P5.16 AN IMPROVED BACKUS-GILBERT SPATIAL FILTER FOR SATELLITE DATA PROCESSING

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1. INTRODUCTION

A computationally efficient discrete Backus-Gilbert (BG) method is derived (Stephens and Jones, 2002) that is appropriate for resolution-matching applications using over-sampled data. The method in its current form is restricted to a resolution-only minimization constraint, but in the future could be extended to use a simultaneous noise minimization constraint using a generalized singular value decomposition (SVD) approach. In 1-D simulated comparisons, the discrete BG (DBG) method is shown to be 250% more efficient than the original BG method while maintaining similar accuracies. In addition, a SVD approximation increases the computational efficiencies an additional 43% to 106%, depending upon the scene. The ability to recompute the modified BG coefficients dynamically at lower computational cost make this work applicable toward applications in which noise may vary, or where data observations are not available consistently (e.g., in RFI contaminated environments).

2. BACKGROUND AND MOTIVATION

The Backus-Gilbert method (Backus and Gilbert, 1970) has been employed by various authors to spatially co-register and invert various data sets while accounting for different spatial and error propagation behaviors. The original work of BG provides a rigorous mathematical basis for the inversion of inaccurate data. Later Stogryn (1976) applied it to the specific problem of microwave footprint matching, and further developed concepts from BG that are the basis of most BG footprint-matching applications today. A key feature of the BG method is that it can be used effectively to trade instrument noise for spatial resolution and vice versa. This flexibility is a fundamental strength of the BG approach.

The mathematics community has made progress in the understanding of regularization methods such as the Backus-Gilbert approach. In particular, this paper builds upon the work of Stogryn (1976) and Poe (1990) and that of Hansen (1994).

Stogryn (1976) introduces an optional additional minimization constraint on the noise amplification so that resolution and noise are both minimized simultaneously to various degrees by varying a parameter, γ. For this work, the case where $\gamma = 1$ is used to simplify the intercomparison analysis. Physically this corresponds to a pure resolution minimization constraint. The inclusion of $\gamma \neq 1$ would require the use of a generalized SVD (GSVD) optimization approach (Hansen, 1994), since several key matrices are no longer purely diagonal, and become interdependent on the gain function. Utilization of a GSVD approach is beyond the current scope of this work. Thus, the practical applications of the DBG methodology as described should only be used where sufficient data overlap occurs (i.e., where noise amplification is not a serious concern (Bennartz, 2000)).

This work has practical implications for the utilization of BG methods within the earth sciences community. For example, a long-standing problem with the application of BG to earth science remote sensing has been the computational expense of calculating the coefficients necessary for the method (Galantowicz and England, 1991). Current applications typically assume that the sensor and noise contributions are stable and that the coefficients can be assumed static. However, in an era of increasing RFI, the relatively benign radiometric operating conditions that the remote sensing community has enjoyed may be part of a passing era. Thus, methods that are more dynamic are needed to cope with such possible changes that threaten the performance of more traditional BG implementations. In addition, certain computationally intensive applications involving remote sensing may impose demanding computational restrictions that traditional BG methods are not able to accommodate. In this vein, a new DBG method is created which is computationally more efficient, and operationally flexible in its configuration. In the new method, it will be shown that computational performance can be dynamically traded for method accuracy. This allows the method to expend CPU cycles where the spatial data analysis is most critical and vice versa.

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In particular, the DBG method explicitly specifies the integration approach, which facilitates several optimization techniques. This paper specifically addresses two optimizations: the diagonalization of several matrices following the Hansen integration approach, and the SVD approximation technique. It should be noted that the optimization improvements are compounding effects, in that the matrix diagonalization increases the computational performance by more than 250% for some scenes, while the SVD performance gains are in addition to that increase in performance. Several other optimizations remain unexplored that could exploit the flexibility of the DBG integration form; these include adaptive grid methods, customized or dynamic quadrature rules, and other possibilities.

3. DBG METHOD

Hansen discretizes the BG method using the simple rule (Hansen, 1994):

$$\int G_i(x)\,dx \approx \sum_{k=1}^{N} w_k G_i(x_k)\,, \tag{1}$$

where N is the number of discrete integration intervals, and w_k are the integration weights. This allows the BG method to be expressed as a product of vectors and matrices. By combining the Hansen discretization with the Stogryn minimization constraints and conditions, a modified set of coefficients, a_i, can be defined. The modified coefficients can be written in matrix form as

$$\mathbf{a}(x_0) = \mathbf{M}\left[\mathbf{v} + \frac{1-\mathbf{u}^{\mathrm{T}}\mathbf{M}\,\mathbf{v}}{\mathbf{u}^{\mathrm{T}}\mathbf{M}\,\mathbf{u}}\mathbf{u}\right], \tag{2}$$

where subcomponents of the matrix $\mathbf{M}$ are now diagonalized, resulting in a more computationally-efficient and flexible form. The reader is referred to Stephens and Jones (2002) for complete details of the DBG derivation.

