

Winston C. Chao, Baode Chen¹ and Wei-Kuo Tao

Laboratory for Atmospheres, NASA/Goddard Space Flight Center, Greenbelt, MD

¹GEST Center, University of Maryland at Baltimore County, Baltimore, MD

1. Introduction

A catastrophe is the sudden transition of a system from one (quasi-) equilibrium state, whose existence can no longer be sustained when an adjustable parameter passes a critical value, to another pre-existing (quasi-) equilibrium state (e.g., Iooss and Joseph 1980, Poston and Stewart 1978). Since the new (quasi-) equilibrium state exists prior to the transition, existence of multiple (quasi-) equilibria prior to the transition is obvious. Suddenness is characteristic of a catastrophe, since the duration of the transition is normally much shorter than the periods the state can stay in the quasi-equilibria before and after the transition. Often there can be multiple parameters whose changes may lead to the transition. Thus in principle these parameters x_i can be put together to form a function, ε , such that changes in x_i which make ε exceed a critical value ε_c initiate the transition; i.e. $\varepsilon(x_i) > \varepsilon_c$. x_i are usually parameters in the boundary conditions. They can also be internal parameters of a system. Until the condition $\varepsilon > \varepsilon_c$ is satisfied, the system stays in the original (quasi-) state. In other words the system is "protected" by an energy barrier and the catastrophe is associated with the disappearance of the energy barrier. In this case the catastrophe is a spontaneous one. See Fig. 5 of Chao (1985) for a simple illustration of this. Catastrophes can be triggered. The trigger, the disturbance that is introduced, has to provide enough energy to knock the system out of the original (quasi-) equilibrium state and get it over the energy barrier.

In general, the characteristics of a catastrophe can be summarized as, for example, in Fig. 1. This figure shows the state of the system, S , as the abscissa and the ordinate is the values of two terms, A and B , whose difference gives the time rate of change of S , dS/dt . As far as understanding catastrophe is concerned, the terms in the governing equation of a system that exhibits catastrophic behavior can be grouped (at least conceptually) into two competing sets. Thus,

$$\partial S / \partial t = A - B \quad (1)$$

Fig. 1 is an example of Eq. (1). Originally, the state is at the stable equilibrium represented by point 1 in Fig. 1. Point 2 is an unstable (quasi-) equilibrium state and Point 3 is a stable one. As one or more parameter changes such that curve A moves right ward and/or the peak of curve B diminishes to such a degree that the point 1 (quasi-) equilibrium can no longer be sustained, the state moves rapidly to (quasi-) equilibrium point 3. The movement is propelled by the difference between A and B in a "free fall" (more about this shortly.) Fig. 1 was used by Held (1983) to explain the catastrophe of the

topographically induced Rossby wave instability of Charney and DeVore (1979). This is a useful conceptual figure. However, this figure, which depicts Eq. (1), needs some clarification. According to Eq. (1) when the state reaches S_3 , $\partial S / \partial t = 0$ and the state should stop at S_3 and no overshooting can occur. Since overshooting is a fact, something is amiss with Eq. (1). The more accurate interpretation of Eq. (1) is that at least one of A and B is really function(s) of both S and $\partial S / \partial t$ and what appears in Fig. 1 is A and B when $\partial S / \partial t = 0$, i.e., steady state forcings. Held (1983) pointed out that curve A he used represents the drag exerted on the zonal flow by the steady forced waves generated by topography in the presence of dissipation. Thus the "free fall" is driven by a forcing greater than what appears in Fig. 1 as $A-B$.

