# 6.4 A Linear Theory of Orographic Precipitation

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## 1. Introduction

A linear orographic precipitation model (Smith and Barstad, 2003) has been developed with the following characteristics.

- Analytically tractable so that its properties can be easily understood
- Applicable to actual complex terrain and arbitrary wind direction so that it can be tested against real data
- Reduces to the classical upslope model so that it can be compared with earlier work
- Includes the basic physical elements: airflow dynamics, condensed water conversion, advection and fallout, and downslope evaporation, leading to a theory of precipitation efficiency

The model includes Fourier Transform solutions of the linear equations of wave dynamics and advection of condensed water. Microphysical processes are represented by time delays between condensation and precipitation. Inputs to the model are: underlying terrain, wind speed and direction, surface temperature and lapse rate. The three factors controlling the precipitation in the model are: 1) the amount of water vapor flux approaching the mountain, 2) the depth of the moist layer in comparison to the depth of airflow lifting, and 3) the advection of condensed water from the production areas (uplift) to the loss areas (downdraft).

## 2. Governing Equations

Based on the steady-state assumption, the advection and transformation of vertically integrated condensed water ($q_c$ = cloud water content and $q_h$ = hydrometeor content) can be written, following Smith (2003):

$$\vec{U} \cdot \nabla q_c = S_{ref}(x,y) - q_c / \tau_c \qquad (1a)$$

$$\vec{U} \cdot \nabla q_h = q_c / \tau_c - q_h / \tau_h \qquad (1b)$$

where $T_c$ and $T_h$ are the characteristic time-scales for cloud water conversion and hydrometeor fall-out. The source term (S) in (1a) can be the classical upslope form

$$S = C_w \vec{U} \bullet \nabla h(x,y)$$

(Smith, 1979), or computed using the moist adiabatic lapse rate and mountain wave theory. The coefficient $C_w$ relates the rate of ascent with the rate of condensation. The last term in (1b) is the precipitation rate at the ground (P). As lifting in front of a mountain drives S positive, the transformation term in (1a) converts cloud water to hydrometeors in (1b). S gets the opposite sign in downslope regions, drying the air and evaporating hydrometeors.

In this brief presentation of the theory, we skip the derivation and display the final result, a transfer function relating the Fourier Transform of the terrain h(x,y) to the Transform of the field of precipitation P(x,y),

$$\hat{P}(k,l) = \frac{C_w \, i \sigma \hat{h}(k,l)}{[1 - imH_w][1 + i\sigma\tau_c][1 + i\sigma\tau_h]} \qquad (2)$$

$$m = \{[\frac{N_m{}^2 - \sigma^2}{\sigma^2}](k^2 + l^2)\}^{1/2} \qquad (3)$$

$$C_w = \rho_{v0}(\frac{\Gamma_m}{\gamma}) \qquad (4)$$

In (2), m is the vertical wave number and $C_w$ the lifting sensitivity factor. In (3) (k,l) are the horizontal wave number components, $\sigma = uk + vl$ is the intrinsic frequency, $H_w$ is the scale height of water vapor, $\rho_{v0}$ is the density of water vapor, $\gamma$ is the ambient lapse rate and $\Gamma_m$ is the averaged moist adiabatic lapse rate. The required transforms can be done analytically in a few cases, but generally, a double Fast Fourier Transform (FFT) is used.

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The first factor in the denominator of (2) describes airflow dynamics. The second and third factors describe cloud delays and advection. The reduction in precipitation efficiency due to downslope evaporation is not contained explicitly in (2), but is present when the predicted precipitation field *P(x,y)* is truncated according to $P_{Trun}(x,y)=Maximum(P,O)$. An interesting property of (2) is that the dynamics and cloud-delay factors in the denominator have a similar form. The appearance of $i=\sqrt{-1}$ in each factor causes a phase shift of the solution, in addition to the amplitude change. The different sign in the dynamic and cloud factors is significant. The negative sign in the dynamics factor gives an upwind shift to the precipitation pattern while the positive signs in the cloud factors cause a downstream shift. These factors also differ significantly in the way that wavenumber enters the definitions of $m(k,l)$ and $\sigma(k,l)$. Note that the two cloud time scales, $\tau_c$ and $\tau_f$ are mathematically analogous. When $m=\tau_c=\tau_f=0$, (2) reduces to the standard upslope model with no airflow dynamics and no condensed water advection (Smith, 1979).

Compared to the standard upslope model, the current model reduces precipitation with two processes. First, depending on the depth of the moist layer, the moist static stability, wind speed and mountain scale and shape, the terrain induced lifting may not penetrate up through the moist layer, thus limiting condensation. Second, depending on the cloud physics time scales, the wind speed and the mountain scale, the condensed water may be advected to the lee side and evaporated instead of precipitating. These two limitations can be monitored by calculating the efficiency numbers,

$$PE_{dyn}=\frac{S_{dyn}}{S_{ref}} \qquad PE_{cloud}=\frac{P}{S_{dyn}} \qquad (5)$$

The $S_{dyn}$ denotes the production rate of cloud water in presence of wave dynamics, and $S_{ref}$ without wave dynamics. $PE_{dyn}$ is therefore a measure of the role played by the wave dynamics in the model. The $PE_{cloud}$ is in the same way, a measure of how the microphysical processes influence the precipitation.

