5.10 SCALED TEMPERATURE SPECTRUM IN THE UNSTABLE OCEANIC SURFACE LAYER

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1. INTRODUCTION

More than thirty years ago a pioneering investigation of turbulence in the atmospheric surface layer was carried out over flat uniform terrain in Kansas (Kaimal and Wyngaard, 1990). The Kansas experiment revealed horizontal wave number properties of scalar and wind fields; when normalized in accordance with Monin-Obukhov (M-O) similarity theory, Kansas spectra of velocity and temperature reduce to a family of curves that depend on the M-O stability parameter at low wave number and reduce to a single universal curve in the inertial sub-range (Kaimal et al., 1972). Several approaches have been used to model surface layer spectra (e.g., Hojstrup, 1982; Kristensen et al., 1989; Mann, 1994; Peltier et al., 1996). Peltier et al. modeled unstable atmospheric surface layer spectra by assuming homogeneity and isotropy in the horizontal directions. Their model has a single form for the two-dimensional spectrum of horizontal velocity, vertical velocity, and a scalar in the horizontal plane.

Observations of turbulence in the oceanic surface layer are relatively rare in comparison to the numerous and extensive investigations of turbulence in the atmospheric surface layer. Recently, Wijesekera et al. (2001) examined horizontal wave number spectra of temperature in the unstably stratified oceanic surface layer similar to that of Kaimal et al.'s study. Wijesekera et al. showed that when spectra were normalized according to M-O similarity theory, the shapes of the oceanic spectra agree qualitatively with observed (e.g., Kaimal et al., 1972) and modeled (Peltier et al., 1996) atmospheric spectra including the wavelength of peaks and the variation of peak wave number with stability. However, spectral levels in the energy containing range differ by as much as a factor of two. Wijesekera et al. also noted a difference between oceanic and modeled (Peltier et al., 1996) near-neutral spectral levels in the inertial sub-range, and suggested that turbulent kinetic energy (TKE) dissipation could be enhanced (up to a factor of three) by surface wave breaking.

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The objective on this paper is to develop a generalized form of the temperature spectrum in the unstable oceanic surface as a function of stability parameter $\xi = z/L$, and surface wave parameter $\gamma = (u_*/gz)^{1/2}$, where z is the depth of the measurement, L is the M-O length scale, u_* is the surface friction velocity, and g is the acceleration due to gravity.

2. SPECTRAL MODEL

Following Peltier et al. (1996), we model the two-dimensional spectrum of a conservative scalar (e.g., temperature, salinity) with the form

$$S(k) = \frac{a_0 l^2 s^2 k}{\left[a_1 + a_2(kl) + (kl)^2\right]^{4/3}} \ . \qquad (1)$$

Note that $a_2 = 0$ for the Peltier et al.'s spectrum. The assumed form of our scalar spectrum (1) is horizontally isotropic. The magnitude of the two-dimensional wave number is, $k = (k_1^2 + k_2^2)^{1/2}$, where k_1 and k_2 are horizontal wave number components. For $kl >> 1$, $S(k) \sim k^{-5/3}$, and for $kl << 1$, $S(k) \sim k$; l and s are characteristic length and intensity scales, respectively; a_1 and a_2 vary with the stability parameter ξ; a_0 is determined from inertial sub-range parameters. By scaling the one-dimensional form of (1) from M-O similarity theory, we obtain the normalized, wave number weighted spectrum

$$S_T^N(n) = \frac{k_1 S_1(k)}{\theta_*^2 \phi_\chi \phi_\varepsilon^{-1/3}}$$
$$= \frac{\beta_T}{(2\pi\kappa)^{2/3}} \frac{n}{\left[c_1 + c_2 n + n^2\right]^{5/6}} A(\gamma)^{-1/3} \ , \qquad (2)$$

where β_T (=0.8) is a universal constant, κ (=0.4) is the von-Karman constant, $a_1 = (2\pi)^2\left(c_1 + 0.25c_2^2\right)$, and $a_2 = 2\pi c_2$. The non-dimensional wave number $n = k_1 z$; ϕ_ε and ϕ_χ are non-dimensional dissipation rate ε, and turbulent temperature variance (TTV) dissipation rate χ; θ_* is the temperature scale based on friction velocity and surface heat flux. The enhancement of ε due to surface wave processes is

$$A(\gamma) = \varepsilon(\ \phi_\varepsilon\, u_*^3 / \kappa\, z)^{-1} \ . \qquad (3)$$

As indicated in (2), the magnitude of the spectral-peak depends on both stability and wave breaking, while the wave number of the peak,

$$n_{\max} = 0.125\left[c_2 + \sqrt{c_2^2 + 96 c_1}\right] \tag{4}$$

depends only on stability.

