P3.2 CONSTRAINTS ON GRAVITY-WAVE-DRAG PARAMETERIZATION SCHEMES FOR SIMULATING THE QUASI-BIENNIAL OSCILLATION

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1. INTRODUCTION

In this paper, parameterization schemes for the drag due to internal gravity waves in the stratosphere are discussed and compared in the context of a simple one-dimensional model of the quasi-biennial oscillation (QBO).

The QBO is an oscillation in zonal wind direction observed in the equatorial stratosphere; it is characterized by alternating easterly and westerly phases that descend with time. The latitudinal profile of the oscillation amplitude is approximately Gaussian with a half-width of about 14°, the average maximum amplitude is $23\,\mathrm{m\,s^{-1}}$ at the equator, and the period of the oscillation is 26–30 months.

In the past few decades, a number of theories have been developed to explain the QBO (e.g., Lindzen and Holton, 1968; Holton and Lindzen, 1972; Plumb, 1977). It is well-known that it is driven by the deposition of momentum by waves propagating upwards from the troposphere. However, there has been considerable debate as to the relative contributions of the different types of waves involved: planetary-scale Kelvin and mixed Rossby-gravity waves, and small-scale gravity waves. It is now understood that gravity waves play an important rôle in driving the QBO. It was noted by Dunkerton (1997) that the presence of the Brewer-Dobson upwelling in the lower stratosphere acts to suppress the descent of the QBO shear zones. Thus, the drag from equatorial planetary waves alone is insufficient to generate a QBO and the additional drag needed must come from gravity waves. However, gravity waves, because of their small scale, are generally unresolved in large-scale general circulation models (GCMs) of the atmosphere and the drag from these waves must be accounted for by means of parameterizations.

Until recently, GCMs had been unable to simulate QBOs; this was seen as a notable failing of the models. In the past few years, GCMs that include parameterized gravity wave drag have had some success in generating QBOs. This suggests that Dunkerton (1997) is correct in stating that gravity wave drag is needed to drive a QBO in the earth's atmosphere, given the strength of the equatorial planetary waves. Unfortunately, there is a lack of quantitative information from observations to infer constraints on gravity wave parameters for use in models.

It must be emphasized also that GCMs that do simulate QBOs seem to do so for various reasons and it is possible that a QBO can be obtained for the wrong reasons by simply tuning the model input parameters. In general, the characteristics of the QBOs generated by GCMs and the requirements to obtain these oscillations are different for each model. This is not surprising, given the fact that the form of the simulated QBO depends on a number of factors, many of which are themselves model-dependent.

All these considerations suggest that there is a need for an understanding of the properties of gravity-wave-drag parameterization schemes in the context of a simulated QBO. We have taken a step towards this by adding parameterized wave drag to a simple one-dimensional zonal-mean model of the QBO. The gravity-wave-drag parameterization schemes used in our study are well-known schemes which are based on the theory of wave breaking and saturation (Lindzen, 1981). Drag due to equatorial planetary waves is also added to the model; this is also represented by means of a parameterization (Holton and Lindzen, 1972, hereafter referred to as HL72). The QBOs obtained from the two types of waves are compared in Campbell and Shepherd (2003a) and the effect of combining drag from both types of waves is examined in Campbell and Shepherd (2003b). An overview of our conclusions is presented here.

The one-dimensional model comprises the single equation

$$\frac{\partial \bar{u}}{\partial t} - \nu \frac{\partial^2 \bar{u}}{\partial z^2} = X(z,t), \qquad (1)$$

where $\bar{u}(z,t)$ is the zonally-averaged zonal velocity and the term $X(z,t)$ represents the forcing due to equatorial planetary waves and/or gravity waves. The parameter ν is the vertical diffusion. The equation is solved numerically in a domain extending from a height of 15 km (around the tropopause) up to a height of 60 km. The source level of the waves is taken to be the lower boundary.

