# 11.8 INFLUENCE OF THE ERROR COVARIANCE STRUCTURE ON SATELLITE-DERIVED ATMOSPHERIC WINDS OVER THE SOUTHERN OCEANS

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Abstract. The mass-conservation technique of Zou and Van Woert is used to retrieve atmospheric wind. In this study we write the variational formalism in the original integral form of Daley. By doing so the derived equation for the atmospheric wind is independent on the differencing scheme. Furthermore the a priori specified weighting function is linked to the expected error covariance of the first-quess wind. We then investigate the influence of the vertical structure of the error covariance on satellitederived atmospheric winds. By appropriately selecting the weight, based on Zou and Van Woert uniform weighting function, this study suggests that the atmospheric general circulation derived from Zou and Van Woert can be further improved, especially near the tropopause.

## **1. INTRODUCTION**

Atmospheric climate studies require winds with reasonable accuracy and high spatial and temporal coverage. Due to the limited available observational data, particularly over the ocean covered areas, wind fields from numerical analyses and reanalyses have been widely used, such as the studies of the Antarctic moisture flux (Bromwich et al. 1995; Cullather et al. 1998). Francis (2002) demonstrated, however, that large bias exist between the wind fields from the European Centre for Medium-Range Weather Forecasts (ECMWF) and National Center for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) reanalysis and (independent) rawinsonde data from two field experiments in the Arctic. Zou and Van Woert (2002) (hereafter referred to as ZVW02) also discussed the advantages and limitations of the

reanalysis winds.

Given the uncertainty in the reanalyses, Slonaker and Van Woert (1999, hereinafter referred to as SVW99) made an attempt to derive a satellite-based, geostrophic-like wind dataset. They used the satellite-based surface wind field and the thermal wind derived from satellite temperature soundings. The derived wind data were then used to estimate the moisture flux and net precipitation for the Southern Ocean region. Zou and Van Woert (2001, 2002) improved the algorithm by including mass conservation in a variational procedure. In ZVW02, the SVW99 wind was used as a first-guess, then a variation procedure was used to force the first-guess wind to conserve mass. In the variational functional formalism, a uniform weighting function was assumed to make the retrieved wind compatible with the radiosonde observations at Macquarie Island. However, the wind structure near the tropopause departed significantly from structure observed in the reanalyses.

In this study the weighting function is linked to the error covariance (Daley 1991). The purpose of this study is to investigate the influence of the vertical structure of the error covariance on the satellite-derived atmospheric winds over the middle and high latitude oceans. By appropriately selecting the weighting functions that depend on the error covariance structure, we demonstrate that the satellite-derived winds at the 300-100 hPa level can be reconciled with the reanalyses winds.

## 2. DATA

The Special Sensor Microwave/Imager (SSM/I) variational analysis wind fields, which are the same as those used by SVW99 and ZVW02, are used for the surface wind. They have an accuracy of  $\pm 2 \text{ m s}^{-1}$  (Atlas et al. 1996).

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Temperature profile data were obtained from the National Oceanic and Atmospheric Administration (NOAA) Advanced Microwave Sounding Unit (AMSU-A). AMSU-A is the first of a new generation of polar-orbiting cross-track multichannel microwave sounders and is available twice daily (ascending, descending) globally on a  $1^{\circ}x1^{\circ}$  grid, and at the standard pressure levels. We have interpolated the data onto a 6-h interval to match the surface wind data, then calculated the winds based on the thermal wind relationship (SVW99). We have also used winds derived from **Television and Infrared Observational Satellite** (TIROS) Operational Vertical Sounder (TOVS) temperature soundings (the same as in ZVW02) for comparison.

#### 3. METHODOLOGY

#### 3.1 Derivation of Mass Conserved Wind

The procedure in this Section is in general the same as ZVW02. However, because we use the integral form of the variational formalism (Daley 1991) rather than the special difference form used by ZVW02, some of the equations need to be rewritten here.