4. DBG RESULTS

A series of simulations were performed in which the number of integration intervals used within the DBG method, N, was modified to determine the computational cost of the method versus the method's RMS accuracy. For this numerical experiment, the number of measures, M, is set to 100. The number of integration points, N, is adjustable. The Stogryn method is used as a control.

Figure 1 presents results for a uniform scene with random noise imposed. The DBG method RMS values are generally lower than the Stogryn RMS values. This is likely due to a more optimal integration pattern for the random distribution, since the DBG integration is evenly spaced. The numerically optimized integration weights used in the Stogryn method may also be amplifying the effect of the various random patterns within the data set series. The trend in the DBG results (Figure 1) is toward lower RMS values with larger N. This corresponds to improved integration accuracy with larger N. The relative CPU consumption for the DBG method increases with N as $\sim N^2$. The computational costs between the methods are approximately equivalent at $N \cong 220$. These results show that significant computational improvements can be made by reducing the number of DBG integration intervals, such that at N =100, the relative DBG computational efficiencies are 250%. This is a result of the matrix diagonalization in the DBG equation form.

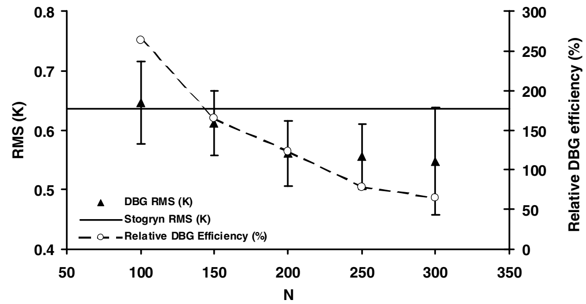

Figure 1. The RMS performance (K) for the uniform scene simulations for the DBG method (DBG) and the Stogryn method versus the number of integration points, N. Also shown is the relative DBG computational efficiency (%) as compared to the Stogryn computational costs.

5. SVD ANALYSIS AND RESULTS

The SVD of a general real $M \times N$ matrix allows the target matrix to be separated into left and right singular vectors with the definition of appropriate singular values. The practical consequence of this is that it allows the matrix to be handled via summations, and additional approximations become available to increase the overall performance of the method. It should be noted that the SVD computational savings are in addition to the computational savings that result from the inherent form of the DBG method over traditional methods. The SVD form of the DBG is an explicit optimization technique that specifically exploits the DBG diagonal matrix form.

The summations in the SVD of the DBG method can be truncated at any desired point. Figure 2 shows a sequence of results in which additional terms are progressively added to the SVD summation, for the case where $M = 100$. It can be seen that the effect is similar to that of a Fourier series. The first few terms give the general average and scaling of the spatial features, and additional terms refine the spatial structure of the results.

The RMS performances of the SVD results are also calculated for three simulated truth scenes with random noise of 5 K imposed (a uniform scene, a step function scene, and a sine wave scene). The results (Table 1) are defined as ratios in which the SVD DBG RMS results are normalized relative to the non-SVD DBG RMS results. Corresponding standard deviations are also calculated. RMS ratio results less than one indicate an improvement using the SVD approach. The computational cost ratio is defined similarly, with the cost ratio being defined as the SVD cost relative to the non-SVD cost. The computational costs are independent of the specific scene. These results show that the RMS errors decrease as the number of summation terms is increased. Table 1 also shows a linear relationship between computational costs and the number of summation terms, which is expected. In practice, 20% of the terms generally gave sufficient structure, low RMS values, as well as minimizing computational costs for the particular simulated scenes explored here. However, it should be pointed out that performance was dependent on the particular scene. For the uniform scene, all SVD RMS results were significantly improved, with RMS values less than 50% of the non-SVD RMS results, while the step function scene results were only marginally improved. The sine wave scene was the most difficult scene for the SVD approach and resulted in errors that were 30% greater than the non-SVD results. The choice of where to truncate the SVD series will in general be dependent on the specific application and its associated error tolerance requirements.

6. CONCLUSIONS

The flexibility of the DBG method allows it to trade computational cost for accuracy, thus lending itself to several challenging research application areas. In particular, the use of a more flexible method would serve well in applications where the BG coefficients need to be routinely recalculated, depending on conditions, such as in a RFI contaminated environment. The flexibility of the DBG method also allowed for several of the optimizations to be performed in a rather straightforward manner. Many additional optimizations are likely possible.

Future work will investigate additional computational enhancements and test the scope and validity of those assumptions. The method will also be used in future cross-sensor data fusion application work (Jones and Vonder Haar, 2002). Research applications in which multi-frequency spatial resolution behaviors are important [e.g., in use with variational satellite data assimilation methods (Jones et al., 2003), and satellite sounding applications (McKague et al., 2003)] will also benefit from the DBG method improvements.

