3.3 VON KÁRMÁN-TYPE VORTEX STREETS AND BAROCLINIC JETS ON A TWO-LAYER BETA-PLANE: CLASSIFICATION AND CRITICAL SCALING

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1. MODEL

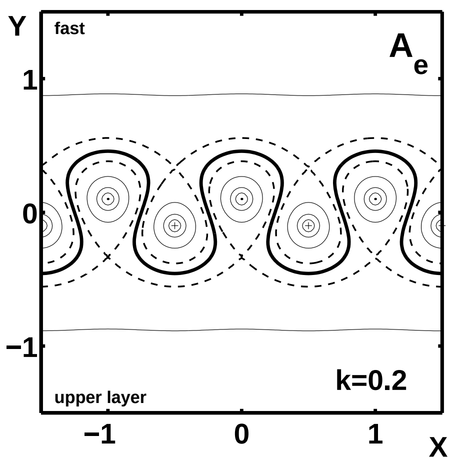

Figure 1: Stream lines in the upper layer (comoving reference frame) of an antisymmetric upper layer vortex street (fast solution for $\epsilon = 0.5$ and $\sigma = 2$) (see also text).

To investigate self-organized periodic coherent vortex structures arising in low Rossby number geophysical turbulent flows a conceptual model based on an analytical theory of von Kármán vortex streets affected by stratification and differential rotation is developed (see Gryanik, Borth & Olbers 2001). The model frame work is given by the standard quasi-geostrophic (QG) potential vorticity (PV) equation on a two-layer beta-plane

$$\begin{aligned} \frac{\partial q_j}{\partial t} &+ J(\psi_j, q_j) + \beta \frac{\partial \psi_j}{\partial x} = 0, \\ q_j &= \nabla^2 \psi_j + (-1)^j F_j (\psi_1 - \psi_2),\ j = 1, 2\,, \end{aligned}$$

with $F_j = f_0^2/g' H_j$ and f_0 the Coriolis parameter at the reference latitude, β the corresponding planetary vorticity gradient, H_j the layer thicknesses, ρ_j the fluid densities, g' the reduced gravity, q_j the vorticity fields and ψ_j the stream functions.

QG vortex streets are exact non-linear periodic solutions of the PV equations moving with constant zonal speed c. They consist of two parallel rows of equally spaced cyclonic and antiyclonic modified baroclinic point vortices which are arranged horizontally as the classical symmetric or antisymmetric von Kármán streets. The two-layer model allows the construction of vortex streets with different transport properties depending on the vertical structure of individual vortices. Three types of streets are considered — barotropic, baroclinic with vortices only in the first layer (upper layer streets) and streets with vortices in different layers (hetonic streets). The QG vortex streets are characterized by five non-dimensional parameters: The aspect ratio $k = b/a$, (with a the distance between vortices in one row and b the distance between rows), the ambient vorticity gradient $\epsilon = 2a^3\beta/\kappa$, (with κ the vortex strength), the stratification parameters $\sigma = a/R_0$ and $\delta = H_1/(H_1 + H_2)$, (with R_0 the Rossby radius), and the propagation speed $C = 2ca/\kappa$.

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2. SOLUTIONS: CLASSIFICATION AND CRITICAL SCALING

On f-plane to every set of parameters $(k, \epsilon, \sigma,\ \delta)$ a QG vortex street solution can be found. On beta-plane on the other hand this is not true anymore. A necessary condition to find von Kármán-type solutions there is that they move to the east ($c > 0$), i.e. not in the range of phase speeds of Rossby waves. An non-linear dispersion relation derived from the PV equation yields sufficient conditions for the existence of QG vortex streets. On beta-plane critical bounds arise in the parameter space beyond which no solutions can be found. For all parameters which lay within the critical bounds two solutions can be found a fast and a slow propagating street.

The regions I-V in figure 2 give the parameter spaces where solutions can be found. In I we have symmetric upper layer streets, in II symmetric hetonic streets, in III we have antisymmetric upper layer and symmetric hetonic streets in IV upper layer symmetric and antisymmetric streets and in V symmetric and antisymmetric hetonic streets. For increasing planetary vorticity gradients ϵ regions III, IV and V will disappear successively. For small ambient vorticity gradients all types of vortex streets can exist as long as the horizontal aspect ratio of the streets and the stratification parameter are in prescribed intervals. For increasing ambient vorticity gradients however only vortex pair-like streets survive (regions I and II). No antisymmetric vortex street can exist on the beta-plane if the vorticity gradient exceeds a critical threshold. The baroclinic jets induced by the vortex streets have zonal mean profiles characterized by several coupled length scales with a typical behaviour in the central part and in the tails. The investigation of the dispersion relation of QG vortex streets shows that there exists a critical maximum meridional distance between vortex rows on beta-plane which can give an explanation for the meridional scale selection process in zonal jets on beta-plane. This maximum critical distance depends on the intermittency properties of the PV field. In the two asymptotic limits of pair-like ($k \ll 1$) and sheet-like ($k \gg 1$) streets we get

$$b_{max} \approx 0.57 \left(\frac{\kappa}{\beta}\right)^{1/3} \quad \text{and} \quad b_{max} = \frac{\sqrt{2}}{e} \left(\frac{\kappa/a}{\beta}\right)^{1/2} \qquad (1)$$

respectively, so that the β-dependence is not universal.

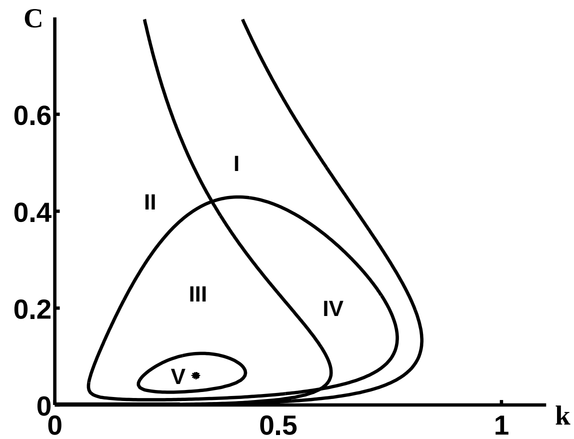

Figure 2: Diagram of states for non-vanishing planetary vorticity gradients ($\epsilon = 0.85$, $\delta = 0.5$ and $0 < \sigma < \infty$). Outside the regions I-V no solutions exist (see also text).

3. STRUCTURE OF FLOW

Figure 1 shows the meandering stream line structure in the upper layer of an upper layer vortex street. The dashed lines delineate the cells of closed fluid motion. The topology can change from the meandering state of interlaced fluid cells to a state with zonally connected fluid cells. It can happen that for the same vortex street different fluid cell topologies can coexist in the upper and lower layer.

QG von Kármán vortex streets can be used to model global zonal jets in the atmospheres or oceans of fast rotating planets. They especially allow the description of transport and mixing properties of meandering jets consisting of strongly localized PV anomalies, which gives a complementary model to the kinematical jet models traditionally used.

References

Gryanik, V. M., H. Borth, and D. Olbers: 2001 submitted, The theory of quasigeostrophic von kármán vortex streets in two-layer fluids on beta-plane. *J. Fluid Mech.*.