13.3 THE NATURE AND EVOLUTION OF BALANCE IN UNSTABLE BAROTROPIC JETS

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1. INTRODUCTION

The nature of balance in the atmosphere is of central importance to the dynamics of both the troposphere and the stratosphere, and unbalanced motions such as inertia-gravity waves play a significant role in many aspects of atmospheric behavior. In light of the importance of upper-tropospheric jets for the generation of inertia-gravity waves in the atmosphere, this study examines the evolution of unstable barotropic jets to assess the nature and evolution of balance in these features. This issue is explored using the simplest non-trivial dynamical framework in which balanced and unbalanced flows can coexist, namely the one-layer shallow-water equations.

In this study, numerical simulations of initially balanced zonal barotropic jets on an f plane are investigated for evidence of the breakdown of balance and the generation of inertia-gravity waves during the life cycles of the instabilities to these jets. In these simulations, the parameters of the basic-state jet (i.e., jet width and speed) are varied systematically in an attempt to elucidate the dependence of balance on the structure and dynamical evolution of the instability.

The presence of unbalanced flow, either in numerical simulations or in atmospheric data, is typically inferred via various quantities that provide indirect measures of imbalance, such as the existence of strong ageostrophy, large Rossby and/or Lagrangian Rossby numbers, and large values of horizontal divergence and its material derivative. Nevertheless, these quantities are based on specific balance constraints (i.e., quasigeostrophy, semigeostrophy, or nonlinear balance), and it should be noted that an assessment of balance based on the inaccuracy of these constraints admits the possibility that the unbalanced flow so identified includes higher-order balanced motions not accounted for in the system under consideration in addition to inertia-gravity waves. In strong jets, neither the Rossby number nor the Froude number are small compared to unity, therefore the applicability of traditional scale analysis is unclear. Nevertheless, it has been demonstrated that even when Rossby and Froude numbers are both O(1), inertia-gravity waves of large amplitude are not typically generated in shallow-water model simulations (McIntyre and Norton 2000).

In the following section, the shallow-water model is described, along with the methodology used for this investigation. Section 3 provides preliminary results from these simulations for a single set of initial jet parameters, and in section 4, a brief discussion and a plan for future research are presented.

2. MODEL AND DIAGNOSTIC CALCULATIONS

A one-layer shallow-water equation model in Cartesian f-plane geometry is used to simulate the life cycles of the instabilities to the basic-state zonal profile

$$U(y) = U_0 \,\mathrm{sech}^2\left(\frac{y}{y_0}\right), \qquad (1)$$

where U_0 is the maximum jet speed, and y_0 is the jet width. Random perturbations of infinitesimal amplitude are added to this basic-state jet, and the model is run to grow an unstable wave. The time t = 0 in the simulation shown here corresponds to the time at which the maximum meridional wind is 1.5 ms^{-1}. Boundary conditions in x are periodic and in y are solid wall with a damping layer to prevent reflection of waves back into the domain. A coordinate transformation is used in the y direction so that the boundaries are far from the jet region. The domain length is equal to one wavelength of the unstable mode. Associated with this jet profile, it is possible to define Rossby, Froude, and Burger numbers as follows:

$$Ro_J = \frac{U_0}{f_0 y_0}, \quad Fr_J = \frac{U_0}{\sqrt{gh_0}}, \quad B_J = \frac{f_0^2 y_0^2}{gh_0}, \qquad (2)$$

where h_0 is the resting depth of the layer, g is the acceleration due to gravity, and f_0 is the (constant) Coriolis parameter. The latitude is set to 40^{o}N for these simulations. In addition, it may be useful to consider local values of Ro and Fr, defined by

