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10.2 INTERACTION OF WAVES AND CONVECTION IN THE TROPICS

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1. INTRODUCTION

This talk attempts to account for the observed dominance of equivalent depths between 12 and 60 m (equivalent depth is a measure of vertical wavelength; the observed equivalent depths correspond to vertical wavelengths in the neighborhood of 8 km) observed by Wheeler and Kiladis (1999) through a somewhat new approach to the interaction of dynamics and deep convection in the tropics. Our approach assumes that deep convection in the tropics is caused by evaporation, but can be patterned by perturbations to low level convergence. Thus, in the presence of a sinusoidal perturbation to convergence, convection will be concentrated in the regions of positive convergence and suppressed in regions of negative convergence. Given sufficient time, such a perturbation will, we assume, completely organize such convection so that the amplitude of the perturbed pattern of convection will equal the mean convection. For purposes of the present study, it is further assumed that the patterning depends only on the sign of perturbation and not on its magnitude.

To crudely analyze this situation, we will take the vertical velocity within the convective boundary layer to be given by $w\sin(wt)$. Let M_c' be the cumulus mass flux responding to wave patterning while $\bar{M}_c$ is the mean mass flux. Let a^{-1} be the characteristic response time of M_c to the patterning provided by the perturbation in *w*. Patterning is taken to concentrate convection in regions where *w* is positive and suppress convection where *w* is negative. The following equation roughly describes how we expect M_c' to behave:

$$\left(\frac{1}{a}\frac{d}{dt}+1\right)M_c' \approx \bar{M}_c \,\mathrm{sgn}(\sin wt) \qquad \text{(1a)}$$

(where sgn(x)=1 for x>0, and sgn(x)=-1 for x<0).

For convenience, we will let $wt = x$, so that the above equation becomes

$$\left(\frac{w}{a}\frac{d}{dx}+1\right)M_c' \approx \bar{M}_c \,\mathrm{sgn}(\sin x)\,. \qquad \text{(1b)}$$

Although M_c' will, of course, be distorted from a sine wave, its impact on the wave will be associated with its projection on the sinusoidal ω component.

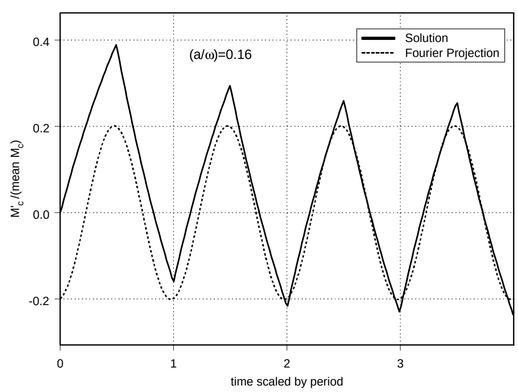

Figure 1a

Figures 1a and 1b show the behavior of $M_c'/\bar{M}_c$ for a/ω= 0.16 and 5. In general, as a/ω becomes large, $M_c'/\bar{M}_c$ approaches one, and the phase lag goes to zero. This is already evident in Figure 1b. On the other hand, as a/ω becomes small, $M_c'/\bar{M}_c$ decreases, and the phase lag for convection approaches 90°.

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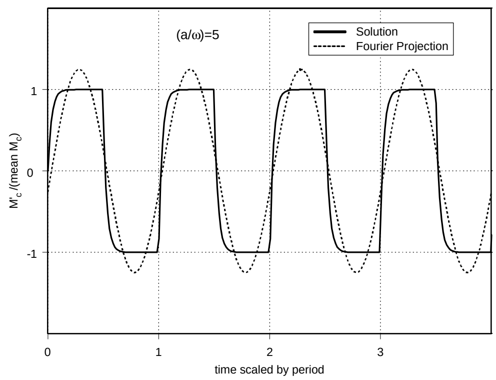


Figure 1b

Figure 2 shows how phase varies with a/ω. Note that for $\left|\frac{a}{w}\right| \geq 1$, the phase lag becomes essentially $\left|\frac{w}{a}\right| \times 90^0$.

