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BAROCLINIC EQUILIBRATION AND THE MAINTENANCE OF THE MOMENTUM BALANCE

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1. INTRODUCTION

In this work we look at the equilibration of the baroclinic problem emphasizing the redistribution of momentum. We show using an idealized model that baroclinic adjustment is essentially a 3D process, and argue that baroclinic adjustment models that only consider vertical adjustments in shear lack some basic dynamics. Surface friction constrains not just the barotropic component of the flow, but also the degree of thermal homogenization at the surface. We also demonstrate that for a forced-dissipative system there must be a nonzero surface temperature gradient over latitudes with surface westerlies.

1. A 2D FRAMEWORK

Consider the equilibration of an unstable mode with mixing depth H in the Charney-Boussinesq problem (figure 1). What we mean by that is that the scale of the mode is such that its fluxes only extend up to the height H throughout the equilibration, leaving an umodified basic state at and above that height.

As the wave equilibrates, the interior PV gradient is reduced and the zonal wind develops some curvature. At the same time, the surface shear is reduced. For the profile shown the reduction in the surface shear is insufficient and there is a remnant temperature gradient at the surface. In order to fully eliminate the surface shear, the flow would need to develop a larger vertical curvature, as indicated by the dash-dotted line on the far right. However, there is a limit to how much curvature the flow can develop, a limit that depends on β. If the curvature is too large, the interior PV gradient becomes negative, which we presume to be unstable. Hence, the interior PV gradient and mixing depth H set up a limit to the maximum reduction in the surface shear.

This constraint can be formalized by considering the vertically integrated PV gradient between the surface and H for the 1D Charney-Boussinesq problem (i.e., neglecting the horizontal curvature of the jet):

$$\int_0^H \overline{q}_y \, dz = \int_0^H \beta \left(1 - \frac{\partial h}{\partial z}\right) dz = \beta \left(H - h(H)\right),$$

where $h = U_z \epsilon / \beta$, and the integral also includes the surface delta function, modeled as a jump in h from 0 to its interior value. Because $h(H)$ remains unchanged, the eddies can only redistribute $\overline{q}_y$, but not change its integrated value over the mixing depth. If $H < h$, the integrated PV gradient is negative.

This argument, hereafter called the mixing depth constraint, suggests that short waves with small H/h are unable to eliminate the surface temperature gradient, even in the inviscid limit. Note that this is precisely the scaling used by Zurita and Lindzen (2001) (hereafter ZL), who choose the half Rossby depth as an estimate for H. They define short Charney modes as modes shorter than the most unstable one ($H/h \leq 3.9$), and argue that those are in fact the only modes allowed by the meridional scale of the jet. ZL showed that (i) the PV flux of short Charney modes peaks at the steering level and (ii) these modes can be neutralized by partial PV homogenization at that level alone. However, they only observed this in their nonlinear model for large enough friction. Otherwise, the strong barotropic acceleration of the jet makes the steering level drop, leading to the eventual homogenization of surface temperature.

These results are also consistent with previous idealized baroclinic equilibration/lifecycle studies that typically find that low friction favors both (i) a strong barotropic acceleration of the jet and (ii)

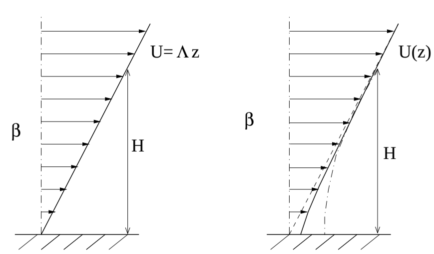

Figure 1: *Sketch illustrating the adjustment of the basic state by a short mode in the Charney problem.*

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enhanced thermal homogenization at the surface. While point (i) is a straightforward consequence of the momentum balance, the physics behind point (ii) is not so clear. We address this issue by discussing the baroclinic equilibration in terms of the redistribution of momentum. For this purpose, the 2D limit provides a useful conceptual benchmark, as momentum is only redistributed along one direction in that problem. It is only in the 2D limit that the mixing depth constraint introduced above strictly applies.