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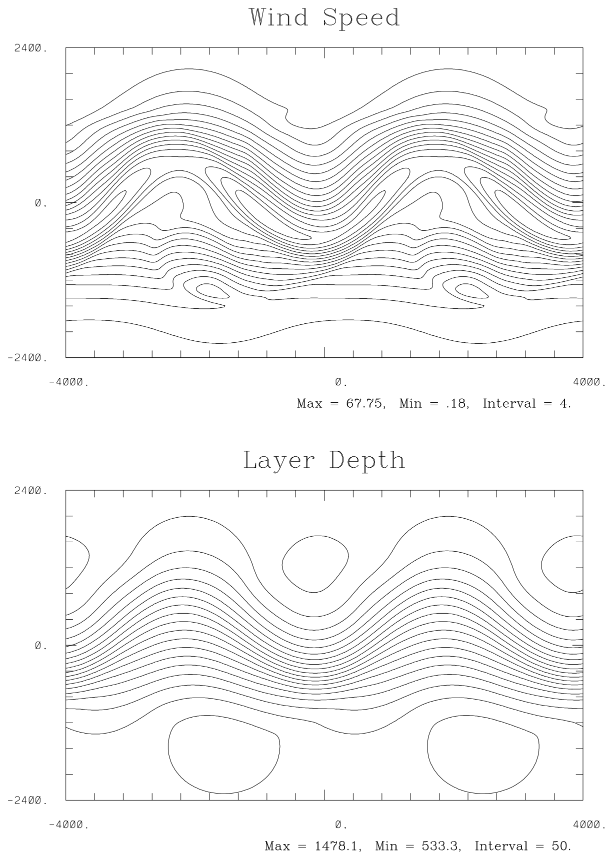

Figure 1. Wind speed (top) and layer depth (bottom) at t = 95 h.

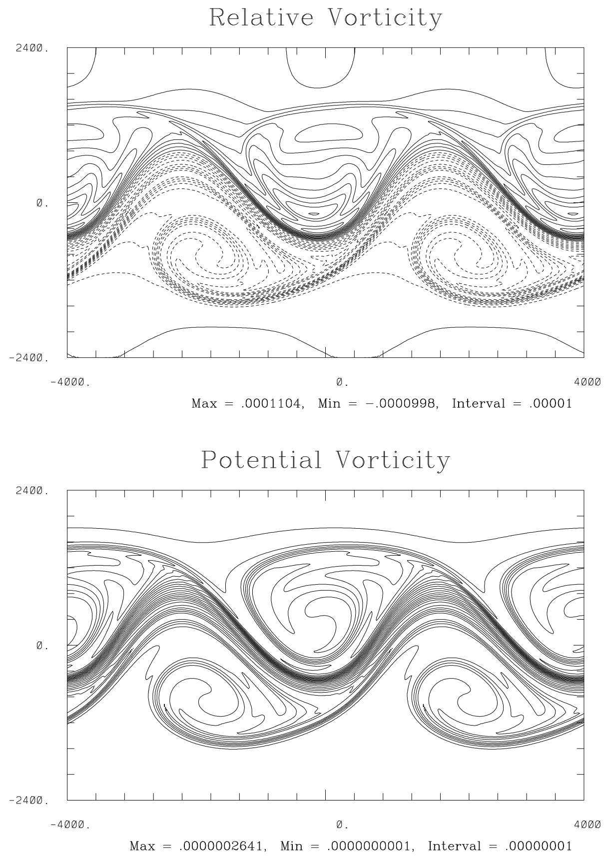

Figure 2. Relative vorticity (top) and potential vorticity (bottom) at t = 95 h.

$$Ro = \frac{\zeta_{\max}}{f_o}, \quad Fr = \frac{|\mathbf{V}|}{\sqrt{gh}} \,, \tag{3}$$

where $\zeta_{\max}$ is the maximum relative vorticity in the domain, and $\mathbf{V}$ and h are the velocity and layer depth, respectively.

Finally, we define a parameter γ,

$$\gamma = \frac{\delta_{\max}}{\zeta_{\max}} \,, \tag{4}$$

where $\delta_{\max}$ is the maximum horizontal divergence in the domain. This parameter provides a crude estimate of the validity of nonlinear balance following the ad hoc scale analysis of Haltiner and Williams (1980), and its relevance may be verified a posteriori by evaluation of terms in the shallow-water divergence equation.

3. RESULTS

In the simulation shown here, $U_0 = 70\,\mathrm{ms}^{-1}$, $h_0 = 1000$ m and $y_0 = 600$ km. Ro_J, Fr_J, and B_J for this simulation are 1.2, 0.7, and 1.7, respectively. The growth rate of the unstable wave in this case is $0.9 \times 10^{-5}\ \mathrm{s}^{-1}$, corresponding to an e-folding time of approximately 30 h. The wind and height fields are shown in Fig. 1 for t = 95 h, at which time the wave has reached finite amplitude. The relative vorticity and potential vorticity fields, shown in Fig. 2, indicate that at this time the wave has the appearance of a von Kármán-like vortex street, with alternating positive and negative vortices.

