4.3 ISENTROPIC ZONAL AVERAGE FORMALISM AND THE NEAR-SURFACE CIRCULATION

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1. INTRODUCTION

Hoskins (1991) proposed dividing the atmosphere into 3 regions: Overworld, Middleworld and Underworld, using potential temperature θ and potential vorticity (PV) as reference (Fig. 1). In the Underworld (θ < 300 K), isentropes intercept the Earth's surface and a direct isentropic zonal average circulation exists in the mid-latitudes. Held and Schneider (1999) suggested that this circulation may be understood as follows: the equator-pole temperature gradient determines the near-surface eddy heat flux via an eddy-diffusion mechanism. The poleward eddy heat flux in turn drives an equatorward mean flow next to the surface whose horizontal convergence in the subtropics forces the mean quasi-isentropic ascent of air into the troposphere. Radiative cooling causes air to sink back to the surface, thus closing the circulation.

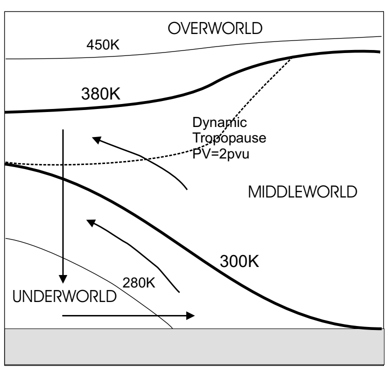

Figure 1 The Overworld, Middleworld and Underworld from Hoskins (1991), separated by 300K and 380K isentropes. Arrows show mid-latitude isentropic zonal average circulation. Solid lines denote isentropes. The dashed line denotes PV=2 pvu (1 pvu=10^{-6} m^2 K kg^{-1} s^{-1}).

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The above view of the general circulation in the Underworld pivots on understanding near-surface dynamics. However, existing theoretical approaches using θ-coordinates, found in e.g. Lorenz (1955), Andrews (1983, 1987) and Held and Schneider (1999), do not have a physically faithful representation of the surface boundary (if included at all). The present paper extends the isentropic zonal average set of equations to include a rigorous treatment of the surface boundary. Applying the extended formalism on a baroclinic-wave dynamics model, we examine the angular momentum. heat transport and the circulation near the surface.

2. CONCEPT OF THE "SURFACE ZONE"

In θ-coordinates, the bottom surface of the atmosphere $\theta=\theta_a$ undulates in zonal and meridional directions and vary with time. We define the "surface zone" at a latitude and time as the region of atmosphere with θ between the maximum and minimum θ_a at that latitude and time. (Held and Schneider (1999) practically defined the same concept.) Within the surface zone, the atmosphere only exists in segments of an isentropic latitude circle where $\theta>\theta_a$ (Fig. 2).

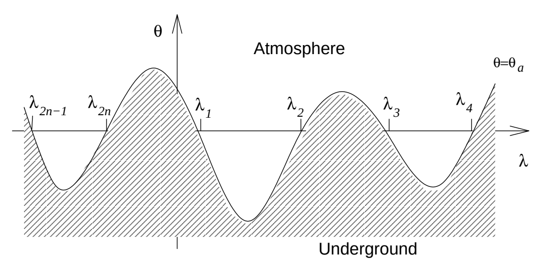

Figure 2 The undulating bottom surface of the atmosphere, shown a latitudinal section in θ-coordinates.

A normalized zonal sum along an isentrope may be constructed as follows:

$$\overline{A}^S = \frac{1}{2\pi}\int_0^{2\pi} A\,\mathrm{H}(\theta-\theta_a)\,d\lambda \qquad (1)$$

where H is the Heaviside step function. Equation (1) allows us to define the zonal mean $\overline{A} \equiv \overline{A}^S / \overline{1}^S$, the mass-weighted zonal mean $\overline{A}^* \equiv \overline{\sigma A}^S / \overline{\sigma}^S$ (where σ is the mass density in θ-coordinates) and their associated anomalies A' and A^{+} respectively.

