3.14 MODELING SURFACE INTERACTIONS IN THE ARCTIC: ACCURACY VS. COMPUTATIONAL COST

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1. INTRODUCTION

This paper examines the accuracy of various methods to parameterize the turbulent surface fluxes of heat and momentum over Arctic sea ice. These parameterizations form an integral part of numerical models that represent geophysical processes in polar regions on a variety of time and space scales. The geophysical modeler has a choice of a variety of parameterization schemes that can be used to represent the turbulent surface interactions that occur over sea ice. Some of the schemes are more numerically intensive, but presumably more accurate, than others. The task of the modeler is to balance the accuracy of a particular scheme against the computational cost. In many cases, computational cost is not an issue, and modelers should strive to use the most accurate methods available. In other cases, such as the General Circulation Models (GCMs) that are used to predict future climate changes, computational cost is a critical issue. The question that arises is how much accuracy is sacrificed by using simpler, cost-effective parameterization schemes?

2. MEASUREMENTS AND DATA PROCESSING

The comprehensive measurements performed by the authors during the Surface Heat Budget of the Arctic Ocean (SHEBA) project provides a means to test the accuracy of various surface turbulent flux parameterization schemes. For this study we will use flux data obtained from the main tower maintained by the Atmospheric Surface Flux Group (the authors) as described by Andreas et al. (1999) and Persson et al. (2002). This tower was 20 meters high and had sonic anemometers at five levels that measured fluctuations of temperature and wind vector as well as instruments for mean wind speed, temperature, and humidity. The fluxes of sensible heat and momentum are obtained by integrating the co-spectra of vertical variations in wind velocity with temperature variations (sensible heat flux) and the horizontal downwind component of the wind vector (momentum flux).

To reduce the random variations and "noise" that inevitably occurs with any type of turbulence measurement, some additional steps were taken beyond what was described by Persson et al. (2002). The co-spectra of all functional levels were compared and a procedure was used to remove spikes at a particular level and frequency. For each frequency in the co-spectra, the median value of all tower levels that were functioning properly was determined. The final fluxes were based on the integration of this "median spectra". In this way, more data points and less noisy values were available than would occur using the data from any one particular level.

3. PARAMETERIZATION SCHEMES

3.1 Transfer Coefficients

Typically, in numerical models, the surface fluxes of momentum, τ, and sensible heat flux, H_s, are determined from average or so-called "bulk" values of wind speed, U, and potential temperature, θ, somewhere within the atmospheric surface layer. These models use the concept of a transfer coefficient to relate the bulk parameters to a surface flux:

$$\tau = \rho C_{DZ} U_z^2 \tag{1a}$$

$$H_s = \rho c_p C_{HZ} U_z (\theta_o - \theta_z) \tag{1b}$$

where C_{DZ} and C_{HZ} are the momentum and heat bulk transfer coefficients, θ_o is the surface potential temperature, and ρ and c_p are air density and specific heat of air at constant pressure. The subscript z refers to the measurement height, and it is included in the transfer coefficient symbols to indicate that they are valid for a particular height in the atmospheric surface layer. This paper will use the SHEBA measurements to examine the accuracy of various formulations of the transfer coefficients and also the accuracy of using just average flux values.

3.2 Monin-Obukhov (MO) Similarity Theory Methods

The most common method of determining surface turbulent fluxes relies on Monin-Obukhov (MO) similarity theory, e.g. Monin and Yaglom (1971). This theory states that the effects of stability on the wind

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vector and scalar profiles in the atmospheric surface layer (where fluxes are constant with height) can be represented by universal functions that depend on ζ = z/L, where L is the Obukhov length scale defined by

$$L \equiv \frac{\tau^{3/2} c_p \theta}{H_s\, k\, g\, \rho^{1/2}}. \qquad (2)$$

Here g is the acceleration of gravity, and k is von Kármán's constant, assumed to be equal to 0.4. This definition does not include the effect of humidity on air density because it is negligible in the Arctic.

There are problems with MO theory, particularly during very stable atmospheric conditions. Guest et al. (1999), Andreas et al. (2002), and Grachev et al. (2002) provide some examples, using the SHEBA data, of when MO theory breaks down. These authors and others are working on developing methods to estimate fluxes that go beyond standard MO theory. However, the goal of this paper is to test simplifications, not complications, to MO theory; and therefore we will start with schemes that are based on MO theory and work toward simpler, more computationally efficient schemes.