We obtain the normalized TTV by integrating (1),

$$\overline{T'^2} = \int_0^\infty S(k)\, dk \ , \tag{5}$$

and then scaling (5) in accordance with M-O similarity,

$$\frac{\overline{T'^2}}{\theta_*^2} = 2.1 \frac{\beta_T}{(2\pi\kappa)^{2/3}} B(\xi) A(\gamma)^{-1/3} \ , \tag{6}$$

where

$$B(\xi) = \frac{\phi_\varepsilon^{-1/3}\phi_\chi}{\left(c_1 + c_2^2/4\right)^{1/3}} \left\{1 - \frac{1.1 c_2 \left(c_1 + c_2^2/4\right)^{1/3}}{3 c_1^{5/6}}\right\} .$$

3. MODEL PARAMETERS

We empirically determined c_1, c_2, and A from observations reported by Wijesekera et al. (2001). Wijesekera et al. used underway measurements of temperature from a bow boom at a depth of 2 m to examine spectral characteristics in the wavelength band from 2 m to 2 km. A total of 306 2-km segments were selected for analysis, in which $0.06 \leq -\xi \leq 4.2$, and $0.0005 \leq \gamma \leq 0.0033$. Two samples of wave number weighted spectra are shown in Fig. 1. In general, the low wave number end of the spectrum was contaminated by meso-scale motions (~ 100 m to 2 km wavelength range), which can be identified from the red spectral slope (e.g., Fig. 1a). The meso-scale contamination was relatively weak for spectra observed at moderate winds, during which the spectra appeared to follow a +1 slope (e.g., Fig. 1b). The meso-scale contribution for a given spectrum was removed by fitting a +1 slope for low wave numbers (Fig. 1).

Independent estimates of A were obtained by matching the inertial sub-range spectral levels of model and observations. The values of c_1 were estimated by matching spectral levels at the +1 slope, and values of c_2 were estimated by forcing the model spectrum to pass through the peaks of the observed spectra. General expressions for c_1, and c_2 as a function ξ, and A as a function of γ were obtained from least square fits. The resulting approximate functional forms are:

$$c_1(\xi) = a_1\left[1 - b_1\, e^{-\xi^{1/2}}\right] , \tag{7a}$$

$$c_2(\xi) = a_2\left[1 - b_2\, e^{-\xi^{1/2}}\right] , \tag{7b}$$

and

$$A(\gamma) = \left[1 + a_3\, e^{-\gamma^{1.2}/b_3}\right]^{-3} \ , \tag{7c}$$

where a_1 = 0.009, b_1 = 0.85, a_2 =-0.047, b_2 = 1.45, a_3 = 110, and b_3 = 0.0015.

The analytical form of (2) for $A(\gamma)$=1 falls within the neutral and the free convection limits of Peltier et al.'s prediction (Fig. 2).

4. COMPARISONS INCLUDING LES

In Fig. 3, we compare *rms* turbulent temperature fluctuations as a function of ξ from three different estimates including the empirical prediction given in (6) and (7), Tillman's prediction (8) (see below), and large eddy simulation (LES) results.

We conducted several large eddy simulation (LES) experiments for surface stress varying from 0.12 to 0.025 Nm^{-2} and surface cooling varying from 10 to 225 Wm^{-2}. TTV was computed by integrating the temperature spectrum. The grid used in all LES experiments was 256×256×90, with resolution of 0.75 m in all directions. The depth of the mixed layer was about 50 m. Since LES spectra for low winds (say, wind stress ≤ 0.05 N m^{-2}) produced a –5/3 spectral slope between intermediate and high wave numbers, we used those spectra to compute TTV after correcting for roll-off at high wave numbers due to sub-grid scale filtering.

In the absence of wave processes, our empirical model follows very well the atmospheric-prediction given by Tillman (1972) (Fig. 3). Based on atmospheric observations, Tillman suggested

$$\frac{\overline{T'^2}^{1/2}}{\theta_*} = C_1\left(\left(\frac{C_1}{C_3}\right)^3 - \xi\right)^{-1/3} \ , \tag{8}$$

where C_1 and C_3 are constants. As shown in Fig. 3, the normalized *rms* temperature fluctuations approach the free convection power law $(-\xi)^{-1/3}$ predicted by M-O similarity theory when $-\xi > 0.4$. The LES results also agree with the atmosphere and oceanic empirical models.

5. CONCLUSIONS

A general form of the temperature spectrum was derived as a function of stability ξ and wave parameter γ. The wave number-weighted one-dimensional spectrum has a +1 slope at low wave numbers and a –2/3 slope at high wave numbers (characteristic of an inertial sub-range).

The spectral level in the inertial sub-range varies with γ, and the spectrum spreads as a function of ξ at low wave numbers. The wavelength of the spectral peak decreases as $-\xi$ (>0) increases. In the absence of waves, the general spectrum spreads at low wave numbers as a function of ξ. The *rms* fluctuations of turbulent temperature are proportional to $(-\xi)^{-1/3}$ for $-\xi > 0.4$, consistent with the predictions of similarity theory when wave processes are negligible and consistent with atmospheric surface layer observations. The results of a large eddy simulation turbulence model agree with the behavior of the temperature variance as a function of stability.

