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1. INTRODUCTION

The Conrad Blucher Institute Division of Nearshore Research operates about 60 platforms along the coast of Texas collecting water level measurements and other meteorological parameters such as wind speed, wind direction, and barometric pressure (Michaud 2001). However, tidal forecasts are available only for the small number of stations for which the National Ocean Service (NOS) provides harmonic constants. Harmonic constants are often not published as there are many factors which cause water level to deviate from tidal predictions. There is a great need to be able to obtain harmonic constants and water-level predictions in support of various research projects (e.g., Tissot 2002, Cox 2002, Drikitis 2002).

The Conrad Blucher Institute for Surveying and Science at Texas A&M University-Corpus Christi has developed the *HarmAn* and *HarmPred* programs to provide tidal forecasts for most of its stations and to make the process of generating harmonic constants and tide forecasts more accessible. The *HarmAn* program implements NOS's harmonic analysis method (Zetler 1982) to compute constants from reliable data sets and a set of tidal constituent waves. To calculate harmonic constants, data sets need to be dependable water level readings of at least one year for the target station. *HarmPred* utilizes the harmonic constants derived by *HarmAn* (or obtained from other sources) to generate water level predictions. These programs are written in Perl/PDL. Perl is a popular programming language for data manipulation and PDL is a Perl module which provides matrix-based numeric calculations.

2. HARMONIC CONSTANT CALCULATION

HarmAn is a Perl/PDL program developed to determine harmonic constants from previously collected data sets. Calculation of harmonic constants requires at least one year of hourly water level recordings, preferably with less than 2% missing data (however up to 10% may be adequate).

HarmAn uses three files as input: a file containing an array of tidal constituent names to be used in the analysis; a file containing a table of constituent names, speeds, node factors and equilibrium arguments; and a third file of water level observations. An array of constituent names, **c**, is created and the number of constituents to be used in the analysis is counted in the

variable *M*. An array of speeds, **a**, is created for the constituents listed in the first file. Matrices of node factors **f** and equilibrium arguments **e** are created. Each row of the **f** and **e** matrices contains one year of node factors and equilibrium arguments, respectively, for the constituents from the first file. The node factors and equilibrium arguments are described in Schureman (1958) and have been obtained from Flater (2001) and Zetler (1982).

Next is the file of observed hourly water levels. Three arrays are created: an array **h** of observed water levels, an array **t** containing the time of observation in hours from the beginning of the year, and an array **y** of indexes into the equilibrium and node factor arrays. The number of hourly observations is recorded in the variable *N*.

A linear least-squares fit is used to calculate amplitudes **H** and phases **k** for the selected constituents. From Zetler (1982), each observed height is made an observation equation

$$h_t = h_0 + \mathbf{S} X_i \cos(a_i t) + \mathbf{S} Y_i \sin(a_i t)$$

where h_t is an observed height at time t , h_0 is the mean height of the tide above a datum, a_i is the speed of constituent i , and X_i and Y_i contain the unknowns **H** and **k** and are defined as

$$X_i = H_i \cos k_i, \quad Y_i = H_i \sin k_i.$$

Each observation equation is linear with respect to h_0 , X_i and Y_i ($2M + 1$ unknowns) and the entire system of N equations can be solved using a linear least-squares fit. We first create an $M \times N$ matrix **z** containing the constituent speeds multiplied by the observation times

$$\mathbf{z}_{M \times N} = \mathbf{a}_{1 \times M}^T \mathbf{t}_{1 \times N}.$$

The equilibrium arguments corresponding to the time of each observation are then added to each row of **z**. From this matrix we create **cosz** and **sinz** matrices containing the cosine and sine of **z**. Each row of these two matrices are then multiplied by the node factors corresponding to the time of each observation. Finally, a $(2M+1) \times N$ matrix containing the system of linear observation equations is created by concatenating a $1 \times N$ matrix of ones with the **cosz** and **sinz** matrices. A least squares fit of this matrix results in a $(2M+1)$ array of elements where the first element is the estimate for H_0 , the next M elements are the estimates for X_i , and the last M values are the estimates for Y_i . The estimates for the constituent amplitudes **H** and phases **k** are then given by:

$$\mathbf{H} = (\cos^2 \mathbf{X} + \sin^2 \mathbf{Y})^{1/2}$$

and

$$\mathbf{k} = \tan^{-1} (\sin \mathbf{Y} / \cos \mathbf{X}).$$

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3. TIDAL PREDICTION CALCULATIONS

