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EDDY-DRIVEN JETS FROM A MEAN-FLOW PERSPECTIVE

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1. INTRODUCTION

An eddy-driven jet is necessarily self-sustaining. Baroclinic eddies generated at the base of the jet propagate vertically and meridionally before dissipating. The resulting momentum fluxes sustain the zonal winds in the core of the jet aloft, while the secondary circulations that arise in response to irreversible eddy transports of heat and momentum sustain the baroclinicity at the base of the jet (cf. Robinson 2000). That baroclinic eddy generation is enhanced in the presence of stronger near-surface temperature gradients, which are in turn enhanced by the action of the eddies, sounds alarmingly like perpetual motion. The eddies, however, must be dynamically possible and they must be sustained against dissipation. Thus, the eddy-driven jet must occur within a broader preexisting baroclinic zone, and its strength is constrained by the requirements that the eddies can exist and can extract energy from the zonal flow.

Within the context of quasi-geostrophic dynamics, eddies gain energy from the zonal flow at a rate (per unit mass) given by,

$$\frac{dE^*}{dt} = -\frac{1}{p_b L}\int_0^{p_b}\int_{-L/2}^{L/2}[u]\left[v^* q^*\right]dy\,dp, \qquad (1)$$

where notation is standard, and the inclusion of the important lower boundary term is implied. For the eddies to gain energy from the zonal flow, their equatorward transport of potential vorticity (Eliassen-Palm flux convergence) aloft must occur where zonal winds are stronger than in the regions at or near the surface where the eddies transport potential vorticity poleward (EP-flux divergence). For a given spatial pattern of EP-flux convergence and divergence, as the strength of these eddy fluxes and the resulting eddy-driven jet increases, zonal winds increase in the eddy-generation region near the surface, and decrease in the eddy absorption region aloft, decreasing the r.h.s. of (1). Thus, as the eddy-driven portion of the jet becomes stronger, the transfer of energy from the zonal flow to the eddies is reduced.

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Here we consider these feedbacks in a very simple context. Explicit calculation of the eddies is avoided. Instead, idealized distributions of eddy generation and dissipation are assumed.

2. WAVE-MEAN FLOW BALANCE

We begin by considering a quasi-geostrophic steady state in which eddy transports of potential vorticity are balanced by Ekman drag at the lower boundary and by linear thermal relaxation in the interior. The zonally averaged potential-vorticity equations in the interior and at the lower boundary become,

$$\begin{aligned}
&\frac{\partial}{\partial p}\left(\frac{S}{\tau_N}\frac{\partial[u]}{\partial p}\right) = \frac{\partial^2\left[v^* q^*\right]}{\partial y^2}, \\
&\frac{S}{\tau_N}\frac{\partial[u]}{\partial p} - \frac{p_b}{\tau_E}\frac{\partial^2[u]}{\partial y^2} = -\frac{\partial^2\left[v^* q^*\right]}{\partial y^2} \ \{p = p_b\},
\end{aligned} \qquad (2)$$

where τ_N and τ_E are radiative and Ekman damping times, S is a static stability parameter (taken as constant in what follows), and p_b is the pressure at the top of the boundary layer. The potential vorticity, q, at the lower boundary has its usual definition, proportional to the lower-boundary temperature.

3. AN IDEALIZED EDDY-DRIVEN JET

We consider eddy forcing and zonal winds that vary sinusoidally with y. The eddy fluxes of potential vorticity are assumed to be confined to the lower boundary, and to one interior layer, $p=p_f$.

$$\begin{aligned}
&\left[v^* q^*\right] = A\left(\cos(ly) - 1\right)\,\delta(p - p_f), \\
&\left[v^* q^*\right] = A\left(\cos(ly) + 1\right)\ \{p = p_b\}.
\end{aligned} \qquad (3)$$

The y-invariant contributions to the fluxes insure that they are poleward everywhere at the surface and equatorward everywhere aloft. These contributions do not affect the solution to (2), but they are important for the energetics.

