

ON THE CONNECTION BETWEEN COHERENT STRUCTURES AND OSCILLATORY WAVE PACKETS IN LARGE-SCALE ATMOSPHERIC FLOW

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1. Introduction

Coherent structures and oscillatory wave packets are among the most striking features of the large-scale atmospheric circulation. Although their role in atmospheric low-frequency variability, storm track morphology, and extended range forecasting is well established, there are many fundamental questions regarding their connection. For example, under what conditions can coherent structures break down into oscillatory wave packets? Can oscillatory wave packets organize into coherent structures and, if so, how? More practically speaking, what are the background flow characteristics that allow for the synoptic-scale transient eddies to be organized into large-scale blocking structures?

Here we present a self-consistent, weakly nonlinear theory that examines the connection between coherent structures and oscillatory wave packets in a barotropic atmosphere. In particular, we examine the weakly nonlinear dynamics of isolated anomalies embedded in a meridionally sheared, zonally varying background flow. An amplitude evolution equation is derived analytically, which has the form of a variable coefficient Korteweg-deVries (VC-KdV) equation, wherein the zonally varying background flow modulates the growth and propagation of the disturbance field. This VC-KdV equation possesses both coherent structure and oscillatory wave packet solutions. We show that the zonally varying background flow serves as a catalyst for the transformation between coherent structures and oscillatory wave packets.

2. The model

We consider a homogeneous fluid confined to a mid-latitude β -plane channel of infinite zonal extent. The upper and lower rigid boundaries are horizontal. The background flow is chosen to be a steady, meridionally sheared, zonally varying current. The evolution of disturbances superimposed on this background flow are governed by the quasigeostrophic potential vorticity equation, which can be written in non-dimensional form as (Pedlosky 1987)

$$\left[\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right] \nabla^2 \varphi + Q_y \frac{\partial \varphi}{\partial x} - Q_x \frac{\partial \varphi}{\partial y} + J(\varphi, \nabla^2 \varphi) = 0, \quad (1)$$

where the background flow satisfies

$$J(\Psi, Q) = F. \quad (2)$$

where F is an external potential vorticity source.

In the zonal direction we demand that the perturbation field be bounded at $X = \pm\infty$. At the channel sidewalls, $y = \pm 1$, we demand that the meridional velocity vanish.

2. Amplitude equation

In order to capture the balance between nonlinearity and dispersion, a requirement for the existence of solitary waves, we introduce the following "long" spatial and temporal scales: $X = \varepsilon^{1/2} x$ and $T = \varepsilon^{3/2} t$, where $\varepsilon \ll 1$.

The background velocity field is chosen as

$$U(X, y) = U_0(y) + \varepsilon U_1(X, y) \quad (3a)$$

$$V(X, y) = \varepsilon^{3/2} V_1(X, y) \quad (3b)$$

The streamfunction field is expanded in the asymptotic series

$$\varphi(X, y, T; \varepsilon) = \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots \quad (4)$$

Insertion of (3)-(4) into (1) and balancing like orders in ε , we obtain a sequence of problems. The $O(\varepsilon^2)$ balance yields the amplitude evolution equation for the disturbance field:

$$A_T + \underbrace{m_d A_{XXX}}_{\text{Dispersion}} + \underbrace{m_p(X) A_X}_{\text{Phase Speed}} + \underbrace{m_g(X) A}_{\text{Growth Rate}} + \underbrace{m_n A A_X}_{\text{Nonlinear}} = 0. \quad (5)$$

This equation is a variable coefficient KdV equation, which describes the evolution of long, low-frequency Rossby waves on zonally varying background flows. These Rossby wave disturbances may manifest as solitary waves or oscillatory wave packets

3. Analytical analysis: locally parallel (LP) background flow

Before proceeding to the numerical solution of the amplitude equation (5), it is instructive to first consider the dynamics of coherent structures in the form of solitary Rossby waves (SRWs) that are embedded in slowly varying, i.e., locally parallel (LP) background flows.

For LP background flows, the following conservation laws governing the dynamics of the SRWs can be derived (Hodyss and Nathan, 2002):

$$\frac{da}{dt} = \frac{2}{3} \left(\frac{\partial m_p}{\partial X'} - 2m_g \right) a, \quad (6)$$

$$\frac{dP}{dt} = - \left(\frac{m_n}{3} \frac{\partial a}{\partial X'} + 2m_g \right) P, \quad (7)$$

$$\frac{dM}{dt} = - \left[\frac{m_n}{3} \left(\frac{\partial a}{\partial X'} + \frac{\Gamma}{2} M_{OWP} \right) + m_g \right] M, \quad (8)$$

where $d/dt = \partial/\partial T' + c_0 \partial/\partial X'$. Here a is the amplitude of the SRW, M_{OWP} is the asymptotic mass contribution of the oscillatory wave packet to the SRW, P is the momentum, and M is the mass of the

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SRW. Among these conservation equations, the mass conservation equation (8) highlights the transformation process that can occur between a SRW and OWP. In particular, as the mass of the SRW decreases the amplitude of the OWP increases. The amplitude and momentum of the SRW are unaffected by the SRW-OWP transformation process.