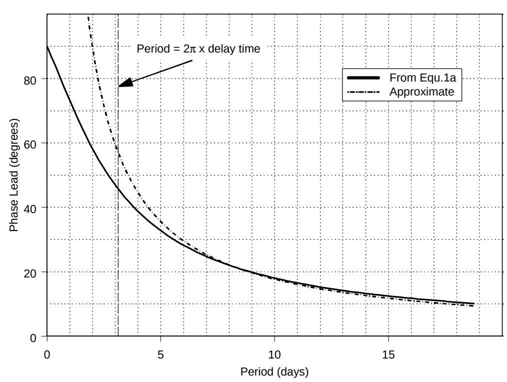


Figure 2

2. APPLICATION TO THE ATMOSPHERIC SEMI-DIURNAL TIDE.

The atmospheric solar semi-diurnal migrating tide is primarily forced by insolation absorption by ozone and water vapor (Chapman and Lindzen, 1970). Such forcing leads to the observed amplitude of the surface pressure oscillation, but phase is one hour off (maxima at 0900, 2100 instead of 1000, 2200). Lindzen (1978) and Hamilton (1981) showed that the observed semidiurnal component of rainfall provided additional forcing that would correct the discrepancy. However, tidal convergence was one order of magnitude less than needed to satisfy the moisture budget. It was already noted by Lindzen (1978) that this implies that patterning rather than direct forcing of the convective pattern is involved. However, in contrast to tropical waves, the tidal component of rainfall is only a fifth of mean rainfall. Assuming that patterning is involved, this would imply that the time needed for the convective response to the patterning perturbation is long compared to the tidal time scale (12 hours/2π). Indeed, one can use the amplitude of the semidiurnal tide in rainfall to estimate the characteristic response time. The simple calculation used to make this estimate also has an additional implication which can be checked in order to test the patterning hypothesis. The ratio of wave time scale (period/2π) to convective response time also determines the phase lag between the *effective* convective heating and the low level convergence responsible for the patterning. Thus one can immediately check if this phase lag is such as to correct the discrepancy in the observed semidiurnal tide.

For the solar semidiurnal tide, Lindzen (1978) finds that $M_c' / \bar{M}_c$. 0.2. Using the results in Section 1 of this extended abstract, it can be shown that this corresponds to a/ω . 0.16, or a^{-1} . 11.94 hours. The phase lag is about 81.8^o. The effective heating associated with this phase lag is, indeed, what is needed to correct the phase of the semidiurnal tide forced by ozone and water vapor heating alone. This offers some confidence that the value of *a* determined by means of the semidiurnal tide is reasonable. It should be added that this value is also compatible with $M_c' / \bar{M}_c$ being on the order of unity for tropical waves with periods on the order of 5 days or longer as observed by Reed and Recker (1971) and confirmed during GATE.

3. APPLICATION TO SELF-MAINTAINED TROPICAL WAVES.

In contrast to the semi-diurnal tide, where the contribution of effective convective heating to the total forcing of the wave is a perturbation to the main forcing which existed independently of the convection, waves of the sort described by Wheeler and Kiladis (1999) are driven by the effective heating of the convection they serve to pattern. Thus, for such waves, the phase of the low level perturbation to convergence must be consistent with the heating pattern needed to produce this perturbation. It will be shown that this condition suffices to determine the equivalent depths observed by Wheeler and Kiladis (1999). However, the condition, itself, provides an independent test of the patterning hypothesis. From the results in Section 2 of this abstract, we estimate the time for patterning to occur to be such that for a 3-5 day wave, the perturbation *w* in the boundary layer would have to lead the effective cumulus heating (which is essentially in phase with *w* at the heating levels) needed to force the perturbation by about 40^o in order for consistent patterning to exist. Straub and Kiladis (2002) analyzed Kelvin waves with such periods in detail, and indeed found the phase lead called for by the hypothesis.

Turning finally to the determination of the equivalent depths implied by the above consistency condition, our approach is to use classical tidal theory assuming an vertically unbounded atmosphere with a radiation condition at the top (following the approach in Stevens et al,1977, and Stevens and Lindzen,1978) to calculate the response to the wave component of the patterned convective heating for a continuous range of equivalent depths in order to see which if any equivalent depths yield self-consistent low level convergence. We used both the approximate basic state for temperature employed by Stevens and Lindzen (1978) as well as a more accurate description (allowing for boundary layer structures as well as an improved depiction of the tropopause region and the stratosphere). For present purposes, the choice was not important.