According to MO theory, the transfer coefficients can be determined by

$$C_{DZ} = \frac{k^2}{\left[\ln(z/z_o - \psi_m(\zeta)\right]^2} \qquad (3a)$$

$$C_{HZ} = \frac{k^2}{\left[\ln(z/z_o) - \psi_m(\zeta)\right]\left[\ln(z/z_{ot}) - \psi_h(\zeta)\right]} \qquad (3b)$$

where z_o and z_{ot} are the roughness lengths for momentum and temperature, and ψ_m and ψ_h are the integrated versions of the MO universal stability functions. Mathematically, the roughness lengths are constants derived from integrating the MO profile functions. As their name implies, they are related to the roughness of the surface. If we remove the stability functions from (3), the transfer coefficients become the neutral transfer coefficients, C_{DNZ} and C_{HNZ}. Specifying the roughness lengths is equivalent to specifying the neutral transfer coefficients.

We can determine the value of the roughness lengths, z_o and z_{ot} during SHEBA using our hourly averaged measurements. However, for the purpose of this paper we can't use the individual roughness lengths from each hour because these were based on the fluxes that we are trying to use as independent checks of the various flux methods. There are methods to determine the momentum roughness length based on characteristics such as ice type (Guest and Davidson 1991), ice concentration, lead and melt pond areas and albedo; see Andreas et al. (2003) in this volume. We don't test these methods here but instead use monthly medians from the SHEBA measurements, using only values when the wind speed was greater than 2 m s^{-1}. This is not a totally independent test, but it does give us an idea of the maximum accuracy that could be expected from any type of flux model. Many GCM models assume a constant ice roughness length over sea ice. To test the maximum accuracy of such an assumption, we also compare the hourly measured fluxes based on a median value determined from the entire SHEBA period. As another more independent test, we use equation (5) in Andreas et al. (2003) during the periods when snow was present on the SHEBA ice floe, and roughness is assumed to be influenced by the drifting action of wind.

Similarly, the temperature roughness length, z_{ot}, used for our tests is determined by monthly and annual medians from the measurements using periods when the wind speed was greater than 2 m s^{-1} and the absolute value of θ_o-θ_z was greater than 0.5°C. We also use the method of Andreas (1987) for determining the value of z_{ot}. This method assumes that the ratio z_o/z_{ot} is a function of the roughness Reynolds number. This method is useful because it can be used in numerical models without any *a priori* knowledge of z_{ot}.

Various forms of the stability functions, ψ_m and ψ_h have been proposed. Here we use the Grachev et al. (2000) forms for unstable conditions ($\zeta < 0$). For the conditions experienced during SHEBA, this form is virtually identical to the Paulson (1970) version used by Andreas et al. (2003). For the stable case ($\zeta > 0$), we use the Holtslag and De Bruin (1988) function that is recommended for use over snow and ice by Andreas (2002). We also test a simpler function proposed by Dyer (1974) that is commonly used for stable conditions:

$$\psi_m = \psi_h = -5\zeta. \qquad (4)$$

The main disadvantage of using (1), (2) and (3) for estimating surface fluxes is that the fluxes cannot be determined explicitly because the ψ_m and ψ_h stability functions are themselves determined by the fluxes, see (2). Therefore, an iterative procedure that cycles between (1), (2) and (3) must be used to determine the surface fluxes. This greatly increases the computational cost of this method compared to methods (see below) that do not require an iteration. Furthermore, in very stable conditions, the method does not converge; therefore checks are required to test for convergence and alternative flux values must be provided in these cases, further increasing computational cost.