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The investigation is aimed at providing answers to the following questions:
(1) For a given parameterization scheme, what are the conditions on the initial configuration and the choice of parameters for the mean wind to be able to evolve to an oscillating state?
(2) What is the mechanism for the descent of the shear layers?
(3) What factors affect the form and, in particular, the period of the oscillation (assuming one is possible)?
(4) What rôle does diffusion play, in particular, in the mechanism for switching between easterly and westerly winds at the lowest levels? Is it needed at higher levels?
(5) What are the differences between the gravity-wave-drag schemes that assume wave breaking and the HL72 scheme for equatorial planetary waves, which is based on the assumption of thermal damping of the waves?

In answering these questions, we derive the constraints that are needed for each scheme to be able to generate a mean-wind oscillation that resembles the QBO in at least the following respects:
(1) Its period is within a range of about 700–900 days.
(2) Its maximum amplitude tends to a steady value within the range of velocities 20–50 $\mathrm{m\,s^{-1}}$.
(3) It takes place over a range of heights from the source level up to a minimum height of about 50 km.
The constraints derived include restrictions on the choice of the relevant parameters, on the initial configuration and, where appropriate, on the gravity wave source spectrum.

2. EQUATORIAL PLANETARY WAVES

A number of the issues raised in questions (1)–(5) were addressed by Plumb (1977) for the HL72 scheme for equatorial planetary waves. Plumb's discussion of the HL72 theory focused on the configuration involving a westerly Kelvin wave and an "anti-Kelvin" wave. The latter is an easterly wave which does not exist in nature; it is assumed to be unaffected by rotation and, thus, has the same form as a Kelvin wave except that its phase speed and momentum flux are in the opposite direction. The term "anti-Kelvin" wave was suggested by Dunkerton (1991). The advantage of using such a wave instead of a mixed Rossby-gravity wave is that the two waves then have identical expressions for their drags:

$$X^{\pm}(z) = \frac{N\mu F_0^{\pm}}{k(\bar{u}-c^{\pm})^2}\exp\left\{-\int\frac{N\mu}{k(\bar{u}-c^{\pm})^2}dz + \frac{z}{H}\right\}, \tag{2}$$

where the source level of the waves is $z = z_0$ and $F_0^{\pm}$ are the momentum fluxes, which are specified at the source level. The phase speeds and horizontal wave numbers of the waves are denoted by $c^{\pm}$ and $k^{\pm}$ respectively and, in all cases, the plus sign refers to the westerly wave and the minus sign to the easterly wave. The other parameters are the Newtonian cooling rate μ and the Brunt-Väisälä frequency N.

Some important points to note about this scheme are the following (Plumb, 1977):

- The strength of the drag and, hence, the period of the oscillation, is determined by the magnitude of F_0 and by the ratio $N\mu/(kc^2)$. This ratio also controls the rate of exponential growth or decay of the drag with height. In Figure 1(a), a time-height plot of $\bar{u}$ is shown for a Kelvin–anti-Kelvin wave configuration in which $N\mu/(kc^2) > 1/H$. The drag decreases with height and so the QBO amplitude also decreases with height.
- In the absence of diffusion, there is no downward propagation of information.
- The downward motion of the shear zones depends on the flow evolution at lower levels, which is controlled by the vertical diffusion at the lowest levels. Diffusion affects the period of the oscillation and is necessary for the switching mechanism at the lowest levels, but it is not needed at higher levels.
- Although $\bar{u} = 0$ is a steady solution, it is unstable and a QBO will develop from any initial condition, if the parameters allow one.
- The above conclusions still hold in the more realistic configuration where the anti-Kelvin wave is replaced by a mixed Rossby-gravity wave. In the expressions for the momentum flux and the drag due to the mixed Rossby-gravity wave, the integrand in (2) is multiplied by a factor of $\left(\frac{\beta}{k^2(\bar{u}-c^-)} - 1\right)$, where β is the latitudinal gradient of planetary vorticity. This results in an asymmetry between the easterly and westerly regimes, as seen in Figure 1(b).