Similar to ZVW02 a first-guess of nonmassconserved atmospheric wind, ( $\tilde{u}$ ,  $\tilde{v}$ ), can be obtained using SSM/I surface winds, and thermal winds derived from AMSU-A temperature profiles.

Following ZVW02, the continuity equation for an atmosphere in hydrostatic balance is:

$$\int_{p_T}^{p_B} \left( \frac{\partial u}{a \cos \varphi \partial \theta} + \frac{\partial (v \cos \varphi)}{a \cos \varphi \partial \varphi} \right) a \cos \varphi dp = 0$$
(1)

where *u* and *v* are zonal and meridional velocities, respectively; *a* is the earth's radius, *p* the pressure,  $\theta$  the longitude and  $\varphi$  the latitude, and  $p_{\tau}$  and  $p_{B}$  are the pressures at the top and bottom of the column, respectively. Here  $p_{\tau} = 100$  hPa and  $p_{B} = 1000$  hPa. Note that (1) is equivalent to Eq. (4) of ZVW02.

As demonstrated in ZVW02, the first-guess wind field ( $\tilde{u}$ ,  $\tilde{v}$ ) is nonmass-conserved. ZVW02 used two methods to retrieve mass-conserved winds. Here we focus only on Method 1 (deriving zonal and meridional winds separately).

Integrating (1) over a zonal circle and over latitude from a pole to a latitude  $\varphi$ , we get (noting  $\cos \varphi$  is zero at the pole)

$$\int_{0}^{2\pi} \int_{p_{T}}^{p_{B}} va \cos \varphi dp d\theta = 0$$
 (2)

The variational formalism for obtaining the mass-conserved meridional wind, v, is to minimize the differences between v and  $\tilde{v}$  in a least-squares sense subject to the mass transport constraint (2),

$$I = \int_{0}^{2\pi} \int_{p_{T}}^{p_{B}} w_{v} (v - \widetilde{v})^{2} a \cos \varphi dp d \theta$$
  
+  $\lambda \int_{0}^{2\pi} \int_{p_{T}}^{p_{B}} v a \cos \varphi dp d \theta$  (3)

where  $\lambda$  is a Lagrange multiplier,  $w_v = w_v(\theta, \varphi, p)$  is an a priori weighting function that needs to be specified. Here an integral form of the variational functional is used instead of the differencing form in ZVW02. In particular, when the integral with respect to p is translated into a summation, the first expression on the right-hand side of Eq. (3)

can be written as  $\int_{0}^{2\pi} \sum_{k=0}^{N} w_{vk} f_k (v_k - \widetilde{v}_k)^2 a \cos \varphi d\theta$ 

( $f_k$  is defined as the pressure depth, a differencing scheme related factor, Eq. (6) in ZVW02). Comparing this to Eq. (11) of ZVW02 it is seen that, mathematically,

$$\alpha_{k} = w_{vk} f_{k}, \qquad k = 1, 2, ..., N$$
 (4)

and  $v_0 = \tilde{v}_0$ .  $\alpha_k$  are the weights used in Eq. (11) of ZVW02.

From (3) we can get the solution for v

$$v = \widetilde{v} - \beta_{v} \lambda \tag{5}$$

$$\lambda = \frac{\int_{0}^{2\pi} \int_{p_{T}}^{p_{B}} \widetilde{\nu} dp d\theta}{\int_{0}^{2\pi} \int_{p_{T}}^{p_{B}} \beta_{\nu} dp d\theta}$$
(6)

where  $\beta_v = 1/(2w_v)$ . Note that, when  $\beta_v(\theta, \varphi, p)$  is appropriately chosen this solution is identical to Eq. (13) of ZVW02 (with a uniform  $\alpha_k$ ).