3.3 The Bulk Richardson Number Method

An alternative to using the MO theory methods described above is to use the bulk Richardson number, R_b, as a stability parameter instead of ζ. The bulk Richardson number is defined as

$$R_b \equiv \frac{g\, z\, (\theta_o - \theta_z)}{U_z^2\, \theta_z}. \qquad (5)$$

If we use (4) and set $z_o = z_{ot}$, then MO theory predicts (Launiainen 1979) that

$$C_{DZ}/C_{DNZ} = C_{HZ}/C_{HNZ} = (1 - 5R_b)^2 \quad 0 \le R_b \le 0.2\,. \qquad (6)$$

If R_b is greater than 0.2, then the fluxes are set to zero. We know from a large body of evidence, including the SHEBA measurements, that setting $z_o = z_{ot}$ is not correct in most situations over sea ice and also that fluxes can occur if R_b is greater than 0.2. However, these approximations may be close enough to reality that reasonable flux estimates can be obtained by using (1) in conjunction with (5) and (6) to estimate turbulent fluxes during stable conditions. The beauty of this method is that (1), (5) and (6) are simple equations that are computationally inexpensive and do not require iteration to produce a flux estimate.

3.4 The No Stability Correction Method

An even simpler and more computationally efficient method for determining turbulent fluxes is to ignore stability variations entirely and just use (1) with constant values of the transfer coefficients. This simplification is used in numerical models, where precise accuracy is not important, especially for momentum fluxes when stability is unknown. We test this method using monthly and annual median values of C_{DZ} and C_{HZ} in (1) and compare these estimates with the hourly measured fluxes.

3.5 The No Transfer Coefficient Method

This method uses constant flux values, ignoring all feedbacks and bulk parameters. This method could not be useful for determining momentum and sensible heat fluxes in a prognostic geophysical model. However, it might be used for short periods or other situations when knowing how surface fluxes may change is not important for the problem being studied. We include it here mainly to see what the total magnitude of the flux variations are to provide a worst case benchmark for the other flux determination methods. We test it using constant monthly and annual mean fluxes and examine the variations of the hourly measured variations about these means.

We haven't mentioned moisture (latent heat) fluxes in this discussion because the direct measurements of latent heat flux during SHEBA did not appear to be reliable. However, based on measurements of bulk humidity over sea ice during SHEBA and other experiments, we know that the humidity over pack ice is almost always near the saturation value with respect to ice, especially during the cold months (Andreas et al. 2002). Therefore, the surface flux of humidity (or latent heat flux) over sea ice floes is almost zero. There will be moisture fluxes over leads, and some of this moisture may be returned to ice surfaces via small downward moisture fluxes or precipitation. But for geophysical models, ignoring the moisture (latent heat) fluxes over ice floes and just setting the near surface humidity to the saturation value represents a vast simplification and savings in computational costs. Therefore, a form of the "no transfer coefficient method" is not unreasonable in the case of moisture and latent heat fluxes over sea ice.

3.6 Summary of Flux Schemes

In Section 5, we will present the results of the comparison of momentum and sensible heat fluxes using the various schemes with the SHEBA hourly measured values. Here we summarize the various flux methods that will be used for comparing with the hourly SHEBA measurements and assign them to a method identifier for easier discussion later. For Methods 1-3, we used the bulk wind speed and temperature from tower level 4, which was approximately 8 m above the surface.

Method 1a: Use (1), (2) and (3) in an iterative fashion. For z_o and z_{ot} in (3), use monthly median values from the SHEBA measurements. For the ψ_m and ψ_h in (3) use Grachev et al. (2000) for the unstable ($\zeta < 0$) cases and Holtslag and De Bruin (1988) for the stable and neutral ($\zeta \geq 0$) cases. Eliminate cases where the iteration does not converge.

Method 1b: Same as Method 1a but use Andreas (1987) to determine the value of z_{ot} .

Method 1c: Same as Method 1a but use Dyer (1974) i.e. (4) for the stable ψ_m and ψ_h functions.

Method 1d: Same as Method 1a but use Andreas et al. (2003) for z_o and Andreas (1987) for z_{ot} . This is only used when dry snow is on the ice floe.

Method 2: Use the bulk Richardson number method, (1), (5) and (6). Calculate C_{DNZ} and C_{HNZ} from the monthly median z_o and z_{ot} values measured during SHEBA.

Method 3a: Use (1) with monthly median C_{DZ} and C_{HZ} values.

Method 3b: Use (1) with the annual median C_{DZ} and C_{HZ} values, i.e. from all the SHEBA measurements.

Method 4a: Use the monthly mean values of momentum flux, τ, and sensible heat flux, H_s.

Method 4b: Use the annual mean (all SHEBA) values of momentum flux, τ, and sensible heat flux, H_s.