3. GRAVITY WAVES

3.1 *Lindzen's parameterization*

Lindzen's theory of gravity wave breaking and saturation (Lindzen, 1981; Holton, 1982) was originally developed for a single wave, but it can be extended to the case of two or more waves. Each wave is assumed to propagate upwards, its amplitude increasing with height, until it gets to a level, which we shall denote as z_b, where it becomes statically unstable and breaks. Using a WKB analysis, Lindzen (1981) derived the following criterion for wave breaking:

$$A(z) \equiv \left(\frac{2N}{k}\right)^{1/2}\frac{e^{(z-z_0)/2H}F_0(c)^{1/2}}{|\bar{u}-c|^{3/2}} = 1, \tag{3}$$

where $F_0(c)$ is the momentum flux at the source level z_0 of a wave with phase speed c. Above its breaking level, the wave deposits momentum in the mean flow

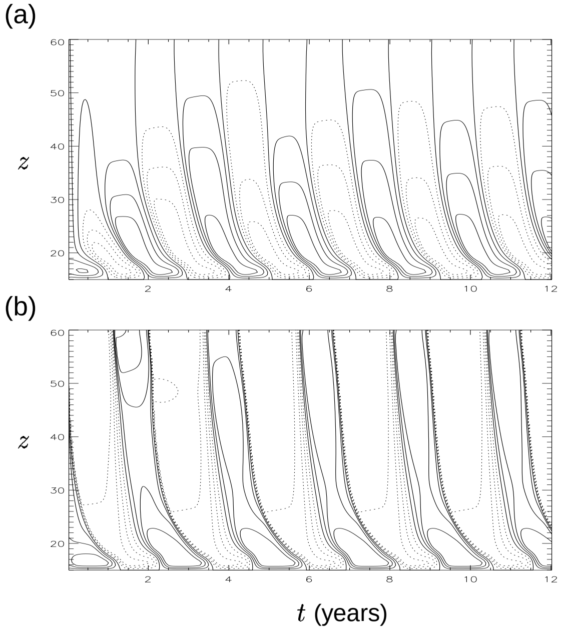

Figure 1: Time-height plot of the zonal-mean wind in QBO simulations using the HL72 parameterization with (a) a Kelvin wave and an anti-Kelvin wave and (b) a Kelvin wave and a mixed Rossby-gravity wave. The solid contours denote westerlies and dotted contours denote easterlies.

to an extent that it remains statically stable, i.e., A does not exceed 1. The resulting drag force is

$$X = -\frac{k}{2}\frac{(\bar{u}-c)^3}{N}\left(\frac{1}{H} - \frac{3\bar{u}_z}{(\bar{u}-c)}\right). \qquad (4)$$

This is often multiplied by a so-called “intermittency factor” usually denoted as ε, which is supposed to measure the percentage of the time that the waves are actually being forced.

To simulate the QBO using this scheme, there must be at least two waves, one westerly and the other easterly. Other points to note about this scheme are the following:

- As in the HL72 scheme, if the wave forcings are equal and opposite (i.e., $k^- = k^+$, $c^- = -c^+$ and $F_0(c^-) = -F_0(c^+)$) then the mean wind must be nonzero initially, otherwise the drag forces due to the waves will cancel each other out. However, a zero initial state may be unstable to small perturbations in the wind.
- The drag, as given by (4), does not depend directly on F_0 and, in this sense, the scheme differs from the HL72 scheme. Changing F_0, however, affects the breaking levels (according to (3)) and, hence, the drag profile.
- The period of the oscillation is controlled by the horizontal wavenumbers $k^\pm$ of the waves (which determine the magnitude of the drag), by the intermittency factor ε and by the strength of the vertical diffusion.
- Vertical diffusion has the effect of reducing the mean wind in the regions of strong shear above the breaking levels; this allows the waves to continue propagating upwards and deposit momentum at higher levels.
- Below the lowest breaking level, there is no drag. Since the breaking criterion is local, this means that, without diffusion, the wind can never change below the lowest *initial* breaking level. Thus, unlike in the planetary wave case, it is not enough to have diffusion only in a boundary layer at the source level; diffusion is needed at all the breaking levels in order to get a QBO.

