13.1 SYNOPTIC AND DYNAMICAL INTERPRETATIONS OF ADJOINT-DERIVED SENSITIVITY STRUCTURE AND EVOLUTION

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1. INTRODUCTION

A forecast sensitivity study typically involves calculating how sensitive a particular differentiable function of a numerical weather prediction (NWP) model forecast state (called a response function, *R*) is to changes in the model control variables. These sensitivities, defined as the gradient of the response function with respect to the model control variables, are most efficiently calculated using the adjoint of an NWP model. Adjoint sensitivities have been used for a variety of applications including data assimilation, model parameter estimation, and targeted observing. Despite their various applications, few studies have provided synoptic and dynamical interpretations of these forecast sensitivity gradients or their evolution. Further, despite the promise these sensitivity gradients have in providing greater insight into synoptic case studies, few studies have exploited their utility.

Over the last two years, we have been observing the structure and evolution of forecast sensitivity gradients in real-time (http://helios.aos.wisc.edu) to develop a 'synoptic intuition' for how these sensitivity gradients may be related to synoptic and larger scale flow features. Toward that end, we have developed a set of tools that we believe allow for the meaningful application of sensitivity gradients in synoptic case studies. These tools represent a generalization of the approach typically taken in adjoint sensitivity studies in that the sensitivity gradients are taken with respect to synoptically and dynamically useful (derived) variables rather than exclusively the control variables of the model being used (*e.g.*, gradients are calculated with respect to relative vorticity and geopotential height instead of with respect to zonal and meridional components of the wind). The evolutions of these sensitivity patterns are also being calculated.

In this presentation, some of the results of this work and the insights developed over the past few years are presented. Specifically, a synoptic and dynamical interpretation of the structure and evolution of a set of forecast sensitivity gradients, particularly useful for understanding mid-latitude weather systems, is provided. Calculations of sensitivities with respect to variables derivable from model control variables will be also be presented.

In section 2, a description of the modeling system, adjoint model and necessary modifications to it, as well as the data sets used in this study is presented. An outline for how the gradients of response functions with respect to derived variables are calculated is found in section 3. Section 4 contains examples of these sensitivity calculations, along with brief interpretations. A description of future work may be found in section 5.

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2. MODEL CONFIGURATION AND DATA

Sensitivity calculations were performed using the MM5 Adjoint Modeling System (Zou et al., 1997). The Tangent Linear Model and the corresponding adjoint of the MM5 include simple physical parameterizations:

- Horizontal and vertical diffusion
- Dry convective adjustment
- Bulk aerodynamic surface flux parameterization
- Kuo and Grell cumulus parameterization schemes

All of the sensitivities to be described were calculated with this system by integrating the adjoint model "backwards in time" (without considering moisture) about a moist basic state derived from a non-hydrostatic MM5 non-linear forecast which utilized more sophisticated physics. All non-linear integrations were initialized using the National Center for Environmental Prediction's Eta model analyses.

In order to remove non-physical oscillations in the output of a time-dependent adjoint model, modifications to the adjoint model code (Zou et al., 2001) were required to output the time evolving sensitivity fields.

3. SENSITIVITY GRADIENTS

As described in the introduction, a typical forecast sensitivity study involves a calculation of the gradient of a response function with respect to model control variables at the initial forecast time. For the MM5, these control variables include u, v, w, T, and p' (the zonal, meridional, and vertical components of the wind, the temperature, and the pressure perturbation respectively). As a consequence, the sensitivity gradients calculated from the MM5 adjoint model integration are gradients with respect to these variables: $\partial R/\partial u$, $\partial R/\partial v$, $\partial R/\partial w$, $\partial R/\partial T$, and $\partial R/\partial p'$. In order to relate the sensitivity gradients to synoptic features (e.g., upper troughs and fronts) which are best characterized by perhaps, the vorticity (ζ) distribution, knowledge of the gradients of the response functions with respect to vorticity would be highly desirable. What is required then, is an operator that relates a derived model variable ($f(\mathbf{x})$), to the basic model variables, $\mathbf{x}$.