Now that the meridional mass-conserved wind v is known, the zonal wind, u, can be obtained using the variational formalism in which the differences between u and  $\tilde{u}$  are minimized in a least-square sense subject to constraint (1)

$$I = \int_{0}^{2\pi} \int_{p_{T}}^{p_{B}} w_{u} (u - \widetilde{u})^{2} a \cos\varphi dp d\theta + \int_{0}^{2\pi} \lambda_{1} \int_{p_{T}}^{p_{B}} \left( \frac{\partial u}{a \cos\varphi \partial \theta} + \frac{\partial (v \cos\varphi)}{a \cos\varphi \partial \varphi} \right) a \cos\varphi dp d\theta$$
(7)

The Lagrange multiplier,  $\lambda_1$ , is a function of longitude, and again  $w_u = w_u(\theta, \varphi, p)$  is the a priori weighting function that needs to be specified.

Considering v as specified in (7) and assuming  $\partial w_u / \partial \theta = 0$ , we can obtain the solution for u

$$u = \widetilde{u} + \beta_u \frac{\partial \widetilde{\lambda}_1}{\partial \theta}$$
(8)

$$\frac{\partial^{2} \widetilde{\lambda_{i}}}{\partial \theta^{2}} = -\frac{\int_{\rho_{T}}^{\rho_{B}} \left(\frac{\partial \widetilde{u}}{\partial \theta} + \frac{\partial (v \cos \varphi)}{\partial \varphi}\right) dp}{\int_{\rho_{T}}^{\rho_{B}} \beta_{u} dp} = H(\theta, \varphi)$$
(9)

where  $\beta_{_{u}} = 1/(2w_{_{u}})$ , and  $\widetilde{\lambda}_{_{1}} = \lambda_{_{1}}/(a\cos\varphi)$ .

The unique solution for  $\partial \widetilde{\lambda}_1 / \partial \theta$  is

$$\frac{\partial \widetilde{\lambda}_{1}}{\partial \theta} = G(\theta_{0}, \theta, \varphi) - \overline{G}(\theta_{0}, \varphi)$$
(10)

where  $\overline{G}(\theta_0, \varphi) = \frac{1}{2\pi} \int_{\theta_0}^{\theta_0 + 2\pi} G(\theta_0, \gamma, \varphi) d\gamma$  and  $G(\theta_0, \theta, \varphi) = \int_{\theta_0}^{\theta} H(\gamma, \varphi) d\gamma$ .  $\theta_0$  is the initial point of  $\theta$  for integration. ZVW02 demonstrated that the solution for  $\partial \widetilde{\lambda}_1 / \partial \theta$  is independent of the selection of  $\theta_0$ .

Equations (5), (6), (8) -(10) provide a complete solution for the mass-conserved winds (u, v), when  $\beta_u$  and  $\beta_v$  are specified. v is solved first based on  $\tilde{v}$  and  $\beta_v$ , then u is solved based on  $\tilde{u}$ , v and  $\beta_u$ .

#### 3.2 The Weighting Function

When applied to real data, Eqs. (6) and (9) are converted into a vertical difference form using the trapezoidal rule (cf. ZVW02). However,  $\beta_u(\theta, \varphi, p)$  and  $\beta_v(\theta, \varphi, p)$  are unknown because  $w_u(\theta, \varphi, p)$  and  $w_v(\theta, \varphi, p)$  are unknown. Daley (1991) suggests that a reasonable choice for the weights might be  $w_u = 0.5 < \varepsilon_u^2 >^{-1}$  and  $w_v = 0.5 < \varepsilon_v^2 >^{-1}$ . Thus

$$\beta_u = \langle \varepsilon_u^2 \rangle$$
,  $\beta_v = \langle \varepsilon_v^2 \rangle$  (11)

where <> is the expectation operator, and  $\varepsilon_u(\theta, \varphi, p)$  and  $\varepsilon_v(\theta, \varphi, p)$  are the first-guess wind errors for  $\widetilde{u}(\theta, \varphi, p)$  and  $\widetilde{v}(\theta, \varphi, p)$ , respectively.