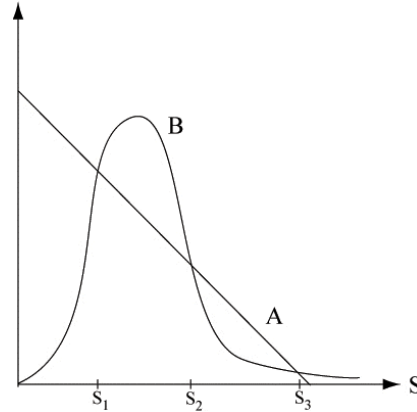


Fig. 1

Other examples of catastrophe are abundant. A simple one is buckling, such as flipping a wall switch and earthquake. Other atmospheric examples are also abundant. A good one is the stratospheric sudden warming. Actually, Fig. 1 is also a representation of it; see Chao (1985) for the physical meanings of curves A and B . Another example is the explosive onset of winter storms in the middle latitudes. Any phenomenon with the name genesis or onset or is associated with multiple equilibria is likely to fit the description of a catastrophe. Tropical cyclogenesis, the transition from a cloud cluster to a tropical cyclone, is a good example. Arguments will be presented to support this claim about tropical cyclogenesis in the next section. Some discussions on previous studies of tropical cyclogenesis from the perspective of the catastrophe notion are presented in Section 3. Experimental work to support our argument using an axisymmetric version of the Goddard Cloud Ensemble model (Tao and Simpson 1993, Tao et al. 2002), is on-going. Section 4 gives a brief discussion and summary.

2. Tropical cyclogenesis as a catastrophe

Tropical cyclogenesis involves the transition from a cloud cluster to a tropical cyclone (TC). Prior to the transition the cloud cluster can last for days without changing its characteristics. Thus one can claim that the cloud cluster is in a quasi-equilibrium state. After the transition, the tropical cyclone can also last for days without changing its characteristics. Thus one can claim that the tropical cyclone is also in a quasi-equilibrium state. Tropical cyclogenesis takes typically only about two or three days. Relative to the duration of either the cloud cluster or the TC this transition period is very short. Tropical cyclogenesis occurs, when the cloud cluster (quasi-equilibrium) state can no longer be sustained (a spontaneous catastrophe) or when a trigger acts on a cloud cluster (a triggered catastrophe). A spontaneous tropical cyclogenesis must be associated with a certain condition being met; and in a triggered tropical cyclogenesis this condition is very nearly being met. Although the precise formula of this condition is still unknown, it is generally associated with what is already known: SST higher than 26.5°C, low background vertical wind shear, sufficiently high Coriolis parameter (Chap. 15 of Palmen and Newton 1969). When the condition is met, all cloud clusters can turn into TC's. It is well-known that a series of TC's often occur concurrently. The experiments of Rotunno and Emanuel (1987), using a cloud resolving axisymmetric TC model, identified two quasi-equilibria (Section 3.b of Rotunno and Emanuel 1987). One is a TC and the other a weak vortex (more about this in the next section). Through these identified characteristics: two quasi-equilibrium states and the rapid transition and so on, tropical cyclogenesis clearly can be identified as a catastrophe.

It is heuristic to construct a schematic diagram similar to Fig. 1 in describing the catastrophic nature of tropical cyclogenesis. One can use the 3-d mass-weighted average temperature in the core region (say within 30 km radius) of a disturbance minus that of the environment; i.e., the degree of warming of the core region, as the state variable S . Curve A represents the diabatic heating, which is mostly cumulus heating, in the core region. Curve B is the dynamic cooling due to upward motion in the same region. Assuming the thermal balance is mainly between A and B, $A-B$ is zero at both quasi-equilibria, cloud cluster and TC. Fig. 2.a shows $A-B$ at the moment that the cloud cluster loses its equilibrium status. $A-B$ is zero at both cloud and TC quasi-equilibria. However, since this is the moment that the cloud cluster loses its equilibrium status, $A-B$ does not cross the abscissa at the cloud cluster quasi-equilibrium but only touches it tangentially. $A-B$ crosses zero at the TC quasi-equilibrium, which is a stable quasi-equilibrium since a perturbation from this quasi-equilibrium will be reduced by $A-B$ to zero. The positive value of $A-B$ on the left side of the TC quasi-equilibrium assures that the state starting from the cloud cluster status will move to the TC status. Curve B can be roughly represented by a linear line through the origin. This is because higher core temperature means a stronger meridional circulation which implies stronger adiabatic cooling in the core region. As a result of these considerations A and B at the moment of cloud cluster's losing its quasi-