3.2 *Alexander and Dunkerton's parameterization*

The parameterization described by Alexander and Dunkerton (1999, hereafter referred to as AD99) is a variant of the Lindzen scheme in which each wave is assumed to deposit all its momentum at its breaking level. For a wave with phase speed c and breaking level z_b, the drag is set to (4) at z_b and to zero at all other heights. Thus, the drag corresponding to each phase speed is a delta function of height and the total drag is obtained by summing up these delta functions over the range of heights. A continuous spectrum of waves over a range of phase speeds is needed in order to have a drag profile that is a continuous function of height.

To implement the scheme, one starts at the source level and removes from the spectrum any waves that would have already been reflected or broken, and then works upwards, at each level, testing each of the remaining waves to determine whether the wave would break or be reflected there. Reflecting waves are removed from the spectrum there and at all subsequent levels; breaking waves deposit their momentum at that level. In the present study, the effect of reflection is neglected.

As in the Lindzen scheme, the period of the oscillation is controlled by k, ε and ν. The width and shape of the source spectrum determines the range of heights over which the oscillation takes place. There are also constraints on the initial configuration. To illustrate this last assertion, it is helpful to examine the special case where the momentum flux at the source takes the form $F_0(c) = (\text{sgn}\ c)\times$ a constant. For each c, the breaking level $z_b(c)$ is determined by Lindzen's criterion:

$$\frac{\alpha e^{z_b/3H}}{|\bar{u}(z_b) - c|} = 1, \qquad (5)$$

where

$$\alpha = \left|\frac{2N}{k}e^{-z_0/H}F_0(c)\right|^{1/3}. \qquad (6)$$

With the above choice of $F_0(c)$, α is independent of c, and c can be written as an explicit function of z_b:

$$c^{\pm}(z_b) = \bar{u}(z_b) \pm \alpha e^{z_b/3H}. \quad (7)$$

Depending on the initial strength of the shear, there are then two possibilities. These are illustrated in Figure 2 for an initial $\bar{u}$ profile in the form of a westerly jet with maximum amplitude at $z = 30$ km. In Case 1, $\bar{u} = 5\,\mathrm{m\,s^{-1}}$ at its maximum (Figure 2(a)) and in Case 2, it is $20\,\mathrm{m\,s^{-1}}$ (Figure 2(b)).