HarmPred is a Perl/PDL program used to predict the tide through harmonic analysis. *HarmPred* takes three inputs: a file of constituent names, amplitudes and phases for the target station (e.g., derived via *HarmAn*); a file of constituent speeds, equilibrium arguments and node factors; and the desired time of prediction.

Arrays consisting of the constituent names, amplitudes and phases are created. The speeds, node factors and equilibrium arguments are extracted and stored as in the *HarmAn* program. Next, the beginning and ending time for prediction is determined. Once all the data are present, a water level prediction h_t for time t can be made by using the formula

$$h_t = \sum H_i f_{i,t} \cos(a_i t + e_{i,t} - k_i)$$

where a_i is the speed of constituent i , H_i and k_i are the amplitude and phase for constituent i from the harmonic constants, and $f_{i,t}$ and $e_{i,t}$ are the node factors and equilibrium arguments for constituent i at time t .

4. PREDICTION COMPARISONS

The National Ocean Service (NOS) has an online tool for generating water-level predictions for stations where harmonic constants have already been published by NOS; this system is available at <http://co-ops.nos.noaa.gov> (National Ocean Service 2002). We have compared the *HarmPred* output with the Co-ops output for the Rockport, Texas, station; the average difference between the two outputs is 0.14mm.

Table 1 compares the amplitudes (in millimeters) published by NOS with the amplitudes derived by the *HarmAn* program for the Rockport, Texas, station. Table 2 compares the phases (in degrees) published by NOS with those generated by *HarmAn*. As Table 1 indicates, the greatest differences in amplitudes occur in the SA and SSA constituents. These constituents are supposed to account for non-uniform changes in the Sun's declination and distance, but in actuality they are a reflection of meteorological variations influencing sea level (Hicks 1999).

TABLE 1

Constituent Tide	NOS amplitude (mm)	HarmAn amplitude (mm)
J1	2	1
K1	30	28
M1	2	1
M2	8	7
O1	30	29
OO1	3	3
P1	8	8
Q1	6	6
2Q1	1	1
S1	4	4
RHO1	1	1
SA	61	60
SSA	90	109

TABLE 2

Constituent Tide	NOS phase (degrees)	HarmAn phase (degrees)
J1	107.4	351.02
K1	104.7	15.27
M1	102.0	57.78
M2	332.0	155.81
O1	99.3	16.97
OO1	123.6	25.64
P1	103.6	12.03
Q1	76.4	3.89
2Q1	93.9	330.73
S1	350.4	259.44
RHO1	97.0	4.44
SA	161.0	162.45
SSA	69.9	57.50

In Table 2, note that large differences in phase values are not significant to the output when a constituent's amplitude is small. Thus the O1 constituent is the only constituent with a significant difference in phase values. The SSA constituent is a slow moving semi-annual harmonic constituent, and thus a phase difference of a few degrees does not significantly change the tidal component.

4.1 Effects of Missing Data in Harmonic Analysis

One of the principal concerns in deriving harmonic constant values and predictions from observational data is the effect of missing data on the resulting output. Ideally the observational record will be complete; in reality, device malfunctions may lead to significant degradations of the output.

The initial results of an investigation into the effect of missing observations in input data are given in Tables 3 and 4 below. This investigation used water-level observations from Bob Hall Pier (Corpus Christi, Texas) to generate multiple sets of input data for use with the *HarmAn* program. In addition to the complete set of observation data, "incomplete" input sets were created by excluding individual months and groups of three consecutive months from the input data. The *HarmAn* program was then used with *HarmPred* to compare the predictions resulting from the harmonic analysis of an incomplete set with the predictions resulting from a complete set.