Substituting (3) into (2) and solving yields,

$$[u] = \frac{A}{p_b}\cos(ly)\left[2\tau_E + \tau_N \frac{l^2 p_b}{S}(p_b - p)\right]\{p \geq p_f\},$$
$$[u] = \frac{A}{p_b}\cos(ly)\left[2\tau_E + \tau_N \frac{l^2 p_b}{S}(p_b - p_f)\right]\{p < p_f\}. \qquad (4)$$

The solution is a jet, with its maximum at y=0. The jet has a barotropic component, and is entirely barotropic above the forcing level (p_f).

As discussed above, the eddies that drive this jet can exist only if there is a background baroclinic flow, here assumed to be a uniform vertical shear with respect to pressure, $U=\Lambda(p_b-p)$. The transfer of energy from the zonal flow to the eddies may be calculated from (1). Including the y-invariant portions of the potential-vorticity fluxes in (3) gives,

$$\frac{dE^*}{dt} = \frac{A}{p_b}\left[U_f - \frac{A}{2}\left(\frac{4\tau_E}{p_b} + \frac{l^2\tau_N}{S}(p_b - p_f)\right)\right], \qquad (5)$$

where $U_f=\Lambda(p_b-p_f)$. The first term represents the energy extraction from the background baroclinic flow, while the negative term results from the interaction of the y-varying part of the eddy forcing with the jet it induces.

The maximum rate of energy conversion with respect to the strength of eddy forcing, A, is,

$$\frac{dE^*}{dt} = \frac{\frac{1}{2}U_f^2}{4\tau_E + \frac{l^2(p_b - p_f)p_b}{S}\tau_N}. \qquad (6)$$

This rate increases with the meridional scale of the jet, and, for Earth-relevant parameters, continues to increase strongly at the largest meridional scales that fit on the planet. Thus (6) conveys no information about the scale of an eddy driven jet. It does, however, include the "barotropic governor" (James & Gray 1986), in that the maximum energy conversion decreases as the Ekman drag weakens (τ_E becomes large). This is a response to the barotropic component of the jet. When the barotropic portion of the jet is suppressed by very strong Ekman dissipation, however, the energy transfer is still limited, for strong eddy forcing, by the reduction of zonal winds on the wings of the eddy-driven jet.

4. AN EDDY-DRIVEN JET WITH DIFFUSIVE HEAT TRANSPORT

To predict a finite meridional scale, the derivation of (6) must be modified to include some dependence of near-surface eddy heat transport on the baroclinicity of the zonal flow, the positive feedback described in the introduction. This is most simply treated as diffusion. Thus,

$$\left[v^* q^*\right] = -\kappa S \frac{\partial[u]}{\partial p} \;\; \{p = p_b\}, \qquad (7)$$

where κ is a diffusion coefficient. If the structure of eddy fluxes assumed in (3) is retained, (7) is consistent with (2) only when,

$$\kappa = \frac{1}{l^2\tau_N}. \qquad (8)$$

The strength of the eddy driven part of the flow is set by requiring,

$$\frac{\partial[u]}{\partial p} \propto 1 + \cos(ly) \;\; \{p = p_b\}, \qquad (9)$$

which must be the case if the diffusive treatment is to apply to the y-invariant as well as to the sinusoidal portion of the flow (i.e. for (3) to be consistent with (8)). Under these constraints, the rate of energy transfer from the zonal flow to the eddies is given by,

$$\frac{dE^*}{dt} = \frac{\frac{1}{2}U_f^{\,2} S}{(p_b - p_f)p_b}\kappa\left[1 - \frac{4S\tau_E}{(p_b - p_f)p_b}\kappa\right] \qquad (10)$$

The value of κ which maximizes this transfer, implies, through (8), a meridional wavenumber given by,

$$l^2 = \frac{8S}{(p_b - p_f)p_b}\frac{\tau_E}{\tau_N}. \qquad (11)$$

For reasonable values of parameters: $S \sim 5\ 10^{-3}\ \mathrm{Pa^2 m^{-2}}$, $\tau_E < \tau_N$, and a middle to upper tropospheric value for p_f, (11) gives plausible values for the width of the jet. It is, however, difficult to know how seriously to take this "prediction" of jet scale, in that the set-up is highly constrained and idealized, p_f is a free parameter, and any direct effect on the eddies of changing dissipation parameters is ignored. On the other hand, this approach appears to be the simplest that takes into account the fundamental positive and negative feedbacks on the strength of an eddy-driven jet.

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