7.9 DYNAMICS OF ISOLATED ANOMALIES IN ZONALLY VARYING BAROCLINIC FLOW: SOLITARY WAVES, NONLINEAR STABILITY, AND CONSERVATION LAWS

Daniel Hodyss* and Terry Nathan
University of California, Davis

1. Introduction

Because local baroclinic instability (LBI) plays a central role in a variety of atmospheric phenomena, including regional cyclogenesis and atmospheric blocking, it has been the focus of much research over the past two decades. Theoretical studies of *nonlinear* LBI have been relatively few, however. Those studies that have examined LBI have traditionally considered the localization of the disturbance to occur from either a zonally varying background flow or from an appropriately chosen initial condition. Few, if any, studies have combined these two localization mechanisms. Here we combine these two mechanisms by examining the finite amplitude baroclinic instability properties of solitary waves (SWs) propagating in *zonally varying* flows having isolated regions of instability.

2. Model and Analytical Development

We consider a two-layer fluid confined to a mid-latitude β-plane channel of infinite zonal extent that is bounded above and below by horizontal, rigid boundaries. We consider a zonally varying background flow that consists of two parts: (i) an O(1) meridionally sheared, *zonally uniform* part, and (ii) a small, meridionally sheared, *zonally varying* part. The O(1) part of the background flow is chosen marginally stable with respect to the necessary condition for linear instability (Pedlosky, 1987). Using a multiple-scale analysis, an amplitude equation is obtained analytically for a disturbance field that is superimposed on the zonally varying background flow. The amplitude equation takes the form of a variable coefficient Boussinesq equation, viz.

$$A_{TT} + \left[m_d A_{XXX} - m_p^2 A_X + m_g A + m_n A A_X \right]_X = 0 \cdot \qquad (2.1)$$

This amplitude equation possesses SW solutions. For zonally uniform background flows, (2.1) reduces to the constant coefficient Boussinesq equation obtained by Helfrich and Pedlosky (1993).

In (2.1) the dispersion term $m_d A_{XXX}$ and nonlinear term $m_n A A_X$ must balance in order to support SWs. The term $m_p^2(X) A_X$ describes the propagation speed of linear long-waves, the speed being $m_p(X)$. The term $m_g(X) A$ affects both the translation speed and amplitude of the long-waves.

3. Conservation Laws

A sequence of conservation laws are obtained by differentiating (2.1) k times (k = 0,1,2,3,…), then multiplying by X^k, and integrating over X and T to obtain

$$\frac{\partial}{\partial T} \int_{-\infty}^{\infty} X^k \frac{\partial^k A}{\partial X^k} dX = \int_{-\infty}^{\infty} X^k \left. \frac{\partial^{k+1} A}{\partial X^k \partial T} \right|_{T=0} dX \qquad (3.1)$$

Consider, for example, the case k = 0, which shows that the integrated amplitude of the disturbance will grow at most linearly with time (T). However, because we are interested in isolated jet flows, we assume that at T = 0 the disturbance is situated far upstream of the zonal variation in the flow. Because the background flow in this region is (marginally) stable, the initial time tendency of the integrated amplitude must vanish. Thus the integrated amplitude of the disturbance must be conserved after propagating through a zonally isolated jet flow.

* *Corresponding author address*: Daniel Hodyss, Department of Land, Air, and Water Resources, University of California, Davis, CA 95616-8627; e-mail: dhhodyss@ucdavis.edu

4. Numerical Experiments

Equation (2.1) was solved numerically using a pseudo-spectral method with the 3rd order Adams-Bashforth scheme in time; the time increment is 0.0005. The spectral expansion was truncated at 128 Fourier modes and the nonlinear term was evaluated using the transform method. The zonally infinite domain is modeled by adding a damping region spanning ten grid points closest to the eastern boundary. The zonally varying background flow, i.e. the zonally varying jet, is Gaussian in form.

a. Linear Analysis

To determine the dynamics and structural properties of small-amplitude waves, we linearize and then solve (2.1) as an initial-value problem with small amplitude ($A<10^{-2}$) random noise as an initial condition. The O(1) background flow is marginally stable.

We find that the linearized, variable-coefficient Boussinesq equation supports an unstable normal mode when either $m_d < 0$ or $m_p^2 < 0$. When $m_d > 0$ and $m_p^2 < 0$, for example, the instability properties are characterized by the following:

i. a growth rate that increases as m_d decreases,
ii. a growth rate that increases as the jet length increases,
iii. growing modes that are oscillatory in space and trapped to the unstable region.