## 3. Triangle ridge

The triangle ridge is a useful example, as the raw upslope condensation value is constant over the windward slope. Thus, it is easy to see modification caused by airflow dynamics. The combined influence of full dynamics and cloud time-delays (2) is shown in Fig. 1, with parameters: $T_0=280K$, $\gamma=-5.8C\cdot km^{-1}$, $U=15ms^{-1}$ so $\Gamma_m=-6.5C\cdot km^{-1}$, $N_m=0.005s^{-1}$, $\rho_{Sref}=7.4gm^{-3}$, $H_w=2500m$. Also $h_m=500m$ and $\tau_c=\tau_f=1000s$, $a=15km$. In Fig. 1, the raw upslope model predicts a constant value of condensation, P=15mm/hr, over the windward slope, and an equal negative value over the lee slope. . The effect of airflow dynamics is to reduce the total condensation and shift the maximum upwind, close to the “slope-break” of the triangle ridge. The source term becomes negative slightly upstream of the hill crest. The effect of cloud delay reduces the precipitation further, and shifts the precipitation peak downstream. The precipitation maximum (3mm/hr) is located close to the hill top. There is downstream condensation in a lee wave, but lee-side descent, drift and evaporation prevent precipitation.

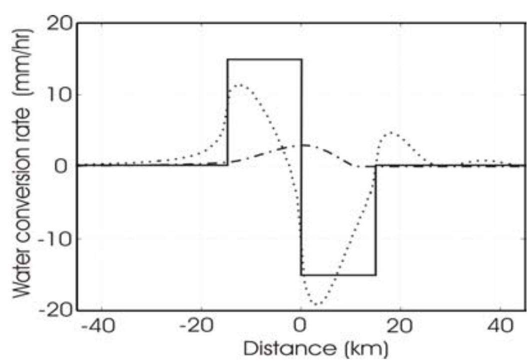


**Figure 1**) Condensation or precipitation rates (mm/hour) over a triangle ridge with three different assumptions: (solid), condensation patterns with no dynamics or cloud delays, (dotted) dynamics only, (dashed) precipitation with dynamics and cloud delays (2). Note the great reduction in total precipitation caused by dynamics and cloud delays. The flow is from left to right. The ridge begins to rise at x=-15 km.

## 4. An application of the linear model

To illustrate the properties of the linear model, we present one example of a predicted precipitation pattern over real terrain. We select the Olympic Range in Washington State as it is compact, complex and relatively well studied. It is one of the rainiest spots in North America, but with a definite rain shadow on the northeast side. Our intention in this section is not to test the model, but only to exhibit its behavior.

For the example, we consider a southwest wind with speed 15m/s and a moist stability of $0.005s^{-1}$. The surface temperature and specific humidity are 280K and 6.2 g/kg. The moist layer depth is 2.5 kilometers. The cloud time delays are each 1000 seconds. The 6 hour accumulated precipitation is shown in millimeters, with a maximum value of about 26 mm just upwind of the highest peak; Mt Olympus (2428m).

Several features can be noted. Four tongues of high precipitation are associated with four southwestward directed ridges. Light precipitation is found well upstream of the mountains, even over the sea. There is some spillover, but mostly the northeast lee slopes are dry. The model predicts that the high peaks in the northeast part of the massif collect no precipitation.

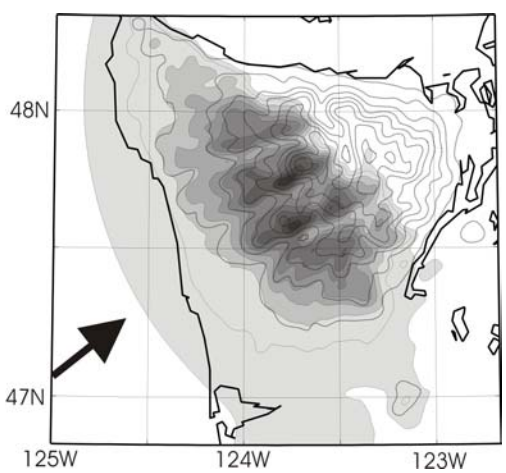


Figure 2) A real-world application of the linear FFT model; the Olympic Mountains under a southwesterly airflow. The 6 hour accumulated precipitation is shown shaded with a 2.5 mm contour interval. The maximum precipitation is 26 mm. The terrain is shown (dotted) with a 200 meter contour interval. The coastline is shown with a dark solid line.

## 5. Conclusions and future work

The new linear model clarifies the roles of uplift penetration and condensed water advection in orographic precipitation. It also provides an extremely quick method for estimating precipitation patterns over complex terrain on fine scales. The disadvantage is the number of strong assumptions required, e.g. linear wave dynamics, near saturation, steady state, etc. Particularly problematic is the assumption of constant cloud physics time scales. Jiang and Smith (2003) discuss a strong non-linearity associated with collection and accretion that may have to be considered. Currently, testing of the linear model is underway using data sets from CALJET (Neiman et al, 2002), IPEX (Cheng, L., 2001), and MAP (Smith et al., 2003)

### ACKNOWLEDGMENTS

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