3. MOMENTUM REDISTRIBUTION

Two-dimensional redistribution

We use the barotropic point jet to look at the 2D redistribution of momentum. This model consists of an easterly triangular jet on the beta plane (figure 2a), and is homomorphic with the Charney problem in the linear regime. The jump in shear at the jet vertex gives a negative delta-function PV gradient, while β gives a constant, positive PV gradient in the interior. As before, we define a dimensionless mode length L/l based on the ratio between the interior and vertex PV gradients. The scale of the dominant mode can then be chosen by zonally truncating the channel, or by changing β for a given channel length. We present results below for the runs with $L/l = 1.25$ and $L/l = 3.9$ (the most unstable mode).

Figures 2b,c show the PV gradient in the final state for both cases. For the shortwave, there is a negative PV gradient over the central part of the channel, and the PV gradient only vanishes at the steering level. For the most unstable mode on the other hand, the PV gradient becomes everywhere positive, and the steering level disappears. These results are consistent with the mixing depth constraint.

Figure 2d shows the mean flow correction in the final state, which is very similar for both cases. There is a westerly acceleration at the vertex and a compensating easterly acceleration in the interior, which results in a reduction of the mean shear. In both cases, efficient PV homogenization was observed at the steering level at all times (not shown). The main difference was that, while for the shortwave the steering level moved slightly away from the vertex during the adjustment, for the MUM it moved inward until it disappeared. We found that the phase speed of the waves changed little during the adjustment (due to the conservation of the mean momentum), and changes in the steering level were mainly due to changes in $\overline{U}(y)$. When the acceleration at the original steering level is easterly, the steering level moves outward, and the reverse is also true. As figure 2d shows, this is the a difference between both runs.

Three-dimensional redistribution

We now turn our attention to the 3D problem. We use a qg model, forced by linear relaxation to a Charney-like basic state (but also modulated meridionally by a Gaussian envelope). Rayleigh friction is included at the lowest resolved level. As before, the channel length is truncated and the dimensionless depth of the mode H/h is changed by changing β

The top panels of figure 3 show results for a run with $H/h = 0.86$ and no surface friction. We show the absolute value of the PV gradient (normalized by β) and $|U - c|$, where c is the phase speed; regions with values of the contoured magnitude outside the range shown are non-shaded. Despite the small value of H/h, the negative PV gradient is eliminated at the center of the channel and the steering level disappears over the same region. The inspection of the time series (not shown) reveals a good agreement between the position of the steering level and the well-homogenized region during the adjustment.

Based on similar results, ZL attribute the failure of the flow to equilibrate through partial PV homogenization to the expansion of the well-homogenized region as the steering level drops. However, it may be misleading to assign causality in this context: more properly, both the changes in PV structure and the evolution of the steering level should be regarded as a result of the eddy redistribution of momentum.

The elimination of the surface shear for this small value of H/h is in violation of the mixing depth constraint. To see why this is the case, it is illuminating to compare how momentum is redistributed in the 2D and 3D problems. In the former, there is a transfer of (easterly) momentum from the jet vertex to the interior, which results in a reduction of the mean shear. Similarly, there is in the 3D problem a vertical transfer of momentum, though now through a very different physical mechanism. The vertical momentum transfer results in this case from the easterly (westerly) tendency that the Coriolis force induces on the upper (lower) branch of an indirect mean meridional circulation. This vertical transfer of momentum should be adscribed to the eddy heat flux, as the role of this circulation is to enforce thermal wind balance with the modified temperature field.

In the presence of the heat flux alone, the situation would be much like in the 2D problem. Momentum would be simply redistributed vertically with no net acceleration of the column, and the mixing depth constraint would still apply. However, what makes this problem essentially different from the 2D one is the fact that there is now an additional horizontal redistribution of momentum by the eddy momentum flux. The net momentum of the column is no longer conserved, but there is a net westerly acceleration over the central latitudes (and compensating easterly acceleration on the margins of the jet). It is this westerly acceleration that makes the steering level drop, as was also the case in the 2D problem when the acceleration at that level was westerly.

The effect of this westerly acceleration can be appreciated by comparing the top and bottom panels of figure 3. With surface friction, the westerly acceleration is less prominent, the steering level drops much less, and the flow fails to eliminate the surface temperature gradient and equilibrates instead through partial PV homogenization. This is also illustrated in figure 4, which shows that in the presence of a strong westerly acceleration the flow is more successful in eliminating the surface shear, thus violating the mixing depth constraint. While in the 2D problem the westerly acceleration at the vertex is accompanied by compensating easterly acceleration along the column, in the 3D problem most of the easterly acceleration occurs on the sides. This allows a more efficient export of easterly momentum out of the baroclinic zone, which is why the mixing depth constraint, that assumes that momentum is just redistributed vertically, no longer works.