The values of  $\langle \varepsilon_{u}^{2} \rangle$  and  $\langle \varepsilon_{v}^{2} \rangle$  are also unknown, thus,  $\beta_{\mu}(\theta, \varphi, p)$  and  $\beta_{\nu}(\theta, \varphi, p)$  need to be specified, similar to the specification of  $\alpha_{k}$  in ZVW02. However, because we have assumed that  $\beta_{\mu}(\theta, \varphi, p)$  and  $\beta_{\mu}(\theta, \varphi, p)$  depend on the error covariance of  $\widetilde{u}$  and  $\widetilde{v}$ , we can provide some general guidance on the form of the solution. Since the surface winds are from observations, they tend to have small (or even zero) errors and thus we can choose  $\beta_{u}$  and  $\beta_{v}$  to be very small at the surface [when zero is chosen for  $\beta_{u}$  (or  $\beta_{v}$ ) no correction is required for u (or v) and  $w_u \rightarrow \infty$ (or  $w_{y} \rightarrow \infty$ )]. Because the upper level wind is based on lower level winds and the vertical wind sheer, the error at upper levels is expected to increase with increasing height and decreasing pressure.

#### 4. EXPERIMENTS AND RESULTS

ZVW02 found reasonable agreement (small biases) between their winds and radiosonde data at Macquarie Island except at 300-100 hPa. The choice of a uniform weighting function by ZVW02 (bear in mind  $\alpha_k = w_k f_k$  except at the surface) is equivalent to choosing  $\beta_u(\theta, \varphi, p) = \beta_v(\theta, \varphi, p) = 0.5 f_k$  and

 $\beta_{u}(\theta, \varphi, p) = \beta_{v}(\theta, \varphi, p) = 0.5 f_{k} \text{ and}$  $\beta_{u}(\theta, \varphi, p = 1000) = \beta_{v}(\theta, \varphi, p = 1000) = 0. \text{ In}$ 

ZVW02  $f_k = 150,175,200,200,100$  at 850, 700, 500, 300 and 100 hPa, respectively. Thus, the ZVW02 weighting function increases with increasing height except at 100 hPa, albeit, somewhat arbitrarily. In the context of the present study, the weighting functions for the 300 and 100 hPa levels are  $\beta_v = 100$  and 50 respectively. Based on the discussion in Section 3.2,  $\beta_v (p = 100)$  is expected to be greater than  $\beta_v (p = 300)$ . Below we examine the impact of a change in the weighting function, on the results of ZVW02.

We ran three experiments with different choices of  $\beta$  (Table 1). (Note: we have chosen  $\beta_{u} = \beta_{v}$ and constant  $\beta$  at each pressure level, however,  $\beta_{u}$  and  $\beta_{v}$  can be different, and  $\beta_{v}$  can vary from one grid point to another on the same pressure level.) The first experiment is equivalent to ZVW02 and the second experiment is similar to the first experiment except that  $\beta_{\mu}(p=100)$  and  $\beta_{\rm w}$  (p = 100) are set to the 300 hPa value. In the third experiment  $\beta_{u}$  and  $\beta_{v}$  are somewhat arbitrarily chosen to increase from 850 hPa to 100 hPa by about 10% at each level. It should be pointed out that the retreived wind is not sensitive to proportional changes in the weighting function. In this regard it can be seen that the differences between experiments 1 and 3 are small at the

Because the retrieved zonal-mean zonal wind does not change (due to the periodic boundary condition and the constant horizontal weighting function used) we do not show the zonal wind here. However we want to emphasize that 'the satellite algorithm yields a zonal-mean zonal wind structure similar to the ECMWF and NCEP/NCAR reanalyses' (cf. ZVW02).

1000 to 500 hPa levels.

For the zonal-mean meridional wind all three experiments produce reasonable wind structure (Figs. 1 and 2). However, relative to ECMWF analysis, the retrieved wind has been improved at the 300-100 hPa level for Exps. 2 and 3. Moreover, the retrieved wind at 700 hPa level has also been improved although further improvement is still required at 40°S (for January) and 35°S (for July). For the latter case it requires the improvement of the first-guess wind. This is due to the fact that when the zonal-mean meridional wind is zero at the surface the correction will be zero at

upper levels. It is interesting to note that when the satellite derived zonal-mean meridional surface wind between 40°S and 60°S is weaker than the corresponding ECMWF analysis, the retrieved satellite winds at upper levels are also weaker. This indicates the very important role of the surface winds in this retrieval method.