The surface zone is characterized by the value of $\varpi \equiv \overline{1}^{S}$ between 0 and 1. $\varpi(\varphi,\theta,t)$ is the probability that a surface air parcel is potentially colder than θ at latitude φ and time *t*. Its time rate of change is a function of eddy diabatic heating and eddy advection of potential temperature at the surface (cf. Koh (2002) for the exact equation).

3. ZONAL AVERAGE EQUATIONS

Taking into account the presence of a surface zone, the zonal average equations of motion may be re-derived. The zonal average angular momentum equation is as follows:

$$\overline{L}_{;t} + a^{-1}\overline{v}^{*}\overline{L}_{;\varphi} + \overline{Q}^{*}\overline{L}_{;\theta} = J + G_F + G_A \qquad (2)$$

where *L* is the absolute angular momentum per unit mass about the Earth's spin axis, *a* is the Earth's radius and the subscript ";*x*" denotes differentiation with respect to *x*. *J* is the flux of isentropic potential vorticity (PV) ξ due to eddy motion v^{+}, eddy diabatic heating Q^{+} and turbulent drag *F*, using the concept of PV substance proposed by Haynes and McIntyre (1987). The acceleration of the mean flow arising from meridional PV transport is well-known.

$$J \equiv \left(\overline{\sigma v^{+}\xi^{+}} - \overline{Q^{+}u_{;\theta}} + \overline{F}\right) a\cos\varphi \qquad (3)$$

G_F is the eddy surface form drag (mainly arising from differences in Montgomery potential *M*) and G_A is the eddy advection of eddy angular momentum at the surface. These terms are non-zero only in the surface zone. The terms G_F and G_A have not been explicitly identified in previous literature, to the best of the authors' knowledge.

$$G_F \equiv (2\pi\varpi)^{-1}\sum_{k=1}^{n}\left[M'_j + \tfrac{1}{2}(u'^2_j + v^{+2}_j)\right]_{j=2k-1}^{j=2k} \qquad (4)$$

$$G_A \equiv (2\pi\varpi)^{-1}\sum_{k=1}^{n}\left[\,\{(a\cos\varphi)^{-1}u'_j - a^{-1}v^{+}_j\lambda_{j;\varphi} - Q^{+}_j\lambda_{j;\theta}\}\,u'_j a\cos\varphi\,\right]_{j=2k-1}^{j=2k} \qquad (5)$$

where $\lambda_j(\varphi,\theta,t)$ is one of 2*n* longitudes at latitude φ and time *t* where the surface air potential temperature θ_a equals θ, and X_j denote the value of *X* at the bottom of the atmosphere at longitude λ_j.

To keep this pre-print brief, we shall not present the zonal average continuity and nonlinear gradient-wind balance equations that were derived. A time average formulation of the isentropic zonal average formalism was also obtained, taking into account the surface zone, now defined with reference to a time interval of interest. All derived equations are valid even when neutrally stable mixed layers are present. The interested reader is referred to Koh (2001, 2002) for more details.

4. BAROCLINIC-WAVE DYNAMICS MODEL

We used the baroclinic-wave dynamics model of Thorncroft et al. (1993) to test the isentropic zonal average formalism developed above. The baroclinically unstable initial basic state (Fig. 3) of the model atmosphere was constructed to be similar to that in the life-cycle LC1 in Thorncroft et al.'s work.

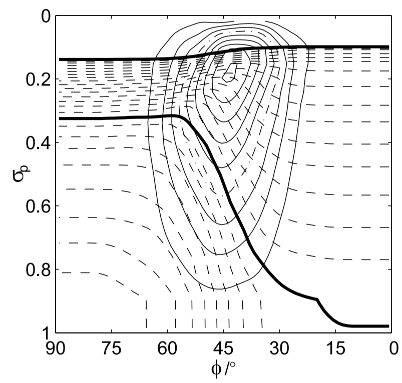

Figure 3 The initial zonally uniform basic state of the model, showing potential temperature θ (dashed lines) and zonal wind *u* (solid lines). Contour intervals are 5 K for θ and 5 m s^{-1} for *u*. Thick lines represent the 300K and 380K isentropes. The dash-dot line is the 20 m s^{-1} contour of zonal wind.