Note that the horizontal redistribution of momentum also has implications for the PV gradient. This is illustrated in figure 5, which shows the contributions to the zonal mean PV gradient resulting from the vertical curvature of the zonal wind (top), the horizontal curvature (middle), and the total PV gradient, also including β (bottom), for the run with $H/h = 0.86$ and damping time scale 3 days. For the simple basic state considered, the radiative equilibrium PV gradient resulting from the vertical curvature term (shown dashed) is zero in the interior and a negative delta function at the surface. This negative delta function reflects the transition from the constant interior vertical shear of the Charney problem to zero shear right underneath the surface. As the wave equilibrates and redistributes momentum vertically, the negative PV jump across the surface delta function is reduced, and part of the adjustment in shear occurs across a finite layer with negative vertical-curvature PV gradient. In order to prevent negative interior PV gradients, the mixing depth constraint would demand that this negative contribution be smaller than β. However, larger values, order $O(2\beta)$ are observed. This is possible because the horizontal curvature term, which is positive, increases, thus allowing larger vertical curvatures and further adjustments in shear while still keeping a non-negative interior PV gradient. In the low friction case, both the horizontal and vertical curvature terms are larger (not shown).

We can account for these effects, by considering a generalized mixing depth constraint based on the areal integral of $\overline{q}_y$. It is then easy to show that when β and/or H are too small for the given shear, the net integrated PV gradient must be negative. However, in the 3D case this does not preclude $\overline{q}_y$ from becoming everywhere positive at the central latitudes, as found in the inviscid runs, provided that the negative PV gradients are enhanced on the sides.

ZL argued that because of the meridional confinement by the jet, tropospheric waves are short Charney modes. They also proposed an equilibration mechanism in which the waves first reduce H/h by narrowing the jet, and then mix down PV at the steering level. The results presented above do show that one of the main effects of the eddy momentum flux is the self-focusing of the jet. However, a more careful analysis reveals that the *dimensionless* scale of the waves H/h increases rather than decreases during the equilibration. The reason is that when the jet narrows, the Held scale h also decreases because the positive PV gradient in the interior is reinforced by the enhanced curvature. This is in fact what ultimately allows modes that were initially short to get rid of the surface shear, by concentrating the positive PV gradient from the sides.

The proposed mechanism relies on the fact that the horizontal and vertical curvatures of the zonal wind are comparable, as shown in figure 5. A careful examination of reanalysis data suggests that the same is the case in the actual troposphere (not shown), despite the frequent claim to the contrary. This is not so surprising: if the jet constrains the depth of the eddies, its width should scale as the Rossby radius, which implies that its horizontal and vertical curvatures should also be comparable. It is inconsistent to assume that the scale of the eddies is controled by the jet width, and yet neglect the PV gradient contribution resulting from its horizontal curvature.

In conclusion, surface friction is what ultimately constrains the scale of the modes in our runs, by preventing the self-focusing mechanism described above. Moreover, by constraining the development of the horizontal curvature, surface friction constrains the development of the vertical curvature, and thus the reduction in the surface temperature gradient. This simple argument explains why surface friction prevents the elimination of the surface temperature gradient, as found in previous studies, and is also consistent with our finding that the flow keeps an interior steering level in such cases.

4. ON THE MAINTENANCE OF THE MOMENTUM BALANCE

We described above the redistribution of momentum as a baroclinic wave equilibrates. In the forced-dissipative case, this redistribution must balance the frictional forcing resulting from the mean flow imbalance. As shown below, this has implications for the sign of the PV gradients in the equilibrated state.

Consider the following equations describing the mean flow balance and eddy enstrophy conservation:

$$\frac{\partial \overline{U}}{\partial t} - f_0 \overline{v}^* = \overline{v'q'} - \alpha_M(\overline{U} - \overline{U}_0) \qquad (1)$$

$$\frac{\partial}{\partial t}\left(\frac{\overline{q'^2}}{2}\right) + \frac{\partial}{\partial y}\left(\frac{\overline{v'q'^2}}{2}\right) + \overline{v'q'}\,\overline{q}_y = Dis \qquad (2)$$

and let us concentrate on the 2D problem first. In that case, the forcing by the residual circulation $f_0\overline{v}^*$ is zero and the eddy PV flux is the only dynamical forcing of momentum, so that in equilibrium:

$$\overline{v'q'} = \alpha_M(\overline{U} - \overline{U}_0) \qquad (3)$$

On the other hand, in the 2D case eddy enstrophy dissipation is negative definite $Dis = -\alpha_M \overline{q'^2}$. The maintenance of eddy enstrophy against dissipation then requires that the time-mean eddy PV fluxes are on average downgradient, i.e., $\int \overline{v'q'}\,\overline{q}_y dy \leq 0$. If we also neglect the eddy advection of eddy enstrophy (second term in equation 2), then $\overline{v'q'}$ is everywhere downgradient, and not just in a global sense.

For downgradient PV fluxes, the PV gradient must change sign in the equilibrated state. Specifically, $\overline{q}_y$ must be negative (positive) over regions of westerly (easterly) mean flow acceleration, so that the downgradient eddy drag $\overline{v'q'}$ can maintain the mean flow imbalance against the frictional drag (equation 3).

However, things are more complicated in the 3D problem. Though eddy enstrophy dissipation is not strictly negative definite, it is still often true that the time-mean PV fluxes are on average downgradient. This again implies that $\overline{q}_y$ must change sign for the equilibrated flow. However, note that this does not prevent $\overline{q}_y$ from becoming one-signed at some latitudes, as found in our forced-dissipative model in the absence of surface friction.

The main difficulty is that $\overline{v'q'}$ is not the only dynamical forcing of momentum, but there is an additional redistribution by the residual circulation $f_0\overline{v}^*$. Hence, it is no longer possible to establish a local relation between the sign of the PV gradient and the mean flow imbalance, as we did in the 2D case. However, the residual circulation only redistributes momentum vertically but exerts no net force on the column. We can thus get rid of this term by integrating equation 1 vertically, provided that we also include the delta-function return flow at the surface (which can be interpreted as a momentum reservoir):

$$\int_0^\infty \overline{v'q'}dz = -\int_0^\infty \frac{\partial}{\partial y}\left(\overline{u'v'}\right)dz = \alpha_M \delta \overline{U}_S, \qquad (4)$$

where $\overline{U}_S$ is the surface wind, and we assumed that friction only acts over a surface layer with depth δ.

Then, the maintenance of surface westerlies against friction requires that the net eddy drag over the column also be westerly: $\int \overline{v'q'}dz > 0$. Assuming downgradient PV fluxes, this requires a negative $\overline{q}_y$ somewhere along the column. This is consistent with the results presented in the previous section, as we always found with friction a nonzero temperature gradient over those latitudes with surface westerlies.

This simple argument thus gives an alternative explanation to the lack of thermal homogenization at the surface in the midlatitudes, which does not rely on the magnitude of the diabatic time scale. Under the conditions specified above, the eddies cannot eliminate the surface temperature gradient over the latitudes with surface westerlies. Our results also point to the formidable complexity of the baroclinic equilibration problem: because the redistribution of momentum in that problem is essentially three-dimensional, baroclinic adjustment models that are concerned with adjustments in the vertical structure alone and only care about the magnitude of the diabatic time scales might lack some essential physics.

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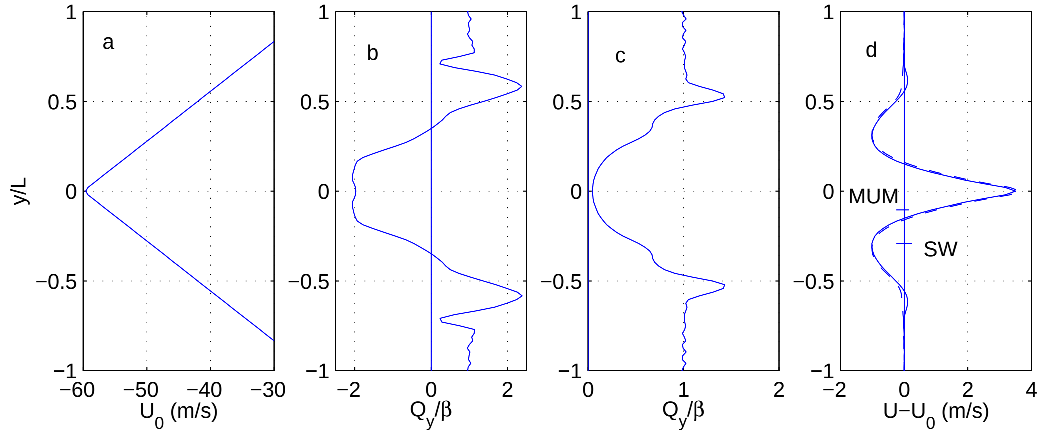