For marginally stable zonally varying jets, for which [$0<m_p^2<<O(1)$], an unexpected, powerful transient (non-modal) growth emerges. This transient growth is strongest when m_g=O(1) and m_d<<O(1). Depending on the sign and form of m_g, two types of waves emerge: 1) waves trapped to the jet region (Fig. 1), and 2) waves repelled from the jet region (not shown).

b. Nonlinear Analysis

We briefly describe two numerical experiments that highlight the uniqueness and importance of the variable-coefficient Boussinesq equation (2.1). In the first experiment we examine the effects of nonlinearity on the linear transient growth shown in Figure 1. In the second experiment we place a SW far upstream of the isolated jet flow and examine its space-time evolution.

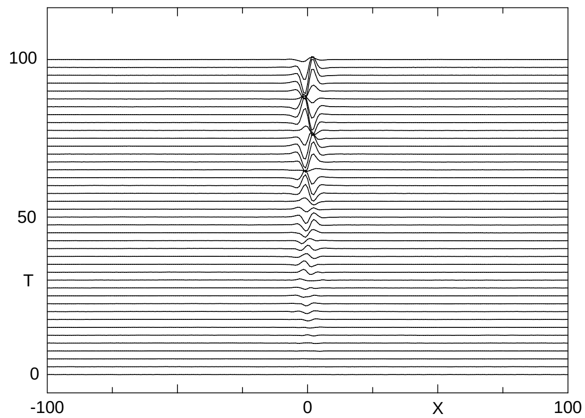

Figure 1. The transient development of an initial field of small amplitude random noise to trapped linear waves. The isolated jet flow is everywhere marginally stable.

The effects of nonlinearity on the transient growth shown in figure 1 are obtained by integrating (2.1) forward in time starting from a small field of random noise. The initial field grows super-exponentially into a coherent structure in 43 time units. Note that the transient disturbance shown in Figure 1 spans 100 time units. This coherent structure is similar in form to the bottom panel of figure 2. Recall, the O(1) background flow in this case is marginally stable; the nonlinear instability that results is due to the relatively large amplitude that occurs after the early, large transient growth of the disturbance.

In the second experiment we choose a flow that is again marginally stable. Figure 2 shows a SW that propagates toward the jet and grows on the upstream side of the jet. This growth is super-exponential, resulting in a rapid increase in amplitude as well as a decrease in horizontal scale. The disturbance field is characterized by the development of a weak "trough" upstream and downstream of the SW. These structural changes can be explained by considering the conservation of mass of the SW (see Hodyss and Nathan 2003).

5. Conclusions

The finite amplitude dynamics of baroclinic disturbances embedded in a zonally varying, marginally stable jet flow is examined. The governing equation for the disturbance field is a variable coefficient Bousinesq equation, which possesses a rich spectrum of behaviors.

A transient stability analysis shows that two types of transient disturbances emerge from a field of small amplitude random noise. One disturbance type remains "trapped" inside the jet region, and the other is "repelled" from the jet region.

Whether the zonally isolated jet flow is stable or unstable, the nonlinearity can eventually organize the initial small-amplitude random noise into a super-exponentially growing coherent structure. Under certain conditions, this super-exponential rate can also occur for a SW that propagates into an isolated jet flow. In the Boussinesq equation (2.1), nonlinearity does not equilibrate the disturbance; the flow is nonlinearly unstable and becomes infinite in a finite time.

These results apply to a wide range of atmospheric phenomenon, including large-scale isolated anomalies in the form of intense jets and split-flow blocks. For example, we have found that combined linear/nonlinear instability is a potentially important mechanism for forming and maintaining isolated anomalies.

The development of large amplitude isolated anomalies from small-amplitude random noise can occur in two ways. First, an isolated unstable jet flow can support a growing normal mode that becomes large enough to excite a nonlinear instability, such as that described above. Second, a stable isolated jet flow may support transient (non-modal) growth that is strong enough to excite the nonlinear instability of the wave.

If stable finite-amplitude disturbances are already present in the fluid, then the isolated jet flow can initiate nonlinear instability.

This ongoing work is currently focusing on determining the specific characteristics of the background flow that control the formation of the coherent structures and nonlinear instability. The robustness of the above solutions is being explored using a fully nonlinear model.

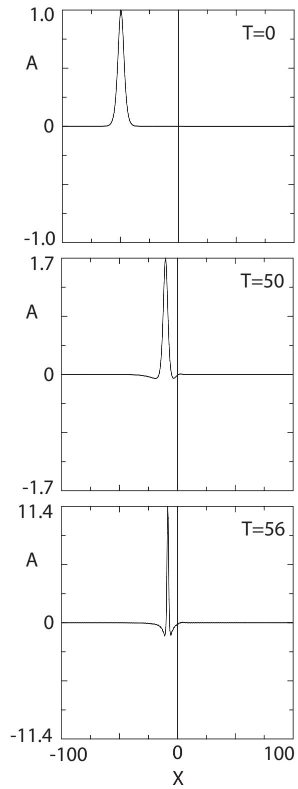

Figure 2. Shown here is the propagation of a SW towards a locally stable, zonally isolated jet. The jet center is denoted by the vertical line.

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6. References

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