Figure 3 shows the evolution of the maximum local Rossby number. It is initially O(1) and increases slightly as the barotropic wave grows. The maximum local Froude number remains O(1) throughout the entire simulation because the jet speed does not vary greatly. From the perspective of a scale analysis (e.g., McWilliams 1985; Spall and McWilliams 1992; McIntyre and Norton 2000), it is required that either Ro or Fr must be small for nonlinear balance to be valid; however, neither of these conditions is met for this jet.

The evolution of the parameter, γ, is shown in Fig. 4. It is apparent that γ remains less than $O(10^{-1})$ for the duration of the simulation, such that the maximum horizontal divergence remains small in comparison to the maximum relative vorticity. This implies that nonlinear balance should remain valid to a good approximation throughout the entire simulation, despite the fact that neither *Ro* nor *Fr* is small enough to suggest a priori that this should be the case. This implication is substantiated by evaluating the terms in the divergence equation that are neglected to obtain the nonlinear balance equation (not shown). Such terms (e.g., the divergence tendency) remain small throughout the simulation (i.e., $\sim 10^{-10}$ s^{-2}), suggesting that inertia-gravity wave activity is negligible in this case.

4. DISCUSSION AND FUTURE RESEARCH

The simulation described here represents a preliminary exploration of the evolution of balance in strong barotropic jets. The results of the diagnostic calculations reveal that nonlinear balance is essentially valid for this particular jet, even though the Rossby and Froude numbers are O(1) throughout the duration of the simulation.

Ongoing and future research considers a large number of simulations over a wide range of Rossby and Froude numbers in an attempt to simulate the generation of inertia-gravity waves and to identify general scaling characteristics of imbalance for a variety of jets. We are also exploring the possibility of employing the potential vorticity inversion techniques described by McIntyre and Norton (2000) to identify imbalance in the vicinity of the jet, along with the methodology employed by Ford (1994) to identify inertia-gravity wave activity in the far field.

5. ACKNOWLEDGMENT

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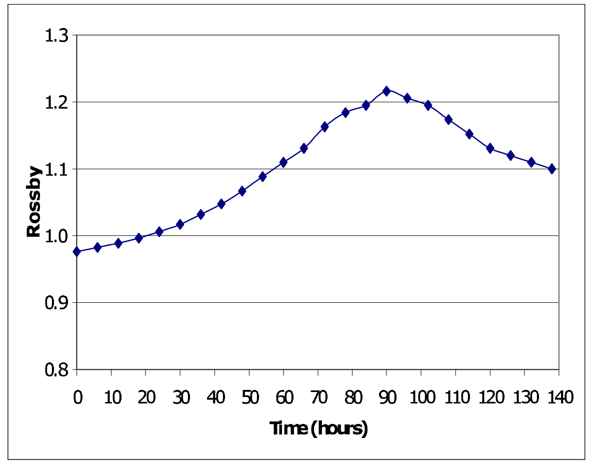

Figure 3. Time series of maximum local Rossby number for the simulation.

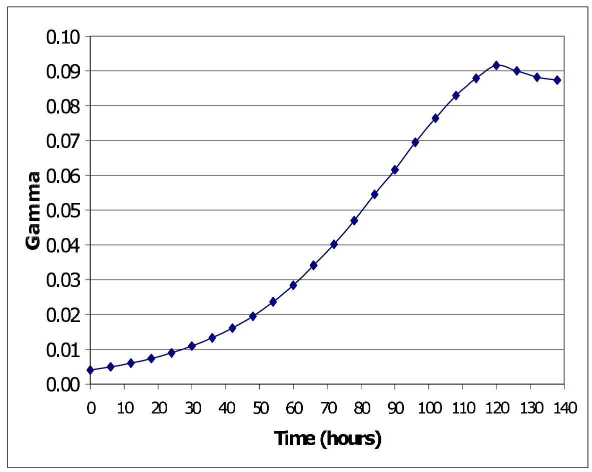

Figure 4. Time series of the ratio of maximum horizontal divergence to maximum relative vorticity.

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