in data assimilation seem to provide an efficient way of accounting for the effects of covariances of spatially varying amplitude, scale and profile shape. In this work, we capitalize on the use of recursive filters to test a simple model for anisotropic background error covariances applied to the moisture field.

2 The Eta 3D-Var system

The 3D-Var problem is that of minimizing the cost-function

$$\begin{aligned} J &= \tfrac{1}{2}\left(\mathbf{x}-\mathbf{x}_b\right)^T \mathbf{B}^{-1}\left(\mathbf{x}-\mathbf{x}_b\right) \\ &+ \tfrac{1}{2}\left(\mathbf{y}-\mathcal{H}(\mathbf{x})\right)^T \mathbf{R}^{-1}\left(\mathbf{y}-\mathcal{H}(\mathbf{x})\right) , \end{aligned} \tag{2.1}$$

to obtain the analysis vector $\mathbf{x} \equiv \mathbf{x}_a$ from which the next forecast is made (*e.g.*, Daley 1991). In (2.1), $\mathbf{y}$ is the vector of observations, which is associated with the error covariance matrix $\mathbf{R}$, and $\mathcal{H}(\mathbf{x})$ is the model equivalent to $\mathbf{y}$. The background state vector and covariance matrix are $\mathbf{x}_b$ and $\mathbf{B}$, respectively. For lack of a better approach, $\mathbf{R}$ is assumed to be diagonal, which amounts to neglecting spatial correlations between the observation errors. The background error covariance matrix, on the other hand, is never assumed diagonal. It depends, among other factors, on the forecast model, the geographical location and the season of the year.

The preconditioned incremental 3D-Var adopted in the ETA 3D-Var system at NCEP is a computationally preferred variant of the original minimization problem (Courtier *et al.*, 1994; Courtier, 1997). It assumes weak nonlinearity of $\mathcal{H}(\mathbf{x})$ and uses a variable transform to obtain a better condition number for the matrix of the equivalent inversion problem. It also provides the advantage of directly operating with $\mathbf{B}$ rather than $\mathbf{B}^{-1}$. The system uses a large-scale minimization algorithm that requires a recipe for evaluating $\delta\mathbf{x} = \mathbf{Bf}$, given the vector of forcing terms $\mathbf{f}$. Due to the large size of $\mathbf{B}(\approx 10^7 \times 10^7)$, this becomes a numerically challenging aspect of the minimization procedure. Computational feasibility is attained by using physical insight into the background error structures: (i) $\mathbf{B}$ is approximated by a number of uncorrelated univariate covariance matrices via a variable transform and the use of balance constraints; (ii) the auto-correlations are modeled as functions of Gaussian shape; (iii) homogeneity and isotropy are assumed for the horizontal background error statistics. In addition, the ETA 3D-Var system uses numerically efficient recursive filters to approximate the convolution of spatial distributions of forcing terms with the Gaussian kernels of quasi-isotropic auto-correlation functions. The use of recursive filters parallels the multiple iteration of a diffusion operator adopted by Derber and Rosati (1989) to produced Gaussian covariances (see also Weaver and Courtier, 2001). However, they yield Gaussian smoothing kernels in fewer operations than are needed with the explicit use of the diffusion operator.

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3 The anisotropic correlation model

The recent work of Wu *et al.*, 2002, removes the constraint of homogeneity in the Eta 3D-Var system and demonstrates how recursive filters are able to efficiently account for spatially inhomogeneous covariances. The present work lifts the isotropy constraint, and thus complements that of Wu *et al.*.

Following Riishϕjgaard, (1998), we use the following anisotropic auto-correlation function for the moisture field:

$$C(\mathbf{x}_1,\mathbf{x}_2) = exp\left\{-\frac{(x_1-x_2)^2}{2L_{\mathbf{x}}^2(k)} - \frac{(y_1-y_2)^2}{2L_{\mathbf{y}}^2(k)} - \frac{(z_1-z_2)^2}{2L_{\mathbf{z}}^2(k)}\right\} \times exp\left\{-\frac{(q_1-q_2)^2}{2L_q^2}\right\}, \quad (3.2)$$