The initial choice for the heating distribution was

$$Q = e^{bx} \sin\left(p\, \frac{x - x_c}{x_T - x_c} \right) \qquad (2$$

which closely follows the form observed by Reed and Recker (1971) and Yanai et al (1973) for b=-0.33, z_c=0.15 , and z_T=2.01. For purposes of determining self consistency, the amplitude of Q doesn't matter. The results are shown in Figure 3.

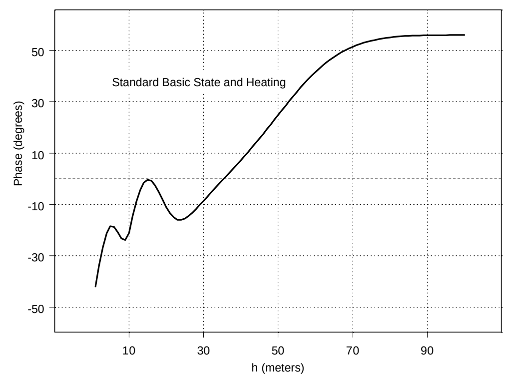


Figure 3

For long period waves, where the delay due to our estimated patterning time is relatively small, the above shows approximate consistency for the appropriate range of equivalent depths. Note that inconsistent phase causes such waves to self-destruct; however, the rate of self-destruction depends on how far the phases are from what is needed. Phases within a few degrees of consistency will be relatively sustainable. However, for waves with periods in the range of 3-5 days, the results in Figure 3 suggest equivalent depths which are too large (corresponding to phase leads on the order of 40^o).

The results presented by Mapes (2000) show that we may have misrepresented the effective cumulus heating. Mapes stresses that the break out of cumulus towers is accompanied and somewhat preceded by the development of congestus clouds whose precipitation is smaller than that of the taller towers (viz Fig. 10 of Mapes). The inclusion of effective heating associated with congestus turns out to be able to bring the above results into excellent agreement with observations.

To show this, we will add the following to our expression for Q:

$$Q_{\text{congestus}} = Ke^{ij}\sin\left(\frac{x - x_c}{x_{tc} - x_c}\right) \qquad (3$$

where $x_{tc} = 0.75$

For K, we will take 0.2. For f , we will try 0 and p/6. The latter choice of phase lead is appropriate for relatively short periods, while zero is more nearly appropriate for longer periods. Note that Mapes (2000) finds a lead time, and a given lead time corresponds to a smaller phase lead as the period gets longer. The results are shown in Figure 4.

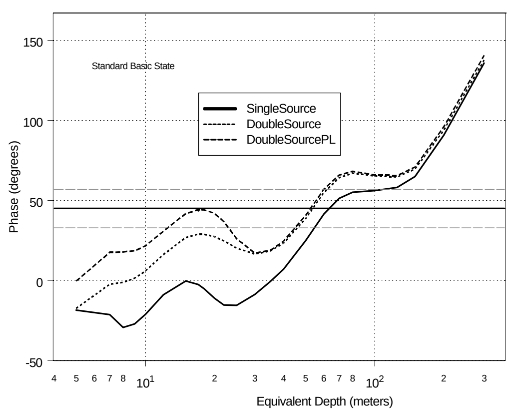


Figure 4

We see that consistent and nearly consistent phases are found in the range of h between 15 and 60 m. It is evident that the presence of a relatively small amount of congestus heating, even in the absence of a phase lead for such heating, significantly changes the phase lead of the low level convergence. The reason is mainly that the shallowness of the congestus heating allows more wave leakage into the boundary layer since leakage depends on the ratio of the vertical wavelength of the wave (in the absence of forcing) to the thickness of the forcing (Lindzen, 1966).

Note that the fact that the congestus clouds play an important role in the cumulonimbus response to dynamic patterning, and that the inclusion of congestus heating is important in order to achieve consistency in phase between patterning and cumulus heating at the observed equivalent depth suggests that the interaction of waves and convection may be more subtle than anticipated.

