10.6 NONLINEAR EQUATORIAL WAVES: ROSSBY VORTICES & KELVIN FRONTS

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1. INTRODUCTION

In recent investigations of equatorial Rossby waves (Boyd, 2002), we found that multiple branches could co-exist at the same phase speed. One branch of contra-rotating vortex solitons has a linear relationship between potential vorticity and streakfunction; the other branch has a strongly nonlinear relationship. Initial-value numerical experiments showed both modes were stable at very large amplitude.

Higher latitudinal Rossby mode solitary waves are "weakly nonlocal" (Boyd, 1998b). Numerical experiments show there is good agreement with theory about the amplitude (exponentially small in the reciprocal of the core amplitude) and wavelength of the weak Rossby waves that radiate away from the soliton core.

Equatorial Kelvin waves exhibit a more-complicated behavior that may be dubbed the "CCB Scenario". That is, for small amplitude, the weak dispersion induced by mean equatorial currents is able to balance nonlinear steepening; the Kelvin cnoidal and solitary waves are approximate solutions of the Korteweg-deVries equation in longitude and time (Boyd, 1984).

For large amplitude, Kelvin waves steepen and break. The ensuing fronts have dispersive ripples radiating westward from the front because of Kelvin-gravity wave resonance so that the fronts are "weakly nonlocal" (Boyd, 1998a, b).

We conjecture that the boundary between traveling waves of permanent form and breaking is a so-called "corner" wave, that is, a wave whose peak is a discontinuity of slope. Stokes knew as early as 1847 that ordinary surface gravity waves exhibited the Cnoidal/Corner/Breaking scenario and in that year showed that the two sides of the crest met at an angle of 120 degrees.

Many other wave species exhibit qualitatively similar behavior as illustrated in Fig. 1.However, it is only in recent years with the advent of fast numerical initial value experiments that it has become clear that the corner wave is an <u>attractor</u> for some wave species, and therefore especially interesting (Shefter and Rosales, 1999). In the presence of small viscosity, large amplitude waves break and then rapidly decay to the corner wave, after which further decay is extremely slow so that the wave remains close to the corner wave for a large portion of its life.

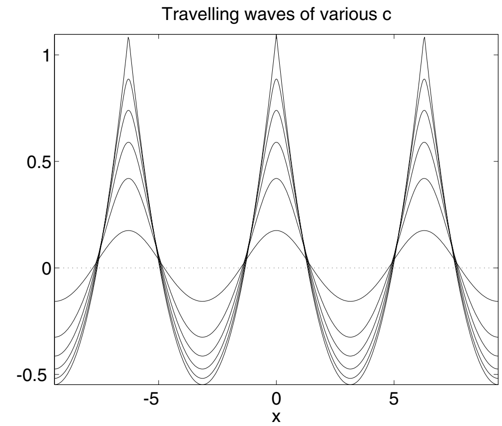

Fig. 1. Travelling waves of the Ostrovsky-Hunter equation; all waves larger than the corner wave (upper curve) steepen and break because wave dispersion is inadequate in this wave equation to counteract such strong nonlinear steepening.

Initial-value experiements with the shallow water wave equations on an equatorial beta-plane suggest that the equatorial Kelvin wave is not as strongly attractive as the one-space dimensional corner wave of Shefter and Rosales. Nevertheless, the boundary between breaking/nonbreaking does seem to be a corner wave, and large amplitude Kelvin waves decay to a coherent travelling wave that resembles the corner wave except for some asymmetry about the crest in the east-west direction.

The most direct way to study the Kelvin corner wave is by solving a two-dimensional nonlinear eigenvalue problem as in some of the computations of Boyd(2002). However, because of the slope discontinuity at the crest, standard numerical algorithms require a rather large number of degrees of freedom. Furthermore, a branch of travelling waves dies at the corner wave with no solutions existing for larger amplitude. At this end-of-branch, the convergence of a Newton-Raphson iteration is more than a little erratic.

We have therefore been forced to develop new analytical and numerical strategies. Some early successes in one space dimension are catalogued in Boyd(2003a,b).

For the Kelvin wave, we report some calculations using the so-called "four-mode" model in which the latitudinal dependence is represented by four Hough-Hermite modes. The four-mode model definitely has a corner wave; interestingly, the velocity and pressure have discontinuous slope, but the north-south velocity has continuous first derivative at the crest of the corner wave (Fig.2)

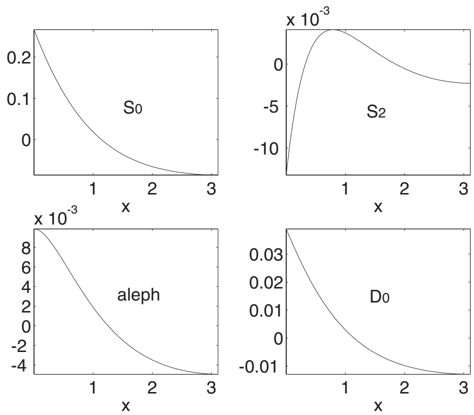

Fig. 2. Plots of the four unknowns in the four-mode model as functions of nondimensional longitude x. S0(x) and S2(x) are the lowest two Hermite coefficients of the Hermite function series in latitude y for the "sum" variable, p+u; D0 is the coefficient of the lowest Hermite function, $\exp(-y^2/2)$, for the "difference" field, p-u; aleph is the x-integral of the north-south velocity. Only positive x (longitude) is illustrated because all fields are symmetric with respect to x=0. All smooth symmetric functions have zero slope at x=0; the nonzero slopes at the origin for three of the four fields imply discontinuities in slope at x=0 for S0, S2 and D0(x).The corner wave is shown for a particular choice of mean currents; the shape and amplitude vary with the mean flow. In the absence of nonlinearity and mean, a Kelvin wave has all amplitude in S0 only; this latitudinal mode is still dominant for the Kelvin corner wave in shear flow.

More ambitious studies which attack the two-dimensional eigenproblem using the so-called Kepler mapping to resolve the crest are in progress.

2. REFERENCES

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