The model was run in two modes: (a) a small initial wavenumber-six disturbance in mid-latitude surface pressure was added to spawn a baroclinic-wave life-cycle and zonal average diagnostics were carried out at day 10 when the eddy kinetic energy is about to saturate; (b) a weak zonally Gaussian wave was added initially in the mid-latitudes to excite baroclinic wave packets and the model was integrated for 300 days. Time average zonal average diagnostics were performed over the equilibrated period from day 105 onwards.

The numerical model is a global pseudo-spectral model at T63 resolution and has 17 σ_p-levels, three of which lie in the planetary boundary layer (PBL) (cf. Bourke (1974)). Radiative cooling is represented by Newtonian relaxation of temperature over 30 days towards the initial basic state. Surface and internal turbulent transport of heat and momentum are parametrized by bulk formulae and act only in the PBL by definition. Dry convective adjustment is included in the model and is active mainly in the PBL and the next higher model level. Moisture and topography are ignored for simplicity.

5. RESULTS AND DISCUSSIONS

Zonal average diagnostics of the baroclinic-wave life-cycle (Fig. 4) shows that the predominant balance in equation (2) within the surface zone is between the meridional advection of planetary angular momentum (i.e. Coriolis torque) and the eddy surface form drag:

$$2\Omega v \sin\varphi \times a\cos\varphi + G_F \approx 0 \qquad (6)$$

where Ω is the planetary angular velocity. Thus, the equatorward mean flow is necessary to produce the westward Coriolis torque that balances the eastward surface form drag in the surface zone.

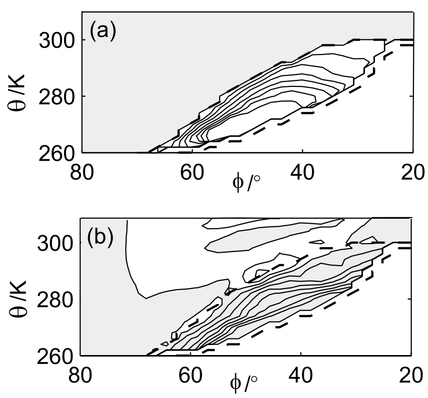

Figure 4 Surface form drag G_F in panel (a) and Coriolis torque $2\Omega v\sin\varphi \times a\cos\varphi$ in panel (b) within the surface zone (demarcated by dashed lines) at day 10 of the baroclinic-wave life-cycle. In both panels, the contoured range is from -0.1 to 0.1 in intervals of 0.02, where the units are Ωa^2 day^{-1}. Shading denotes negative or zero values.

Quasi-geostrophic Transformed Eulerian Mean formalism (Andrews and McIntyre (1976)) can be used to show that equatorward near-surface mean flow is due to poleward surface heat flux. Held and Schneider (1999) also showed this in the isentropic zonal average formalism, assuming constant isentropic mass density σ and vertically uniform geostrophic velocity v_g in the surface zone. (Note that in practice, σ and v_g are not constant in the surface zone.)

Using the isentropic formalism for the surface zone developed in this paper, it was proven that

$$\int_{surface\ zone} \varpi G_M \, d\theta = fa\,\overline{v_g \theta_a}^{a} \qquad (7)$$

where G_M is the (dominant) part of G_F that comes from Montgomery potential differences in equation (4), and the overbar with superscript "*a*" denotes zonal average along the bottom of the atmosphere. We may understand equation (7) as follows: equatorward advection of potentially cold air depresses the bottom surface in θ-coordinates, creating a "valley" in the surface zone (Fig. 5). Geostrophic balance means that the western "valley wall" is at higher pressure than the eastern "valley wall" at the same θ-level. Hence, there is an westward positive difference in Montgomery potential *M*, giving rise to eastward form drag. Therefore, poleward geostrophic heat flux at the surface causes eastward surface form drag, which in turn forces equatorward mean flow in the surface zone.