Finally, Figures 5 and 6 show the vertical

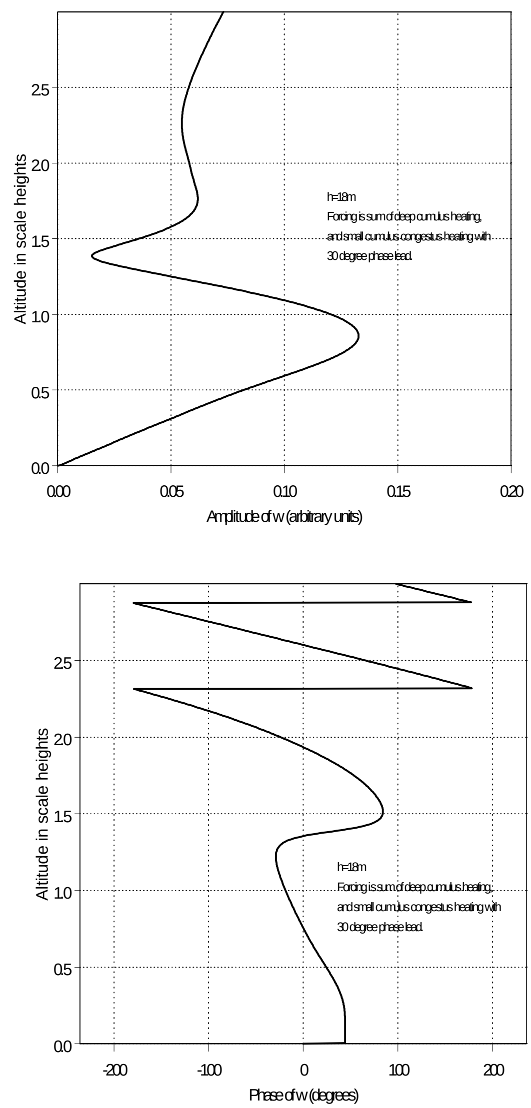
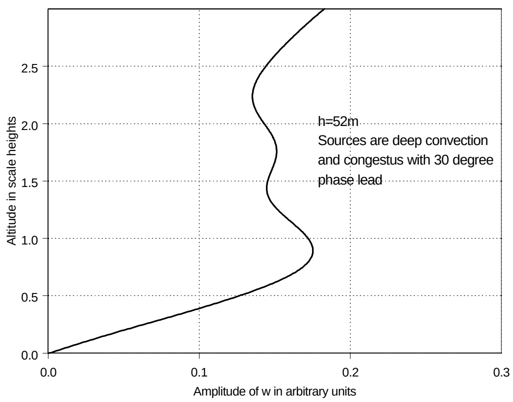


Figure 5

structure of waves with h= 18m and h= 52m. We see that within the troposphere, the waves pretty much follow the distribution of the effective heating, but above this region they behave like vertically propagating disturbances. This is exactly what is found by Straub and Kiladis (2002) (viz their Figure 3). The mathematical reason for this is simply that for thick forcing relative to the vertical wavelength,



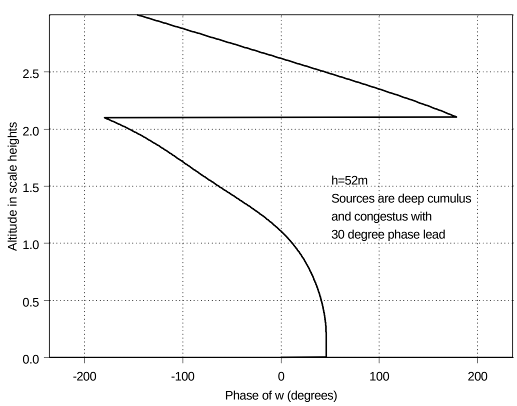


Figure 6

the solution is dominated by the particular solution which follows the forcing, while above the forcing, the homogeneous wave solution dominates. Note as well, that there is more wave leakage for h= 52m than for h= 18m; here again this is because the ratio of the vertical wavelength to the thickness of the forcing is greater.

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