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TABLE 1

SVD DBG RMS Performance Relative to the Non-SVD DBG RMS Performance

	Uniform Scene		Step Scene		Sine Scene		Relative Cost	
%*M*	RMS	σ	RMS	σ	RMS	σ	AVERAGE	σ
100	0.50	0.09	0.991	0.003	1.30	0.03	0.699	0.009
50	0.47	0.05	0.993	0.004	1.31	0.03	0.580	0.006
20	0.43	0.07	0.992	0.004	1.31	0.03	0.519	0.010
10	0.43	0.07	1.062	0.004	1.46	0.03	0.486	0.002

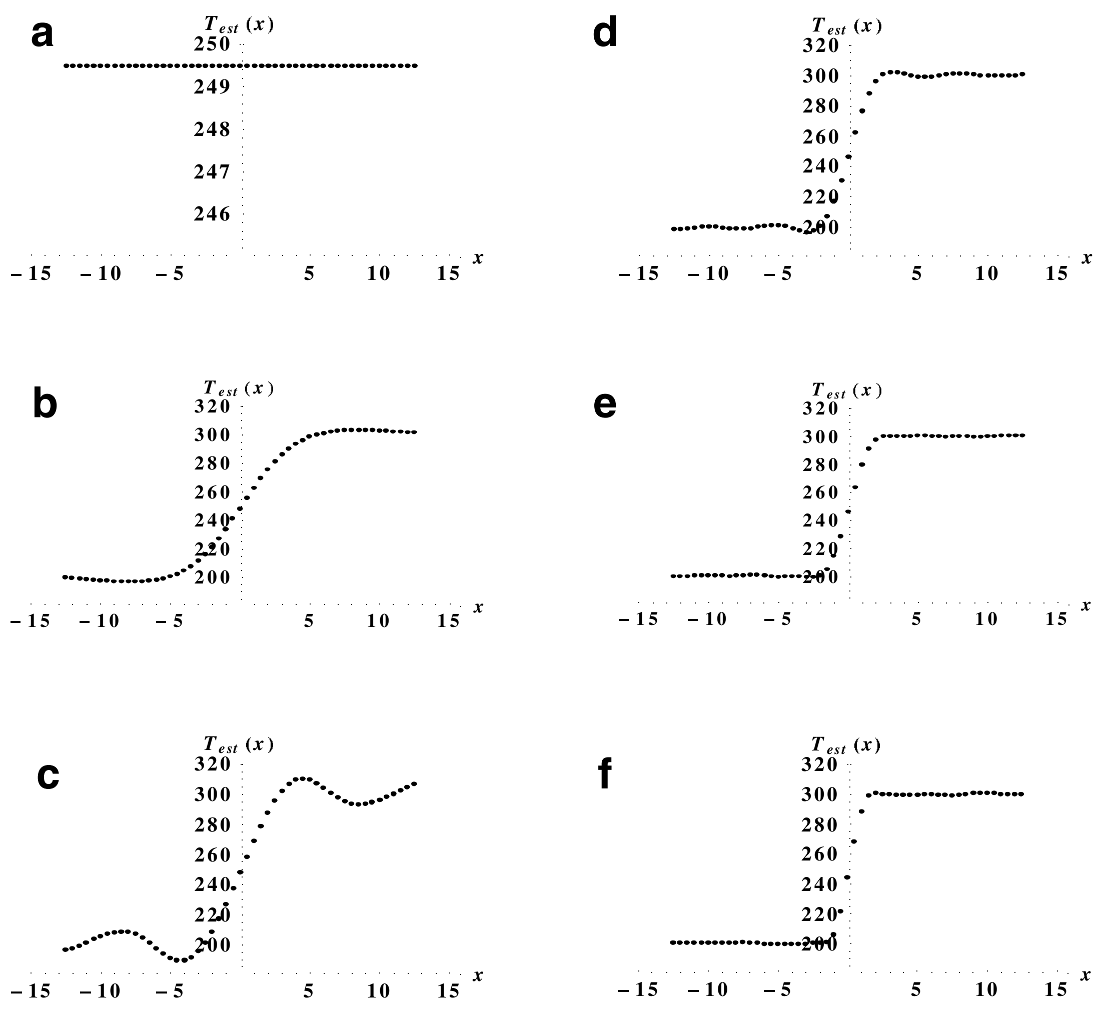

Figure 2. The summations in the SVD of the DBG method can be truncated at any desired point with reduced fidelity. A sequence of results are shown in which additional terms are progressively added to the SVD summation, where M = 100, and a) has 1% of M terms, b) 3%, c) 5%, d) 10%, e) 15%, and f) 100%.