Table 3 displays the mean difference, standard deviation, and maximum difference in millimeters of the predictions generated from an incomplete data set as compared to the predictions generated from a complete data set. All values are in millimeters. As Table 3 illustrates, omitting a month of data from the input data set affected the resulting output by as much as 41 millimeters for some predictions, but in general the overall impact of omitting the month is relatively small. Omitting three months of data produce more profound effects, with maximum changes of as much as 132 millimeters and much higher variance in the differences.

TABLE 3

Starting Month	Missing 1-month			Missing 3-months		
	Mean	Stdev	Max	Mean	Stdev	Max
Jan	5.38	11.91	41	17.91	34.05	126
Feb	-3.48	8.98	-33	-7.40	20.92	-75
Mar	3.25	12.34	40	0.51	13.50	-41
Apr	-2.04	4.95	-18	10.67	21.28	70
May	0.84	7.54	20	15.51	32.18	110
Jun	3.59	7.67	27	22.03	39.87	-132
Jul	-3.36	9.42	-35	-16.41	31.18	-112
Aug	-3.34	10.57	-38	7.32	22.78	84
Sep	3.38	10.48	38	15.38	30.46	104
Oct	0.18	7.53	-21	-19.08	31.94	-98

Finally, an analysis was performed on the effects of missing data on the amplitudes of the harmonic constants. Table 4 shows the standard deviation maximum differences resulting from omitting one or three months of data from the 2001 input sets. Again, this table shows that missing a single month of data has relatively small impact on the constituent amplitudes regardless of the missing month (maximum difference of 10 millimeters for the SSA component), while 3 months of missing data will have greater impact. As expected, missing data has the greatest impact on the longer-period harmonic constituents such as SA and SSA.

5. CONCLUSION

HarmAn and *HarmPred* are two Perl/PDL scripts that have been developed to support the harmonic analysis and prediction of water levels from the Texas Coastal Ocean Observation Network. These programs simplify the tasks of producing harmonic constants from water-level observations and generating water-level predictions from harmonic constants. The outputs of the two programs have been compared to results published on the National Ocean Service's Co-Ops web site. A preliminary investigation into the impact of missing data on harmonic predictions has been performed. The results of this investigation have set initial guidelines for continued research into the use of harmonic analysis to support water-level forecasting along the Texas coastline: data sets used to create harmonic constants should be a year in length and have not more than one month of missing data.

6. ACKNOWLEDGEMENTS

The work presented in this paper is funded in part by the Texas General Land Office (TGLO) and the National Oceanic and Atmospheric Administration Coastal Management Program (CMP). The views expressed herein are those of the authors and do not necessarily reflect the views of TGLO, NOAA or any of their subagencies.

TABLE 4

Constituent Tide	1-month omission		3-month omission	
	StDev	Max	StDev	Max
J1	0.43	1	1.06	2
K1	0.49	1	2.06	3
K2	0.49	1	0.71	2
L2	0.00	0	0.00	0
M1	0.00	0	0.82	1
M2	0.49	1	0.48	1
M3	0.47	1	0.42	1
M4	0.28	1	0.00	0
M6	0.50	1	0.53	1
M8	0.28	1	0.48	1
N2	0.37	1	0.88	2
2N2	0.43	1	0.42	1
O1	0.80	2	1.62	4
OO1	0.28	1	0.67	1
P1	0.60	1	1.65	3
Q1	0.43	1	0.74	1
2Q1	0.64	2	1.29	2
R2	0.37	1	0.70	1
S1	0.49	1	1.23	2
S2	0.50	1	0.70	1
S4	0.28	1	0.48	1
S6	0.28	1	0.42	1
T2	0.00	0	0.57	1
LDA2	0.00	0	0.42	1
MU2	0.49	1	0.70	1
NU2	0.00	0	0.32	1
RHO1	0.43	1	0.79	1
MK3	0.28	1	0.42	1
2MK3	0.00	0	0.00	0
MN4	0.00	0	0.32	1
MS4	0.28	1	0.48	1
2SM2	0.28	1	0.48	1
MF	3.63	5	5.00	10
MSF	4.48	8	9.40	19
MM	3.24	6	5.38	9
SA	3.86	7	14.69	27
SSA	4.23	10	16.32	34

7. REFERENCES

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