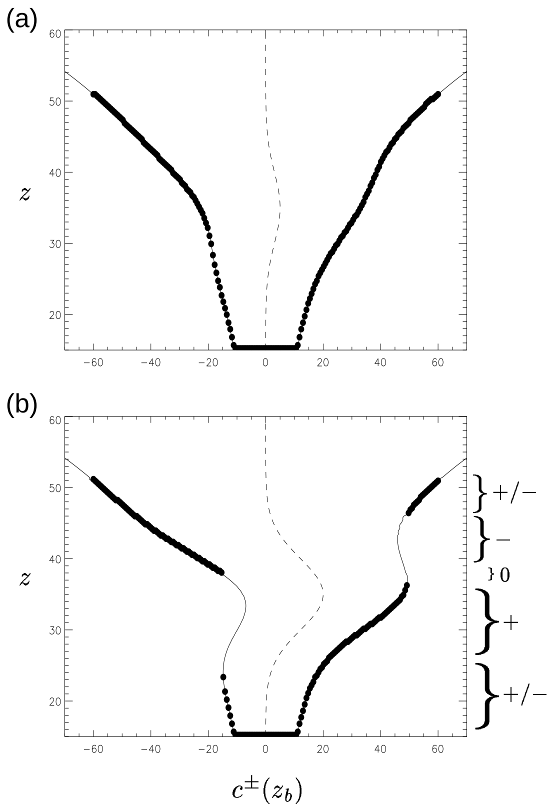

Figure 2: AD99 parameterization with the momentum flux spectrum $F_0(c) = (\mathrm{sgn}\ c)\times$ a constant: (a) The dotted line shows the mean velocity with maximum value of $\bar{u}_{max} = 5\,\mathrm{m\,s^{-1}}$. The thin solid line shows the graph of $c^{\pm}(z_b) = \bar{u}(z_b) \pm \alpha e^{z_b/3H}$ and the black circles show the actual breaking levels of the waves. (b) The same as for (a), but with stronger shear: $\bar{u}_{max} = 20\,\mathrm{m\,s^{-1}}$. The profile of breaking levels is now a discontinuous function of c. The sign of the drag over each range of heights is shown at the right of the plot.

Case 1. Weak shear: If the initial shear is so weak that there is no level at which $\partial c^{\pm}/\partial z_b$ changes sign (i.e., c^+ is an increasing function and c^- is a decreasing function of z_b), then, at every level, exactly two waves contribute to the drag, a westerly wave with phase speed c^+ and an easterly wave with phase speed c^- (Figure 2(a)). At the levels where $\bar{u}$ is zero, $c^- = -c^+$ and so the total drag is zero, but at the levels where $\bar{u}$ is nonzero, $c^- \neq -c^+$ and, since $\bar{u}$ is positive, $c^- > -c^+$. For the westerly wave,

$$\bar{u} - c^+ = -\alpha e^{z/3H}, \quad (8)$$

so, from (4), the drag is

$$X^+ = \frac{\varepsilon k}{2N}\left(\frac{\alpha^3 e^{z/H}}{H} + 3\alpha^2 e^{2z/3H}\frac{\partial \bar{u}}{\partial z}\right). \quad (9)$$

For the easterly wave,

$$\bar{u} - c^- = \alpha e^{z/3H} \quad (10)$$

and the drag is

$$X^- = \frac{\varepsilon k}{2N}\left(-\frac{\alpha^3 e^{z/H}}{H} + 3\alpha^2 e^{2z/3H}\frac{\partial \bar{u}}{\partial z}\right). \quad (11)$$

Thus, the total drag at height z is

$$X = \frac{3\varepsilon k}{N}\alpha^2 e^{2z/3H}\frac{\partial \bar{u}}{\partial z}. \quad (12)$$

In the absence of diffusion, the equation for the time-evolution of $\bar{u}$ then takes the form

$$\frac{\partial \bar{u}}{\partial t} + f(z)\frac{\partial \bar{u}}{\partial z} = 0, \quad (13)$$

where

$$f(z) = -\frac{3\varepsilon k}{N}\alpha^2 e^{2z/3H}. \quad (14)$$

The solution to (13) can be found quite readily by the method of characteristics to be

$$\bar{u}(z,t) = \bar{u}_{\text{initial}}(\tau), \quad (15)$$

where τ is given by

$$e^{-2\tau/3H} = e^{-2z/3H} - \frac{2\varepsilon k \alpha^2}{NH}t. \quad (16)$$

This solution describes a jet moving downwards with no change in amplitude, as shown in Figure 3. The speed $f(z)$ of downward propagation of the jet increases exponentially with height; so the downward propagation is faster at higher levels. Clearly, the evolution of $\bar{u}$ shown in Figure 3 cannot lead to an oscillating state, since $\bar{u}$ remains non-negative everywhere.