equilibrium status can be presented as in Fig. 2.b. Before that moment, the picture is depicted in Fig. 2.c. In this figure the cloud cluster corresponds to point 1 and the TC point 3. Point 2 is an unstable quasi-equilibrium. As the boundary conditions change (say, for example, the SST increases), curve A moves upward and points 1 and 2 merge and then both disappear; i.e., the cloud cluster can no longer be maintained. The system rapidly moves toward the TC state pulled by a forcing greater than $A-B$ in a "free fall". There is overshooting.

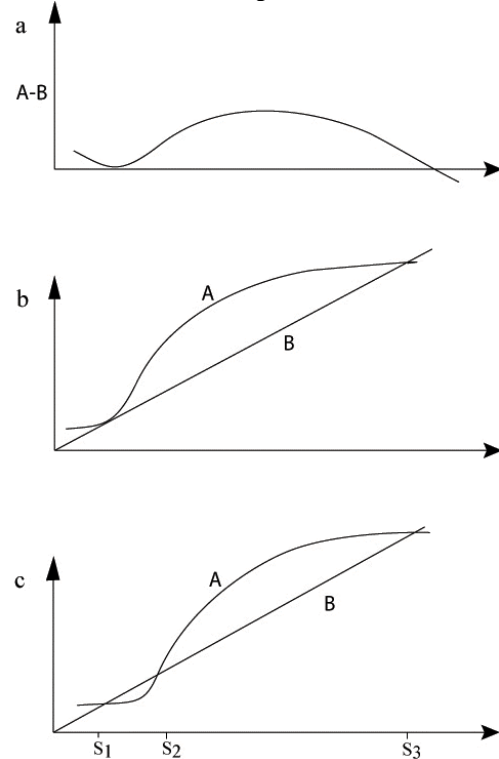


Fig. 2

The remaining question is to explain the shape of curve A. As S increases from zero to a state between points 1 and 2, curve A increases modestly, because higher S means stronger meridional circulation which brings in more moisture and causes more evaporation through the associated stronger surface tangential wind and thus more convective heating. After that the increase of A takes on a greater rate as a result of the increase in tangential wind in the boundary layer and the corresponding increase in evaporation as the vortex spins up. Further increase of A, as the state gets close to that corresponding to point 3, takes on a reduced rate as a result of higher temperature in the core region which reduces the vertical instability and hinders convective activity. Also, since the humidity in the boundary layer air becomes high, evaporation rate cannot keep increasing; thus limiting further intensification of convective activity. Next we will discuss some implications of the catastrophic view of tropical cyclogenesis.

3. Implications of the catastrophic view of tropical cyclogenesis

3.1 Tropical cyclogenesis as an instability

As a catastrophe, tropical cyclogenesis can be either a spontaneous or a triggered event. Spontaneous tropical cyclogenesis means that a certain condition in the environmental parameters is met and the cloud cluster (quasi-) equilibrium disappears. Triggered events are quite common. It is often observed that the passage of an upper-level trough in the neighborhood of a cloud cluster can act as a trigger. Another trigger is the disturbances caused by the passing of the cloud cluster over islands. Of course the strength of the trigger matters. A weak trigger will not suffice. The required strength of the trigger depends on how close the system is to the point where it loses its equilibrium. Of course, if a system is about to lose its equilibrium state, a smaller trigger suffices.