where $\mathbf{x}_1 = (x_1, y_1, z_1)$ and $\mathbf{x}_2 = (x_2, y_2, z_2)$ are the position vectors of the two points being correlated, and q_1 and q_2 are the relative humidity of the background fields at these points. $L_x(k)$, $L_y(k)$ and $L_z(k)$ are the local spatial correlation lengths along the model x, y and z directions, respectively. They depend on the model level k. For simplicity, we assume that $L_x(k) = L_y(k) \equiv L_h(k)$. L_q is the function correlation length, which is assumed constant over the model domain. The first term on the *r.h.s.* of (3.2) represents the original quasi-isotropic model, and the second term is aimed at improving the anisotropies in the resulting auto-correlation function. We note that this term is one along lines of zero gradient of the background field and less than one otherwise. Hence, it acts to stretch the auto-correlation function along the contour lines of the background field.

4 Results

We use the "NMC method" (Parrish and Derber, 1992) to define the background error statistics and evaluate the correlation lengths L_h and L_z. L_h is calculated from the error variances and their Laplacian, while L_z is estimated from the statistics of the covariance matrix for the vertical coordinate. For L_q, we experimented with values ranging from 90% down to 15%. As expected, the stretching of the auto-correlation function along the contour lines of the background field was found to be more pronounced as the value of L_q decreased. For the present work, we (arbitrarily) adopted the value L_q=20.

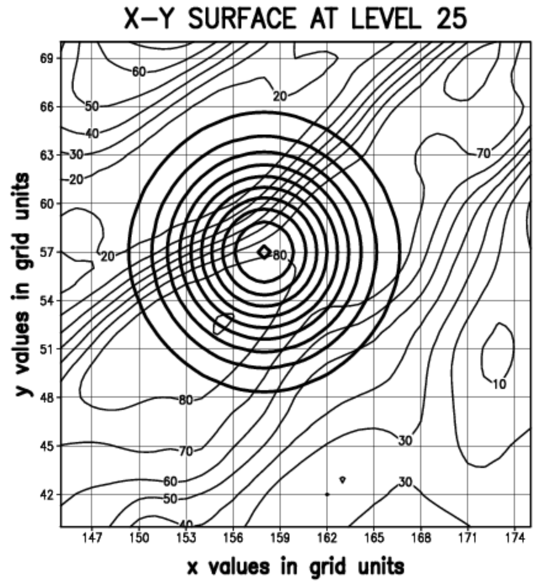

Figure 1: The auto-correlation function for the quasi-isotropic model at 0.1 contour interval (thick contour lines) and the background relative humidity in % (light contour lines) on the x-y surface. The outermost contour value for the auto-correlation function is 0.1. The full analysis grid posses 226 points in the x-direction, 184 in y-direction and 50 vertical levels.

The effect of the background error covariance matrix on the forcing terms is approximated by applying a recursive filter. If $\mathbf{F}$ is the linear operator of the recursive filter, then we write $\mathbf{B}_{rh}$=$\mathbf{EFF}^T\mathbf{E}$, where $\mathbf{B}_{rh}$ is the auto-covariance matrix for the moisture field and $\mathbf{E}$ is a diagonal matrix of error variances. We note that, under the assumption of isotropy and Gaussian correlation model, the filtering proceeds in the model x, y and z directions only. In the presence of anisotropies, however, the filtering directions become six, and are determined by the appropriate local "aspect tensor" (Purser *et al.*, 2002b).

We test the response of the filter by inputting column test vectors $\mathbf{v}$ comprised of zeros, except at a selected row, where the value is one. That row corresponds to a given test point in the model grid, selected by visual inspection of the background field. Of interest are points that simultaneously lie close to regions of large and small gradient of the background field. For one such point (x=158, y=57 and vertical level 25), Figs. 1, and 2 show the filter response ($\mathbf{w}=\mathbf{F}\mathbf{F}^T\mathbf{v}$) for the original quasi-isotropic correlation model. The background field is also shown, and corresponds to the Eta-analysis for 1200 UTC 1 February 2002. The fields are displayed on the x-y, and y-z surfaces. For clarity, only portions of the analysis grid are displayed. As expected, the auto-correlation function is perfectly isotropic on the x-y surface and quasi-isotropic on the y-z surface. The quasi-isotropy results from the fact that $L_h \neq L_z$.