It is also encouraging to see that winds derived from TOVS data have also been improved in Exp. 2 (Table 2). This includes Method 2 (not discussed here, cf ZVW02 and Wu et al., 2003). In particular, the bias has been reduced at all levels.

**Table 1.** Experiments for the values used for  $\beta_u$  and  $\beta_v$ . Note that when  $\beta_u$  or  $\beta_v$  changes any constant factor for every grid point it does not affect the retrieved wind.

	p (hPa)	1000	850	700	500	300	100
1	$\beta_u = \beta_v$	0	75	87.5	100	100	50
2	$\beta_u = \beta_v$	0	75	87.5	100	100	100
3	$\beta_u = \beta_v$	0	100	110	120	130	140

**Table 2.** Statistics between TOVS retrieved (Exp.1 and Exp.2) and radiosonde observed (obs) meridional wind (m  $s^{-1}$ ) at Macquarie Island (cf. ZVW02) for 1988.

p (hPa)		1000	850	700	500	300	100
obs. mean- wind		-2.25	-1.33	-1.15	-1.07	-2.50	-1.00
	$\widetilde{v}$	-0.01	-1.05	-1.44	-1.83	-1.27	-1.53
Bias	Exp.1	-0.01	0.42	0.27	0.13	0.70	-0.53
	Exp.2	-0.01	0.32	0.15	0.00	0.57	0.34
	$\widetilde{v}$	2.79	5.69	6.47	9.06	11.83	8.52
RMS	Exp.1	2.79	5.64	6.34	8.92	11.85	8.40
	Exp.2	2.79	5.63	6.33	8.92	11.84	8.42



**Fig. 1.** Zonal-mean meridional wind for January 1999, (a) first-guess, (b) Exp.1, (c) Exp.2, (d) Exp. 3, and (e) ECMWF. The contour values are -1.5, -1.0, -0.5, -0.25, 0, 0.25, 0.5 and 1.0 (m s<sup>-1</sup>). Southerlies (negative values) are shown as dashed lines.



Fig. 2. As in Fig. 1 but for July 1999.

## 5. DISCUSSION AND CONCLUSION

Recently, Zou and Van Woert developed a technique to retrieve atmospheric wind profiles from satellite observed temperature profiles and ocean surface winds. The technique uses the satellite surface wind and thermal wind as the firstguess field (SVW99), and then a variational procedure to force the first-guess wind to conserve mass. In their variational formalism, a uniform weighting function was assumed. In this study we write the variational formalism of ZVW02 in an integral form (Daley 1991) and link the a priori weights to the expected error of the first-quess wind. This further leads to the discussion of the role of the error covariance in the atmospheric wind retrievals from satellite soundings. Here the first-guess, nonmass-conserved atmospheric wind is from SSM/I surface wind, and thermal winds derived from AMSU-A temperature soundings. The mass conservation technique presented here requires two weighting functions,  $\beta_{\mu}(\theta, \varphi, p)$  and

 $\beta_{\scriptscriptstyle \rm v}(\theta,\varphi,p)$  , which are assumed to be a function of

the error covariance  $\langle \varepsilon_u^2 \rangle$  and  $\langle \varepsilon_v^2 \rangle$ . In

general,  $\beta_{u}$  and  $\beta_{v}$  need to increase (decrease)

when  $\langle \varepsilon_u^2 \rangle$  and  $\langle \varepsilon_v^2 \rangle$  are expected to be large (small). In this study, by appropriately selecting the unknown error covariance (basically physically meaningful), it suggests that the atmospheric general circulation pattern derived by Zou and Van Woert (2001,2002) can be improved, especially near the tropopause through a judicious choice of vertical weighting functions.

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