Figure 2: *For the barotropic point jet runs: (a) Basic state, (b) Equilibrium PV gradient for a shortwave ($H/h = 1.25$), (c) Same for the most unstable mode ($H/h = 3.9$), and (d) Mean flow correction for both cases; the horizontal marks show the initial position of the steering level.*

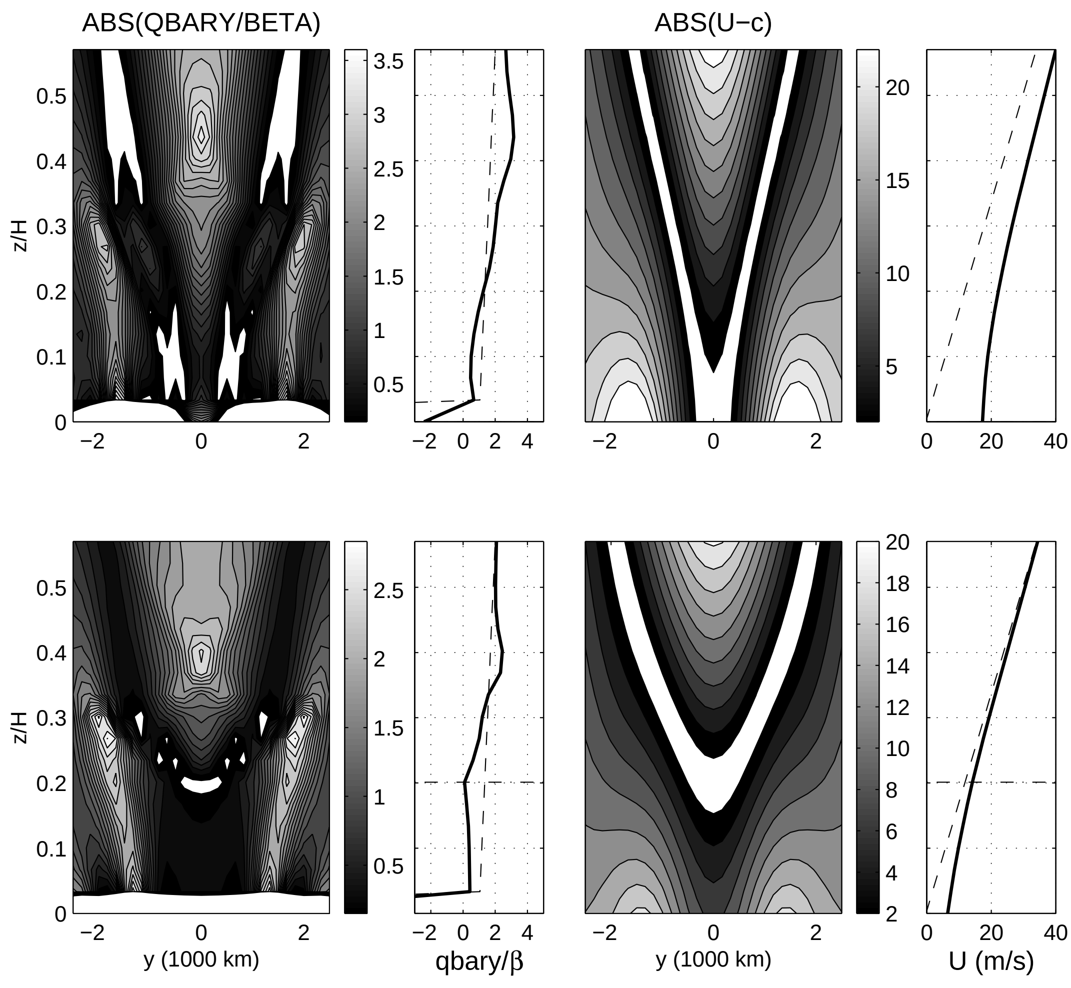

Figure 3: *For the runs with $\beta = 1.6 \times 10^{-11}\ m^{-1}s^{-1}$ ($H/h = 0.86$): Absolute value of the zonal-mean PV gradient normalized by β (left, contour unit 0.15β) and of $\overline{U} - c$ (right, contour unit $2\ m/s$). Also shown are the profiles at the center of the channel at equilibration (solid) and radiative equilibrium (dashed). (Top) With no surface friction. (Bottom) With surface friction of time scale 3 days. Note that non-shaded regions denote values of the contoured magnitude outside the specified contour range.*