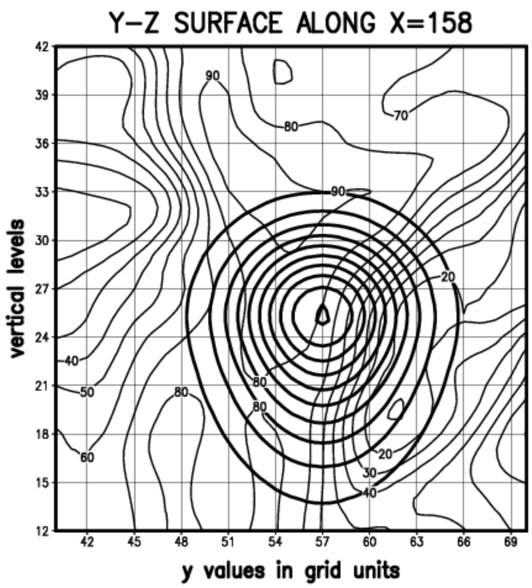

Figure 2: As in Fig. 1, but for the y-z plane

The response of the filter for the above anisotropic model at the same test point is shown in Figs.3 and 4. We see, on both surfaces, the desirable anisotropy of the contour lines, which now stretch

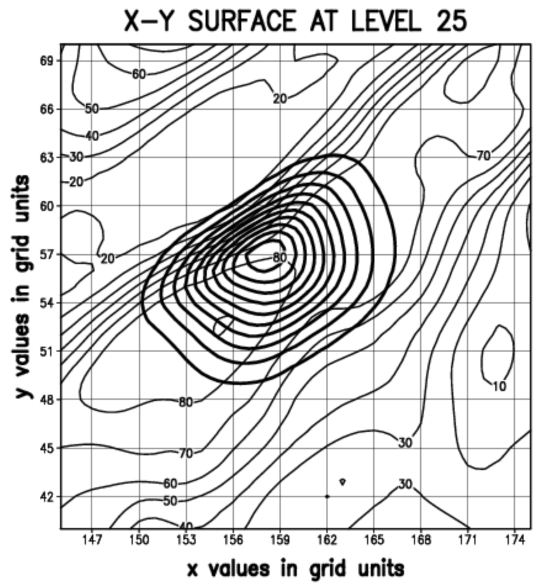

Figure 3: As in Fig. 1, but for the anisotropic model

along the contours of the background field. In the regions of sparse gradient, however, the response resembles in shape that for the isotropic model.

5 Summary

Relaxing the homogeneous and isotropic assumption is an important step toward improving the Eta 3D-Var system as the observation density and quality is location dependent. An anisotropic correlation model has been used that captures the observational evidence that auto-correlations are large along contours of constant background field and short along the direction of the field gradient. Results from the use of the anisotropic model in a 12-hour assimilation cycle followed by a short-range forecast will be presented at the conference. A discussion will be offered on how to "optimally" choose the filter parameters as well as extend the correlation model to the mass and wind analysis variables. Some thoughts will also be presented on the use of

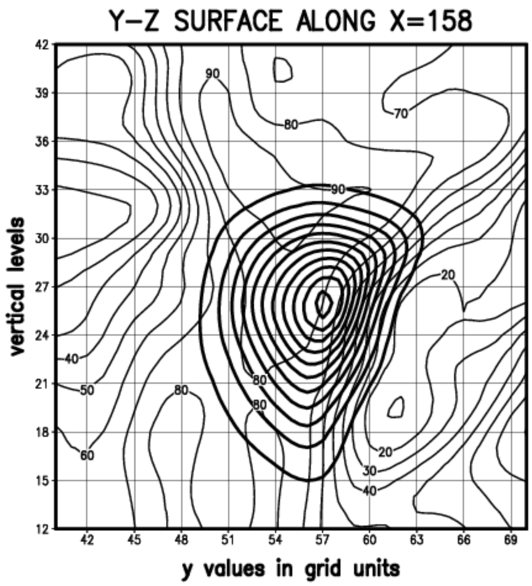

Figure 4: As in Fig. 2, but for the anisotropic model

ensemble forecasting to determine the appropriate filtering directions.

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