4. Transformation between coherent structures and oscillatory wave packets

Examination of the amplitude equation (5) reveals that if the phase speed coefficient is symmetric and the growth rate coefficient is anti-symmetric, then the amplitude equation is invariant to the transformation, $X \rightarrow -X$ and $T \rightarrow -T$. In other words, the amplitude equation is reversible in space and time, which has important implications for the transformation between coherent structures and oscillatory wave packets, as shown in Fig. 1.

Consider first Figure 1a, which shows the space-time evolution of an initial coherent structure having the form of a SRW. The SRW slowly drifts downstream until it enters the local jet region, at which time an imbalance occurs between dispersion and nonlinearity. Because of this imbalance, the initial SRW emits an oscillatory wave packet (OWP). The original SRW and the OWP continue to propagate downstream. We note that the distinction between the SRW and OWP, which is difficult to see in Fig. 1a, becomes increasingly evident with time, i.e., as the OWP continues to disperse and separate from the SRW.

Figure 1b uses the results from the model run shown in Fig. 1a as the initial condition. The initial disturbance field, which contains a SRW and OWP, propagates downstream into the local jet region, which serves to organize and transform the disturbance field into a SRW, the same SRW that was used to initialize the model run shown in Fig. 1a.

Figure 1c uses the same initial condition as in Fig. 1b, but the background flow is zonally uniform. In this case the initial disturbance field, which contains an OWP, is unable to organize into a coherent structure. The zonally varying background flow is solely responsible for the transformation between coherent structures and oscillatory wave packets.

5. Conclusions

We have presented preliminary results showing that a zonally varying background flow can serve as the catalyst for the transformation between coherent structures and oscillatory wave packets in large-scale atmospheric flow. The breakdown from a coherent structure into an oscillatory wave packet is quite robust, and is due to the local imbalance between dispersion and nonlinearity that occurs within the zonally varying background flow. The reverse transformation, whereby an oscillatory wave packet transforms into a coherent structure, depends on the detailed nature of the zonally varying background flow and the initial structure of the wave packet. Work is currently underway to determine the specific background flow characteristics and packet structure that can organize an oscillatory wave packet into a coherent structure.

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6. References

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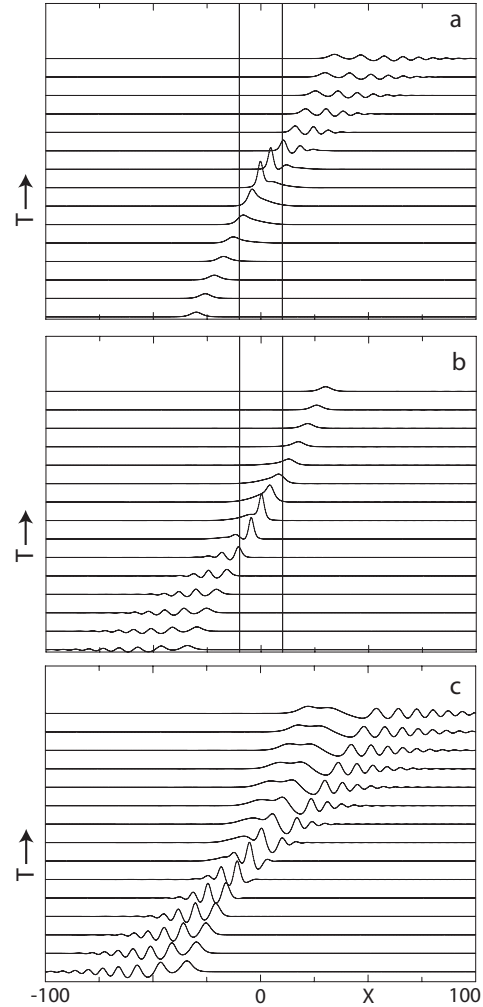


Figure 1. (a) An initial coherent structure in the form of a SRW breaks down into a SRW and an oscillatory wave packet. (b) An initial SRW and oscillatory Rossby wave packet are organized into a single SRW. This figure was initialized with the result from (a). (c) The same integration as in (b), except the zonally varying portion of the background flow has been removed; the background flow is zonally uniform.