6. REFRENCES

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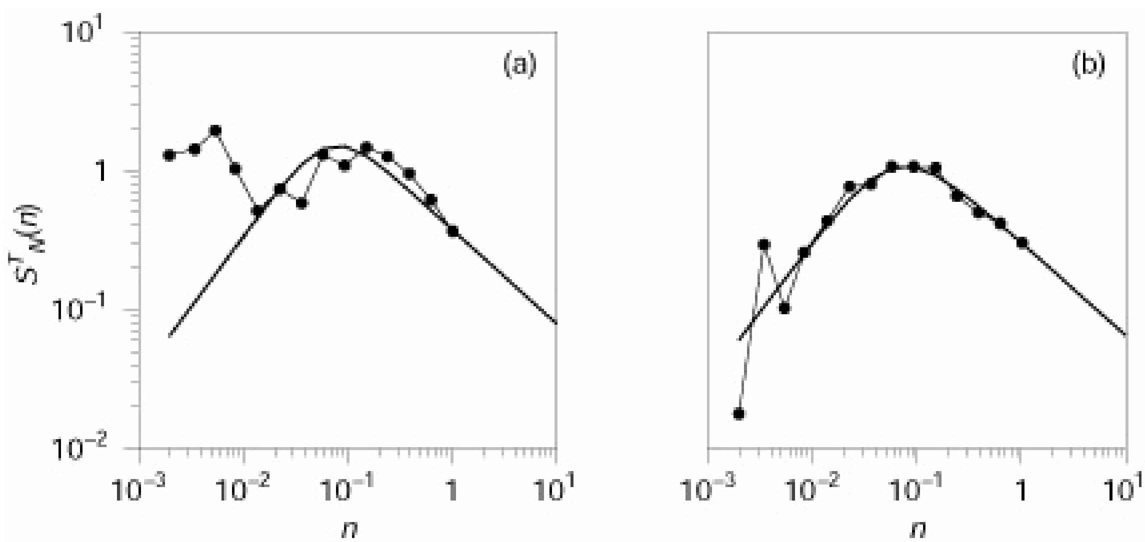

Fig.1: Two examples of observed wave number weighted spectra (bullets) and the corresponding model fits (thick solid line). (a) $\xi = -2.822$ and $\gamma = 6.71\times10^{-4}$, and (b) $\xi = -0.066$ and $\gamma = 3.127\times10^{-3}$.

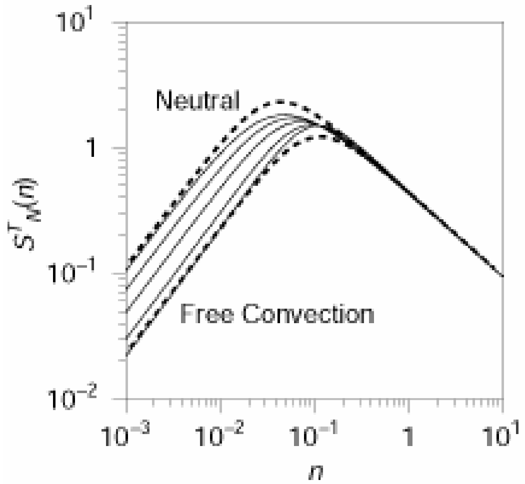

Fig. 2: Modeled temperature spectra (2) (thin lines) based on the empirically derived values of c_1 and c_2 (7a,b) in the absence of wave processes (i.e., A = 1). The thick dashed lines denote Peltier et al's spectra for free convection (bottom curve) and for neutral (top curve) conditions. The five thin curves plotted from the free convection limit to the neutral limit are $-\xi$ = 50, 1, 0.1, 0.01, and 0.0, respectively.

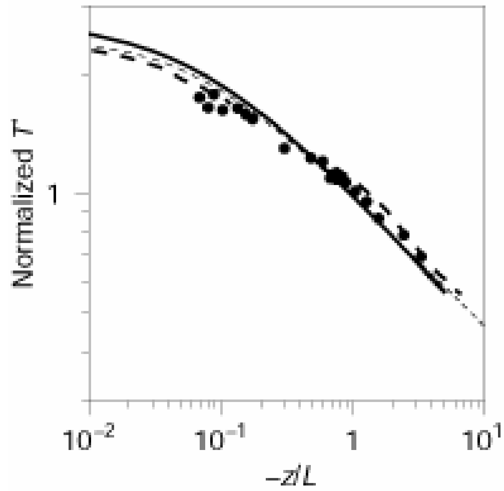

Fig. 3: Normalized *rms* turbulent temperature fluctuations as a function of ξ. The thick solid denotes the estimate from (6) with A = 1. The thin dashed line denotes Tillman's estimate (7) for C_1 = 1.0 and C_3 = 2.5. The thick dashed line represents the LES estimate, and the bullets are oceanic data averaged in bins.