With time, $\bar{u}$ goes to zero at the higher levels. Also, $|\partial \bar{u}/\partial z|$ increases at the lower levels and may eventually become large enough that

$$\frac{\partial \bar{u}}{\partial z} = \mp \frac{\alpha}{3H} e^{z/3H} \quad (17)$$

at some level. This would mean that $\partial c^{\pm}/\partial z = 0$ at that level, which is the situation in Case 2. The drag would then become zero for some distance above that

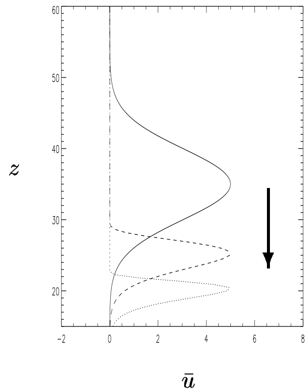

Figure 3: AD99 parameterization with the momentum flux spectrum $F_0(c) = (\text{sgn } c)\times$ a constant and the initial mean velocity profile shown in Figure 2(a): Evolution of $\bar{u}$ with time in the absence of diffusion, according to the exact solution; $t = 0, 2000, 4000$ days.

level and it would then be possible to get a QBO (see the discussion of Case 2 below).

If the vertical diffusion term is restored to (13), the jet still moves downward, but its amplitude decreases with time. With too strong diffusion, $\bar{u}$ could be damped to zero too fast, thus preventing the system from ever reaching a state where $\partial c^{\pm}/\partial z$ could be zero somewhere. Instead, it would tend towards a steady state with zero wind.

It is interesting to note that in Case 1, the drag profile and the evolution of the mean wind are similar to those obtained by Lindzen and Holton (1968). In their model, there is a continuous spectrum of upward-propagating waves (over a range of phase speeds) and each wave is completely absorbed at its critical level, i.e., the level where $\bar{u}(z) = c$. They obtained an approximate expression for the drag due to the wave absorption; this took the form $g(\bar{u})\,\partial\bar{u}/\partial z$. As in the AD99 weak-shear case described by (13), it is impossible to get a spontaneous QBO-like oscillation with such a drag profile. The shear zones propagate downwards, but the wind cannot change direction. Lindzen and Holton got around this by applying an oscillating upper boundary condition around the level of the stratopause to mimic the semi-annual oscillation.

Case 2. Strong shear: With sufficiently strong shear, $\partial c^{\pm}/\partial z_b$ changes sign at one or more places, as shown in Figure 2(b). Thus, the range of levels affected by the westerly waves does not coincide exactly with the range affected by the easterly waves, although they could overlap. There are four possible regimes: the overlapping regions in which the total drag is given by the expression (12), the regions affected by only westerly waves and by only easterly waves, where the drag is given by (9) and (11) respectively, and the regions where the drag is zero. These regions are indicated in Figure 2(b) by the labels $(+/-)$, $(+)$, $(-)$ and (0) respectively. In the $(+/-)$ regions, $\bar{u}$ propagates downwards with speed $f(z)$, given by (14), and remains positive, as in (15). In the $(+)$ and $(-)$ regions, expressions (9) and (11) imply that the equation for $\bar{u}$ is of the form

$$\frac{\partial\bar{u}}{\partial t} + \frac{f(z)}{2}\frac{\partial\bar{u}}{\partial z} = \pm h(z), \qquad (18)$$

where

$$h(z) = \frac{\varepsilon k \alpha^3 e^{z/H}}{2NH} \qquad (19)$$

and the plus sign on the right-hand side of (18) applies in the $(+)$ region and the minus sign in the $(-)$ region. Thus, the jet moves downwards, but with half the speed as in the $(+/-)$ regions. The nonzero term on the right-hand side of (18) means that $\bar{u}$ is of the form

$$\bar{u}(z,t) = \bar{u}_{\text{initial}}(\tau) \pm \int_0^t h(z(t,\tau))dt, \qquad (20)$$

where z is related to t and τ by an expression similar to (16). Since the function h is always positive, this means that, in the $(+)$ regime, $\bar{u}$ becomes more positive and, in the $(-)$ regime, $\bar{u}$ becomes more negative. Thus, $\bar{u}$ eventually becomes negative in the $(-)$ regime and an oscillation in $\bar{u}$ would then be possible. In Figure 4, the time-height contour plot of $\bar{u}$ is shown from a simulation corresponding to the initial state shown in Figure 2(b). The range of QBO heights is from the source level up to a height of about 52 km. This is because all the waves break below this level (as seen in Figure 2(b)) and so there is no drag above.