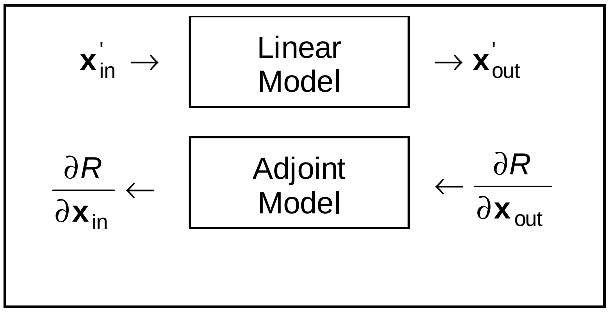

Figure 1. Schematic showing relationship between input and output of linear model and its adjoint. Note that the "input" to the adjoint model is actually a gradient of the response function with respect to the model output.

A model is an operator having an input and an output. The linearized model maps perturbations of the input ($\mathbf{x}'_{in}$) to perturbations of the model output ($\mathbf{x}'_{out}$) (Fig. 1, upper). The adjoint of this linear model has as its input the gradient of a pre-defined response function with respect to the model output variables and as its output, the gradient of that same response function with respect to the model input variables (Fig. 1, lower). Finally, in order to relate the output of the adjoint model, $\partial R/\partial \mathbf{x}_{in}$, to $\partial R/\partial f(\mathbf{x}_{in})$, the adjoint of the inverse of $f(\mathbf{x})$ is required (Fig. 2).

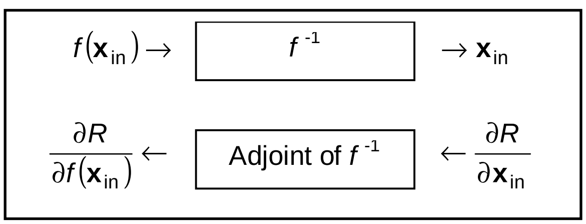

Figure 2. Schematic outlining procedure for obtaining the gradient of the response function with respect to a derived variable, which is a function of the model variables. If the derived variable can be inverted to obtain model variables, one can derive the gradient of the response function with the adjoint of the inverse.

As an example, in order to determine the gradient of a response function with respect to the relative vorticity ($\partial R/\partial \zeta$) given $\partial R/\partial u$ and $\partial R/\partial v$ as output from the MM5 adjoint modeling system, we need an operator that calculates u and v from ζ. This compound operator is defined by two steps: 1) the vorticity inversion operator which calculates the streamfunction (ψ) from ζ and 2) an operator which calculates the two-dimensional, non-divergent wind, $\mathbf{v}_{\psi}$ from the streamfunction (Fig. 3, upper). The adjoint of this two-step procedure allows for the calculation of $\partial R/\partial \zeta$ from $\partial R/\partial u$ and $\partial R/\partial v$ (Fig. 3, lower). Formally, an additional operator that relates the non-divergent wind to the full wind is required, but it can be shown that gradients with respect to the full wind are equal to gradients with respect to the non-divergent wind.

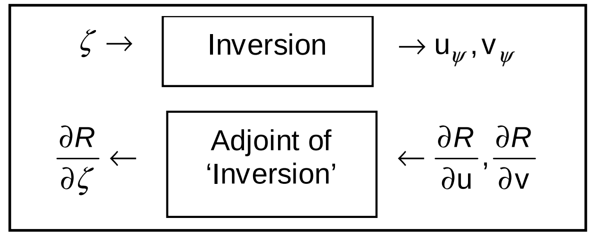

Figure 3. Schematic outlining procedure for obtaining the gradient of the response function with respect to relative vorticity, from gradients with respect to the zonal and meridional wind.

4. EXAMPLES AND INTERPRETATION

As was previously mentioned, adjoint-derived forecast sensitivities have been calculated in real-time over the past two years. One of the response functions studied intently is the 36h forecast of average temperature in a two-dimensional box over Wisconsin on the σ = 0.85 model surface (lower troposphere). For this response function, the gradient with respect to all model variables other than temperature at the 36h forecast time is zero (*i.e.*, $\partial R/\partial u = \partial R/\partial v = 0$). However, $\partial R/\partial T$ is a constant on the level and in the box for which the response function is defined (Figure 4d), and serves as the "input" for the adjoint model integration.