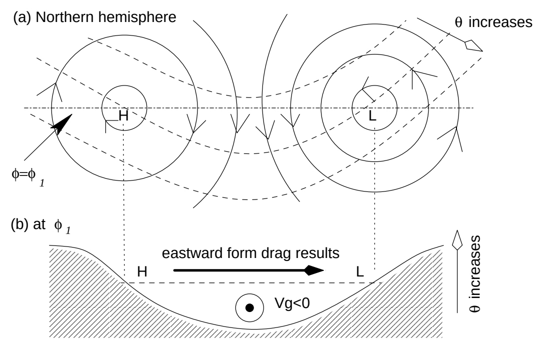

Figure 5 Geostrophic winds (arrows) in a cyclone-anticyclone pair in the northern hemisphere deform potential temperature contours (dashed lines) in panel (a). Panel (b) is a latitudinal section showing a "valley" at the bottom of the atmosphere in θ-coordinates, with high (H) and low (L) pressure on the western and eastern "valley walls" respectively.

The above results do not change when time average zonal average diagnostics were performed on the results of the long-time model run. We observe that the equilibrated surface-zone circulation has a strong anti-symmetric component about the median potential temperature θ_m of surface air (not shown): for $\theta > \theta_m$, equatorward flux of PV forces a poleward mean flow; for $\theta < \theta_m$, eastward surface form drag forces at stronger equatorward mean flow. Held and Schneider (1999) noted a similar anti-symmetric structure in the surface-zone circulation in their GCM simulation, and suggested that the presence of mixed layers is necessary for such anti-symmetry to exist. However, our investigations using the formalism developed above show that mixed-layer contributions to near-surface mean flow are insignificant. Thus, another explanation to the anti-symmetric structure of the surface-zone circulation is needed. It may be shown that

$$< \sigma v > \approx \int_{\min(\theta_a)}^{\theta} \mu \, \mathrm{E}(\sigma v) \, d\theta_a \qquad (8)$$

where <σv> denotes the equilibrated meridional mass flux at level θ and latitude φ, $\mu \, d\theta_a$ is the probability that the potential temperature of a surface air parcel at latitude φ is between θ_a and $\theta_a+d\theta_a$, and E(σv) is the expectation value of surface-zone σv when the surface air at latitude φ is at potential temperature θ_a. As the isentropic densities in poleward-moving warm air are generally higher than those in equatorward-moving cold air, E(σv) is not purely anti-symmetric about the median θ_m (i.e. it contains symmetric components). Meanwhile, μ is observed to be largely symmetric about θ_m, and so <σv> has an anti-symmetric component about θ_m.

6. CONCLUSIONS

The isentropic zonal average formalism, found e.g. in Andrews (1987), is extended to include the lower boundary condition of the atmosphere as a "surface zone", where isentropic latitude circles are interrupted by the Earth's surface. We found that the equatorward mean flow in the surface zone is forced by eastward surface form drag, which in turn arises from poleward geostrophic potential temperature flux at the surface.

In the equilibrated model run, we noted a significant poleward mean flow in the upper region of the surface zone. We attribute this warm branch of the surface-zone circulation to a secondary maximum in equatorward PV flux in the troposphere near the surface. The anti-symmetric surface-zone circulation about the median potential temperature of surface air is primarily due to the warm poleward mass flux σv being generally stronger than the cold equatorward mass flux σv in baroclinic waves. The surface-zone circulation effectively represents a quasi-horizontal transport pattern next to the Earth's surface, where poleward recirculation of air and tracers in warm sectors of surface cyclones occur in tandem with cold equatorward surges in anticyclones.

ACKNOWLEDGEMENTS

This work was supported by the National Science Foundation, under grant ATM-98190992.

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