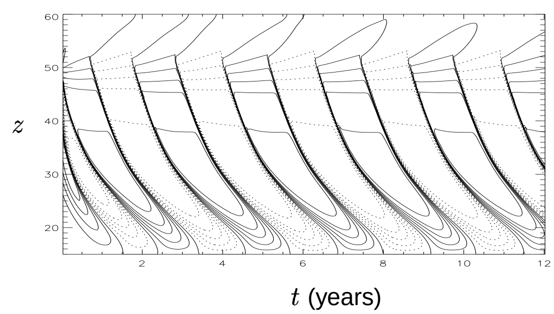

Figure 4: Time-height plot of the zonal-mean wind in QBO simulations using the AD99 parameterization with the spectrum $F_0(c) = (\text{sgn } c)\times$ a constant and the initial mean velocity profile shown in Figure 2(b). The solid contours denote westerlies and dotted contours denote easterlies.

As in Case 1, vertical diffusion acts to reduce $\bar{u}$ in regions with large shear. With excessively strong diffusion (i.e., large ν), $\bar{u}$ could be reduced to the extent that the situation in Case 1 would result. It would then be impossible to obtain an oscillation between positive and negative values of $\bar{u}$.

In summary, in both Cases 1 and 2, the ability of the model to generate a QBO-like oscillation depends on the balance between the strength of the vertical diffusion and the initial strength of the shear. This is true in general, i.e., for other $F_0(c)$ profiles, even those for which there is no explicit relationship between c and z_b.

4. DISCUSSION

In this paper, we have described some of the requirements for simulating a QBO with a simple one-dimensional model using parameterized wave drag. The discussion provides some answers to the questions (1) to (5) that were posed in Section 1.

There are a number of differences between the gravity-wave-drag schemes and the HL72 scheme for equatorial planetary waves. It was noted in Section 2 that, in the HL72 scheme, the strength of the drag, and hence the period of the QBO, depends directly on the amplitude of the waves at their source. In the gravity-wave-drag schemes, the amplitude of the waves determines the levels at which the waves break and thus determines the vertical profile of the drag, but does not affect the strength of the drag directly. The drag profile is, in general, discontinuous in height and the levels where the discontinuities occur are determined by the vertical profile of the mean wind. A local variation in the wind at a given level can have a profound effect on the profile of wave drag at higher levels. In the AD99 scheme, the initial strength of the shear (relative to other input parameters) is important in determining whether it is possible to generate a mean-wind oscillation at all. In the HL72 planetary wave scheme, on the other hand, local variations in the mean wind are not so important, since the drag at any level is determined by the cumulative effect of the wind over the whole range of heights below.

In all the schemes discussed here, the strength of the vertical diffusion is an important factor that affects the period and structure of the QBO. Increasing the strength of the diffusion acts to shorten the period. To generate a QBO using the HL72 scheme, vertical diffusion is only needed at the lowest levels, as Plumb (1977) pointed out. With the gravity-wave-drag schemes that were considered here, however, it is necessary to include vertical diffusion in the vicinity of each of the breaking levels.

It was shown that the AD99 scheme is similar to the original two-wave Lindzen scheme in the sense of the drag profile being discontinuous in height (Case 2 in Section 3). However, in the particular configuration where the mean shear is weak (Case 1), the evolution of the wind is similar to that in the scheme described by Lindzen and Holton (1968).

When drag from both types of waves is used in our model, there are additional constraints on the phase speeds of the waves (Campbell and Shepherd, 2003b). The one-dimensional model used here has a number of obvious deficiencies, one of which is that it does not include the effect of upwelling. The next step in this study involves adding parameterized wave drag to a two-dimensional balance model that simulates the seasonal cycle, including upwelling in the tropics (Semeniuk and Shepherd, 2001). The ultimate goal is to extend the constraints derived using these simple one- and two-dimensional models to provide some guidance for the use of gravity-wave-drag parameterization schemes in GCMs.

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