The time evolution of the forecast sensitivity gradients with respect to temperature for this response function is shown in Fig. 4 at 12 hour intervals. These gradients identify those regions at the initial (Fig. 4a) and 12 and 24 hour forecast times (Figs. 4b and c) of the model integration where changes in the analyzed or forecasted model temperature would have the greatest impact in changing the final time (36h) forecast temperature over Wisconsin. A positive perturbation to the analyzed temperature on the σ = 0.85 surface over the region over and to the west of James Bay in Ontario, where the forecast sensitivities are positive, would be associated with an increase in the average forecasted temperature over Wisconsin 36 hours later.

Our experience in monitoring the evolution of the sensitivity of this response function to the initial and forecasted temperature suggests that horizontal temperature advection is the principal mechanism in determining the average temperature over Wisconsin on the σ = 0.85 surface. Evidence for this interpretation may be seen in the evolution presented in Fig. 4. For the case shown, which occurred over the period 1200 UTC 28 March 2003 through 0000 UTC 30 March 2003, a cyclone propagated northeast from northern Wisconsin into Quebec and in its wake, cold advection dominated much of the north-central Midwest. As suggested by the evolution of the sensitivity pattern, the wind and temperature fields, one might surmise that the 'source' region for the air mass to reside over Wisconsin 36 hours from the model initial time of 1200 UTC 28 March would be found south of Hudson Bay. Further, for other cases not shown, we have seeded trajectories within and outside of these sensitive regions and