Conditional instability of the second kind (CISK) (Charney and Eliassen 1964, Ooyama 1964), the first major theory for tropical cyclogenesis, is now losing popularity and it is replaced by the wind induced surface heat exchange (WISHE) theory. CISK does have some important elements of truth. It points out the collaboration between cumulus scale and cyclone scale convection. However, its emphasis on low-level convergence is misplaced. Low-level convergence is a result of convective instability rather than a cause. From the perspective of the catastrophe view of tropical cyclogenesis, the CISK theory has a problem. It assumes a resting atmosphere as the basic state and are presented as an instability starting from infinitesimal disturbance. The early attempts using the CISK theory to account for the horizontal scale of the TC are clearly off the mark. WISHE also started from a resting basic state; however this shortcoming was corrected by the recognition of the necessity of finite amplitude initial condition (Rotunno and Emanuel 1987, Emanuel 1989). What is really needed is a theory that starts from individual clouds and ends up with a cloud cluster. In addition the new theory needs to account for how the cloud cluster is maintained and how it becomes unstable (or losing its quasi-equilibrium status) and the process of transforming an unstable cloud cluster into a TC. Another requirement is to be able to account for the triggering mechanism, which starts the genesis process when the initial cloud cluster is still stable. Both CISK and WISHE theories have their positive contributions. The challenge is to preserve what is correct in the two theories and add what is missing.

3.2 Initial conditions used in the 2-d tropical cyclogenesis models

The catastrophe view of tropical cyclogenesis provides a good perspective to look at the work of Emanuel on the finite-amplitude nature of tropical cyclogenesis (Rotunno and Emanuel 1987, Emanuel 1989). Using a 2-d axisymmetric cloud resolving model and a balanced (gradient wind balance between wind and pressure field, not taking into account surface friction) initial condition, he and Rotunno found that to get TC the initial condition has to be of sufficient amplitude. With a weak initial vortex the TC does not

develop, the simulation end up with a weak vortex. So they showed that for the same boundary conditions and model parameters there are two final states, one resembling a TC and the other a weak vortex. (As Emanuel stated there may not be two final states; but for practical purposes one can consider there are two final states, see Section 3.b of Rotunno and Emanuel 1987.) This second weak vortex state corresponds to a cloud cluster (acknowledging the fact that cloud clusters have strong non-axisymmetric component.) Since there are two final states, it is not surprising that the amplitude of the initial vortex matters. The initial high enough amplitude is often achieved through meso-scale vortex merger. Simpson et al. (1997) concluded that nearly all tropical cyclogenesis in monsoon environments involve vortex merger.

Of course starting from a gradient balanced initial vortex really does not resemble what occurs in nature. With such initial condition, as the simulation starts surface friction reduces the low-level wind and thus the Coriolis force; the resulting imbalance between the Coriolis force and the pressure gradient force creates a low-level convergence and therefore intense convection. This may be a strong enough disturbance to trigger the “free fall” toward the TC state; but it is hardly the way that tropical cyclogenesis starts. Tropical cyclogenesis starts from a cloud cluster, which has a balance among the Coriolis force, pressure gradient force *and surface friction*. However, this might resemble the triggered event when a cloud cluster passes over islands. The increased surface friction can lead to enhanced Ekman convergence.

4. Discussion and summary

While the catastrophe theory does not provide the detailed explanation for tropical cyclogenesis, it provides an important framework and a good perspective to understand past theoretical work on tropical cyclogenesis. Among the past work that catastrophe theory sheds light on is the finite amplitude nature of tropical cyclogenesis and the way the initial conditions of the TC model should be set. It also points out that there are multiple equilibria--cloud cluster and TC--in the balancing between convective heating and dynamic cooling due to upward motion at the center of the disturbance. Thus cloud cluster and TC are both maintained by the balance of the same two processes.

In summary the tropical cyclogenesis is identified as a catastrophe whose cause is the loss of stability in the balance between two main forcings on the core temperature. The two forcings are the adiabatic cooling due to the upward branch of the meridional circulation and the diabatic heating which is the sum of cumulus heating, surface heat flux, and radiative cooling. Experimental work based on the Goddard Cloud Ensemble model to support our arguments is on-going.

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