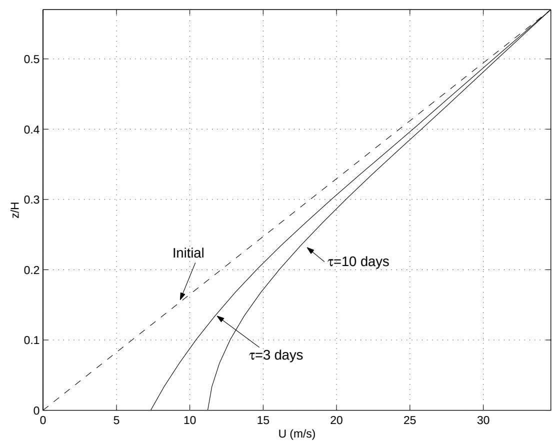

Figure 4: *Comparison of the zonal mean flow for the initial profile (dashed), and the equilibrated states for the runs with $H/h = 0.86$ and the surface damping time scales indicated (note that a barotropic component, different for both cases, has been substracted).*

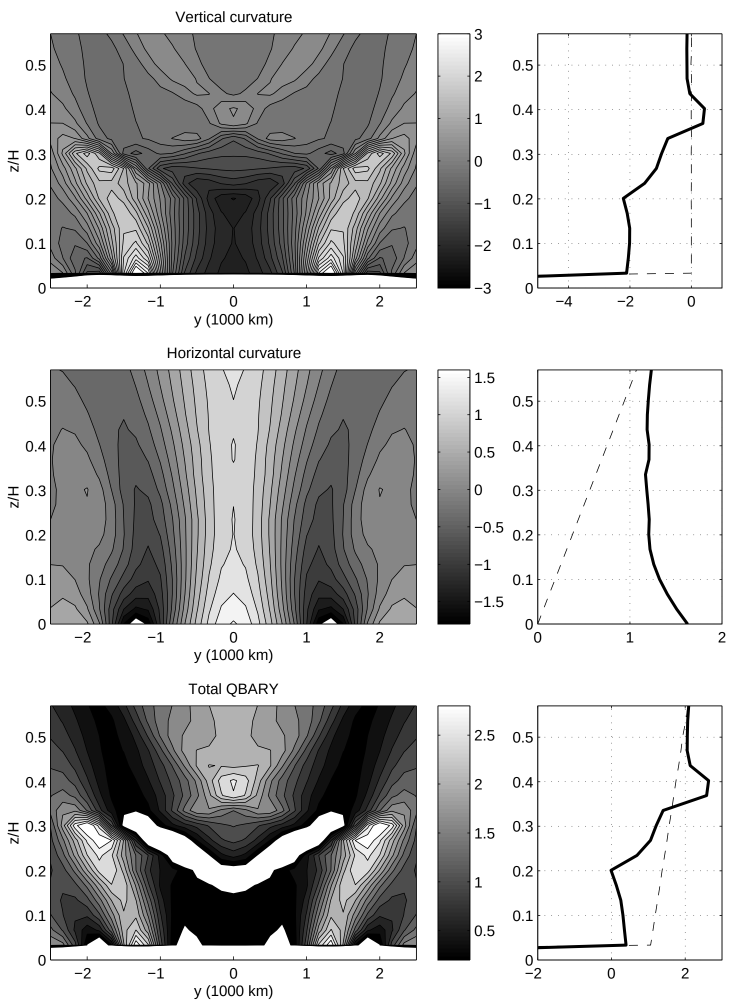

Figure 5: *(Left) Contributions to the zonal mean PV gradient resulting from: (top) vertical curvature, (middle) horizontal curvature, and (bottom) total PV gradient, normalized by β for the run with $\beta = 1.6 \times 10^{-11}\ m^{-1}s^{-1}$ $(H/h = 0.86)$, and surface damping time scale 3 days. (Right) Same but for the vertical profiles at the center of the channel, with the radiative equilibrium distributions shown dashed.*