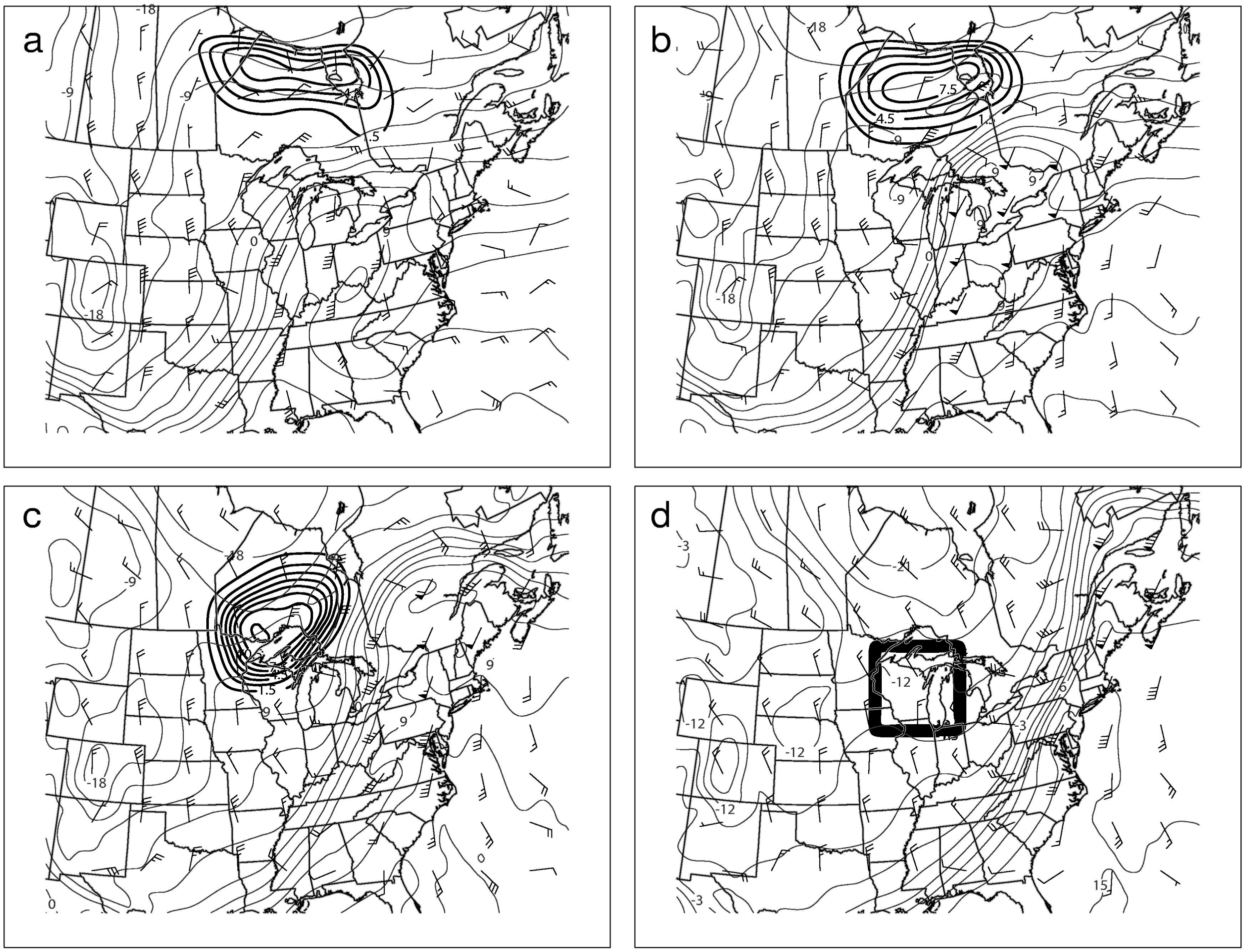

Figure 4. Time evolution of wind (barbs, knots), temperature (thin contours, 3°C) and sensitivity (heavy contours, interval 1.5 x 10^{-3}) of 36h forecast of average temperature over Wisconsin with respect to temperature on the σ = 0.85 surface at selected forecast times: (a) 0h; (b) 12h; (c) 24h; and (d) 36h for forecast initialized at 1200 UTC 28 March 2003.

observed that those trajectories originating within the sensitive regions end up in the region defining the response function at the final time, and those originating in zero sensitive regions (along the zero gradient contour) do not reach the box. These trajectories approximately remain on the same horizontal level as well.

As part of our goal to identify physically meaningful interpretations of the forecast sensitivity gradients, we have also calculated sensitivity gradients with respect to derived model variables. Fig. 5 shows the distribution of 700 hPa forecast sensitivities with respect to u, v, ζ, and ϕ (geopotential) along with the geopotential height at 30h in a forecast initialized on 0000 28 March 2003 (12 hours earlier than the previous map). At this time, the maximum sensitivities happen to be coincident with or flanking a geopotential trough. To interpret the sensitivities with respect to u or v, consider a perturbation to the 30h forecast of horizontal wind that would lead to a higher temperature 6h later in the region defined by the response function. Increasing (decreasing) the zonal and meridional components of the wind in the regions of positive (negative) sensitivity indicated on Figs. 5a and b respectively would lead to an increase in the average temperature. Such perturbations would be associated with the addition of anti-cyclonic vorticity in the vicinity of the trough axis – a weakening of that trough. The sensitivities, $\partial R/\partial \zeta$ and $\partial R/\partial \phi$, calculated from $\partial R/\partial u$ and $\partial R/\partial v$, provide a more succinct interpretation of the information found collectively in $\partial R/\partial u$ and $\partial R/\partial v$. By inspection, one can immediately see that weakening the cyclonic vorticity (or increasing the geopotential) in the model forecast in regions where $\partial R/\partial \zeta$ ($\partial R/\partial \phi$) is negative (positive) leads to an increase in the final time forecasted temperature. Synoptically, such perturbations to the relative vorticity and geopotential would imply a weakening of the 700 hPa cyclonic flow and a concomitant weakening of the cold temperature advection in the wake of the cyclonic circulation. Alternatively, such perturbations could also imply an increase in the thickness of the column, and as a consequence, indirectly suggest a warmer average temperature on the σ = 0.85 surface 6 hours later.