4 ACCURACY METRICS

We use two metrics to quantify the error associated with each flux determination method, recognizing that differences can also be due to experimental error, the latter discussed in Persson et al. (2002).

The first metric is the square of the correlation coefficient, R^2, using the SHEBA hourly fluxes described in Section 2 as the independent variables and the fluxes determined by each of methods (“method fluxes”) summarized in Section 3.6 as the dependent variables. The value of R^2 represents the portion of the variance in the flux measurements that is captured by the a particular method flux.

The second metric is a root-mean-square (r.m.s.) value defined as

$$\left[\frac{\sum (X_{meth} - X_{mc})^2}{n}\right]^{1/2} \qquad (7)$$

where X_{meth} represents an individual method flux data point of wind stress or sensible heat flux, X_{mc} is a "corrected" measured value from the SHEBA tower, and n is the number of data points.

The correction applied to the data is now explained. For determining the r.m.s value, all the available data points were used. However, the roughness lengths used for the flux methods were not based on all the data points and they were determined from the *medians* of monthly or annual values. For these reasons, the *mean* value of the difference between the measured flux and the method flux was not equal to zero. The purpose of this study is not to verify the value of z_o and z_{ot} for the Arctic, but rather to determine how well various flux methods work at capturing variations in the fluxes. Therefore, before calculating the r.m.s. value, we subtract the mean difference between the method flux and the measured flux ("bias") from every data point. In this way, the r.m.s. value represents a measure of the variations between the values, and biases are not included. The biases were not large compared to the variations, and therefore this correction did not have a large effect on the final r.m.s. value.

5. RESULTS

5.1 Momentum Flux

All of the methods that use a momentum transfer coefficient ("drag coefficient") capture between approximately 91% and 93% of the variance in the winds stress variations (Table 1, Figures 1 and 2). The most accurate, in terms of both the highest R^2 value and the lowest r.m.s., is Method 1b, which uses the Andreas (1987) temperature roughness scale, z_{ot}, algorithm. This result shows that there is a connection between variations in z_o and z_{ot}. The Andreas (1987) algorithm is the only published method for determining z_{ot} based on z_o. Therefore, this result supports the use of this method for determining z_{ot} over methods based on empirically-derived z_{ot} values. The main drawback to this method is that it requires extra calculations over the methods that used a fixed z_{ot} and is therefore the most computationally expensive. Because this method is based on the roughness Reynolds number, which is a function of a wind stress, it must be used in an iterative scheme.

Table 1
Correlation Coefficient Squared, R^2, R.M.S. Value and Number of data points, n, for Momentum Flux

Method	R^2	R.M.S. (Nm^{-2} x 100)	n
1a	0.9297	2.213	4496
1b	0.9302	2.204	4496
1c	0.9241	2.234	4279
1d	0.9102	2.674	2975
2	0.9127	2.442	4522
3a	0.9208	2.622	4522
3b	0.9101	2.638	4522
4a	0.0213	7.860	5570
4b	0.0000	7.950	5570

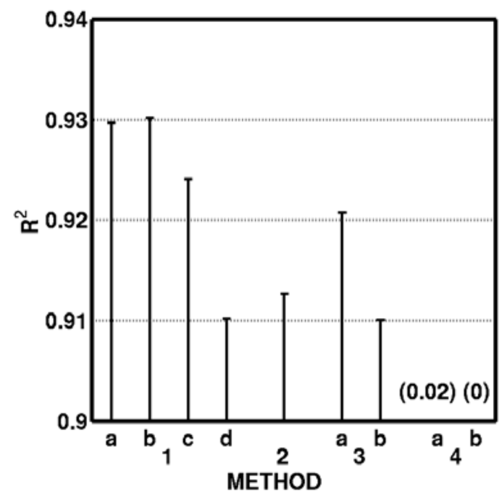

Figure 1. Correlation coefficient squared, R^2, for the various momentum flux methods, repeating values in Table 1 graphically. Method 4 values are below the scale and are indicated in parentheses.

Methods 1a and 1c are just slightly less accurate than Method 1d. Method 1a, which uses the more recent Holtslag and De Bruin (1988) stability function is a little more accurate than the older but simpler Dyer (1974) function.