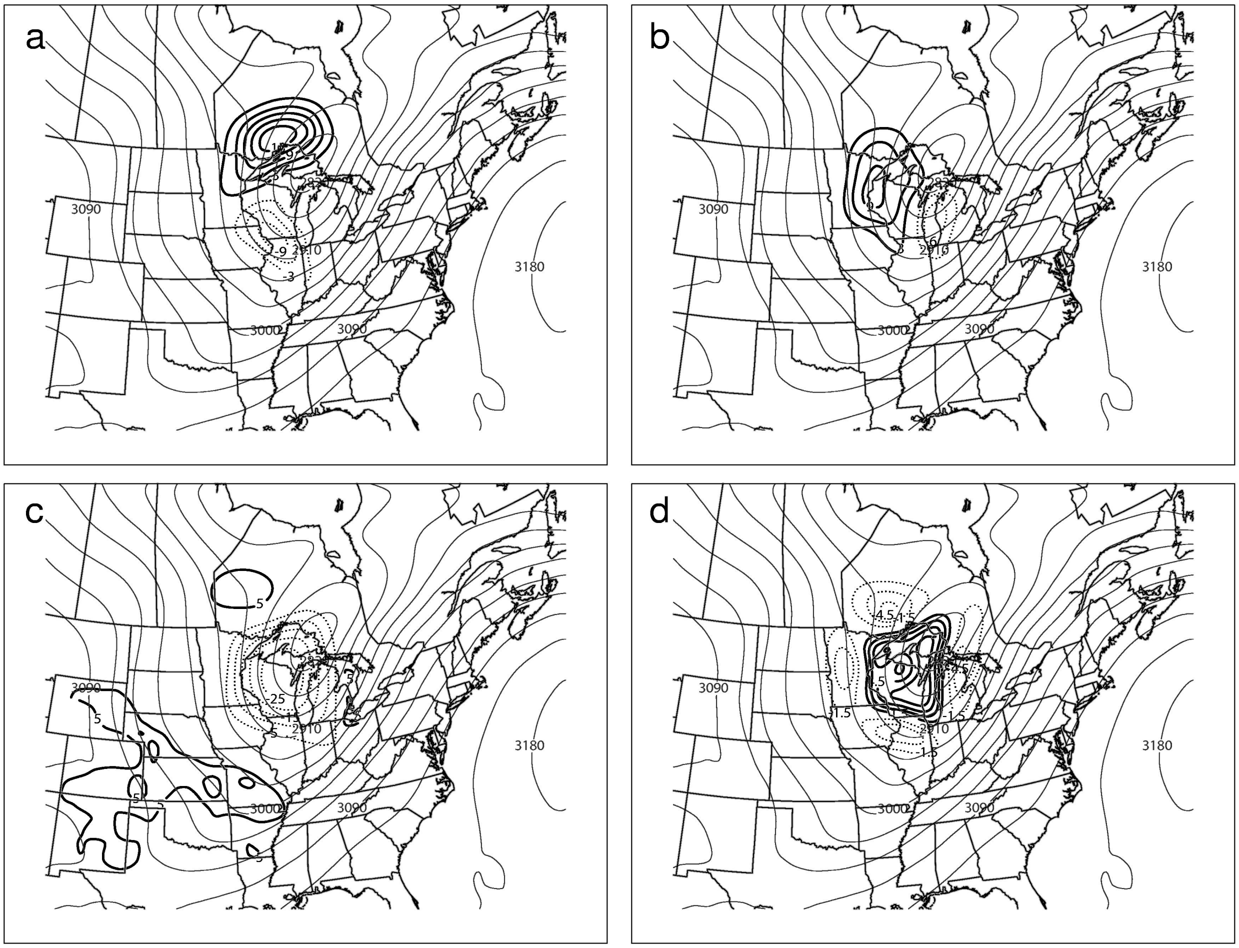

Figure 5. 30h forecast of 700 hPa geopotential height (thin contour, interval 30 m) and sensitivity of 36h forecast of average temperature over Wisconsin with respect to 30h forecast of (a) zonal component of wind (heavy contour, interval 3 x 10^{-4} $Km^{-1}s$); (b) meridional component of wind (heavy contour, interval 3 x 10^{-4} $Km^{-1}s$); (c) relative vorticity (heavy contour, interval 5 x 10-1 Ks); and (d) geopotential (heavy contour, interval 1.5 x 10^{-5} $Km^{-2}s$) for forecast initialized at 0000 UTC 28 March 2003. In all panels, negative values of sensitivity gradient are dotted.

5. FUTURE WORK

This work is part of a larger project aimed at providing meaningful synoptic and dynamical interpretations for forecast sensitivity gradients. Other synoptically relevant response functions such as energy-weighted error, circulation, horizontal frontogenesis, and vertical motion are being considered for operational calculations or synoptic case studies. Also, calculating forecast sensitivity gradients of a response function with respect to distributions to potential vorticity (or perhaps other variables) should provide a more complete means of interpreting these forecast sensitivity fields. Lastly, adjoint-derived sensitivities can be used with differences between operational analyses to generate an ensemble of forecasts for a particular forecast aspect (Kleist et al., 2001).

6. REFERENCES

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Acknowledgements

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