Method 1d does not use z_o and z_{ot} values that reflect monthly variations in roughness due to ridging and other factors. Therefore, it is not surprising that this method has slightly higher r.m.s values than the other Methods 1-3. The accuracy of this method is more realistic for conditions in a typical model that is not based on *in situ* roughness measurements.

Considering its simplicity, Method 2, based on the bulk Richardson number, does very well. Its r.m.s. value is only approximately 10% less accurate than the methods that used the more sophisticated iterative MO theory stability corrections.

The even simpler Methods 3a and 3b are approximately 20% less accurate than the iterative methods. These methods are very similar to using the wind vector alone to predict ice motion. This method still captures 91% of the variance in wind stress, giving some justification to the rules-of-thumb that relate ice motion to wind speed, without considering stability effects.

Method 4b has an R^2 value of zero because the method flux does not vary. The r.m.s. value is the same as the standard deviation of the wind stress during the whole SHEBA period. The Method 4a R^2 value shows that only 2% of the variance in wind stress was due to monthly differences.

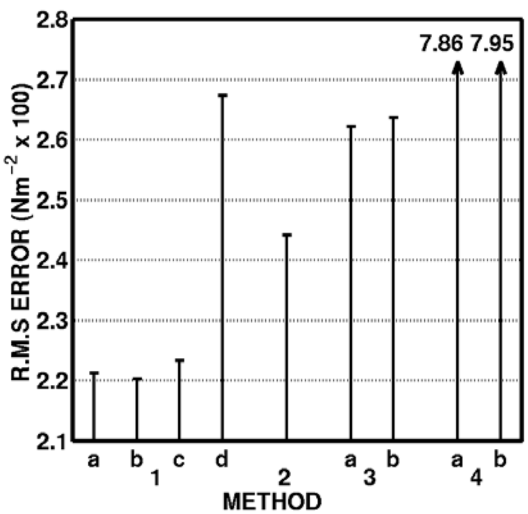

Figure 2. R.m.s differences for the various momentum flux methods, repeating values in Table 1 graphically. Method 4 values are above the scale and are indicated above the arrows.

5.2 Sensible Heat Flux

The heat flux correlations are significantly less than the momentum flux correlations for all the flux methods (Table 2, Figures 3 and 4). Much of this has to do with uncertainty in the surface temperature beneath the tower, θ_o, which we estimate is approximately 0.5 °C. Leads, melt ponds, and other inhomogeneities in upwind surface conditions also contribute to problems resulting from using one temperature to describe surface conditions. These uncertainties have a much larger effect on the heat flux methods than the momentum flux methods.

Similar to momentum flux, there are not large differences between the various Methods 1a, 1b, 1c and 1d. Method 1c has the lowest r.m.s. and highest correlation of the iterative methods. However, this method also converges less often than the other iterative methods (note n value in Table 2) showing that it does have problems during very stable situations.

Method 2 did remarkably well, having the lowest r.m.s. and the second highest correlation coefficient of all the methods, despite its simplicity in predicting stability effects.

Methods 3a and 3b, which have no stability corrections, perform much more poorly for sensible heat flux than for momentum flux. This is because during stable conditions there is a point when an increasingly cooler surface, relative to the air, results in decreased downward heat flux. This effect cannot be simulated by using a constant heat transfer coefficient.

The r.m.s value from Method 4b shows the total standard deviation of the heat fluxes during SHEBA. The Method 4a R^2 value indicates that only 12% of the heat flux variation is associated with monthly differences. Using some type of heat flux climatology in numerical models is not a good idea.

Table 2
Correlation Coefficient Squared, R^2, R.M.S. Value, and Number of data points, n, for Sensible Heat Flux

Method	R^2	R.M.S. (Wm^{-2})	n
1a	0.8270	3.585	4496
1b	0.8113	3.745	4496
1c	0.8386	3.548	4279
1d	0.8054	3.760	2975
2	0.8363	3.484	4522
3a	0.5716	6.186	4522
3b	0.5297	6.781	4522
4a	0.1208	8.064	5570
4b	0.0000	8.601	5570

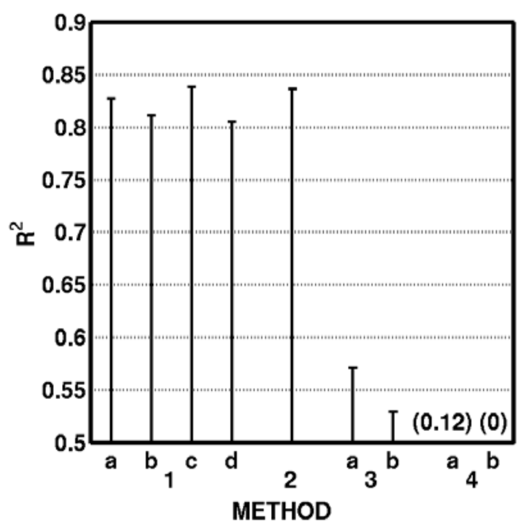

Figure 3. Correlation coefficient squared, R^2, for the various sensible heat flux methods, repeating values in Table 2 graphically. Method 4 values are below the scale and are indicated in parentheses.

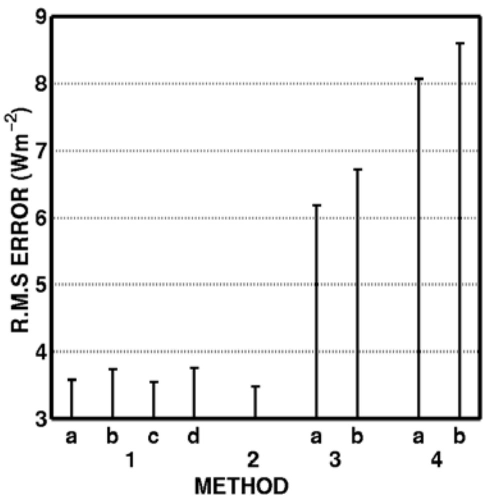

Figure 4. R.m.s differences for the various sensible heat flux methods, repeating values in Table 2 graphically.

6 DISCUSSION

We have shown how various flux determination schemes for momentum and sensble heat flux compare to direct measurements during the SHEBA project. Because the flux methods used roughness lengths that were based on medians from the SHEBA experiment, this was not a completely independent comparison. However, the purpose of this paper is to provide estimates of the relative accuracies of various flux methods, not to quantify actual errors to be expected in a particular numerical model.

For momentum, the iterative methods (Methods 1) gave the most accurate results. However, methods that used bulk Richardson number stability corrections (Method 2) or a constant transfer coefficient (Method 3) did not result in large degradations in accuracy and may be appropriate for modeling situations where limiting computational cost is important.

A word of caution regarding these results is in order. Because momentum flux is proportional to the square of the wind speed, the r.m.s. calculation tends to put more importance on high wind periods than low wind periods. The errors in estimating momentum flux in low wind situations wouldn't significantly affect modeled ice motion because the momentum flux is low in these situations anyway. However, even if the overall magnitudes are relatively small, variations in wind stress during stable conditions when a very shallow boundary layers is present may have a significant effect on modeling the generation of turbulence and mixing.

The biggest surprise from this study is how well the bulk Richardson number performed for predicting sensible heat flux. This method's accuracy is comparable to the iterative methods, and it is much less expensive.

Unlike momentum, not using any stability correction results in very inaccurate sensible heat fluxes, especially during the stable conditions that are common during the Arctic night.

Similar to the momentum fluxes, the heat flux results give more weight to high wind speed cases when variations in the difference between the surface and the near-surface air temperature causes relatively large differences in sensible heat flux. We know from the SHEBA flux measurements that small heat fluxes can exist even when the bulk Richardson number is greater than 0.2, but these cases do not have a large effect on the overall results presented here. There is a danger in using the Richardson bulk method as formulated here when the winds are light and the surface is colder than the air (R_B > 0.2). In these situations, the modeled decoupling of the snow/ice surface from the air in the surface layer may cause the surface temperature to continue dropping due to radiative cooling. The result could be an unrealistic feedback effect that causes the surface to become too cold and the atmosphere too stable. We suggest that models using a bulk Richardson number stability correction allow a small transfer of heat from the surface when R_B is greater than 0.2 by setting a minimum value of the downward sensible heat flux to 1 Wm^{-2}.

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http://www.weather.nps.navy.mil/~psguest/sheba/people_list.html

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