2A.3 USING RADARS TO MEASURE CLUSTERING IN CLOUDS AND RAIN

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1. REMARKS

Because of Bragg scatter and signal statistics considerations, clustering in clouds and rain is of intrinsic interest to the remote sensing community (Kostinski and Jameson 2000; Jameson and Kostinski 1999a). In clouds, it likely plays a role in physical processes such as snowflake aggregation, raindrop formation, and cloud radiation (Kostinski 2001; 2002). Furthermore, recent results (Jameson and Kostinski 2000a) suggest that clustering, at times, is important for aircraft icing. Recent preliminary wind tunnel experiments using airfoils (Koenig et al. 2003) support this finding. In rain, clustering broadens the distributions of rainfall rates (Jameson and Kostinski 1999b), slowing the convergence to the true mean and increasing the inaccuracies of the estimates. Moreover, differences in clustering at dissimilar scales (resolutions) also confound comparisons between observations by different instruments such as radars and rain gages (Jameson and Kostinski 2002). At times, clustering also likely affects remote sensing signal statistics of radars as they scan (Jameson and Kostinski 1996; 1999a).

Clustering, however, is not all the same. An important variable for quantitatively describing how clumpy rain or clouds are is the clustering intensity parameter, $\aleph$, (Jameson and Kostinski 1999a; 1999b; 2000b) defined by

$$\aleph \equiv \frac{\overline{\delta N^2}}{\mu^2} - \frac{1}{\mu} \tag{1}$$

where μ and $\overline{\delta N^2}$ are the global mean and variance of the number of droplets from sample volume to sample volume over the entire observation domain. Thus, when there is no clustering but only Poissonian fluctuations, $\overline{\delta N^2} = \mu$, and ℵ60.

The observed $\aleph$ obviously depends upon the intrinsic clustering. Secondly, however, it also depends upon the sample volume size or resolution which, in the case of radars, means the beam size and pulse length. This is known because of the Ornstein-Zernike (1914) relation (more completely discussed in Kostinski and Jameson 2000). The third point to note is that even for modest sizes of sample volumes, $1/\mu$ often quickly becomes negligible so that $\aleph \to \frac{\overline{(\delta N)^2}}{\mu^2}$. This parameter can then be used to characterize the clustering of continuous variables as well.

In rain, data indicate that the statistics of drop counts at one size are consistent with the arrival of random 'patches' of random duration (Kostinski and Jameson 1997). This conclusion appears to apply to entire distributions of drop sizes as well (Jameson and Kostinski 2000a) and apparently even to clouds (e.g., Jameson et al 1998; Kostinski and Jameson 2000). Moreover, as discussed in those articles, observations suggest that the number of drops in a sample volume can often be well described in terms of probability mixtures. If we now use the appropriate continuous gamma distribution representation of the discreet negative binomial distribution of the number of drops in a sample volume and if we also consider a novel scanning procedure in which a radar rapidly gathers just one pulse measurement per beam width over the observational domain (see Jameson 2003 for detailed discussions), the resulting probability distribution of radar observed intensities is given by (Jameson 2003)

$$P(I)dI = \frac{2}{\aleph^{\frac{\aleph+1}{2\aleph}} \mu_I \Gamma\left(\frac{1}{\aleph}\right)} \left(\frac{I}{\mu_I}\right)^{\frac{1-\aleph}{2\aleph}} Bessel1K\left[1-\frac{1}{\aleph}, \frac{2\sqrt{I}}{\sqrt{\mu_I \aleph}}\right] dI \tag{2}$$

where *Bessel1K* is the modified Bessel function of the second kind, $P(I)dI$ is the distribution of the observed intensities, μ_I is the global mean intensity over all the observations and Γ is the gamma function.

Obviously, the distributions no longer obey the exponential distribution of Rayleigh statistics. Moreover, these deviations from Rayleigh statistics grow as $\aleph$ increases because the tails of the distributions (see Fig.1 in Jameson 2003) become more and more stretched, while the frequencies at smaller values decrease. This turns out to be very fortunate. Specifically, letting <> denote the mean of an ensemble of measurements and letting *q* be an integer, the ratio $\frac{\langle I^q \rangle}{\langle I \rangle^q}$ is a convenient tool for measuring deviations from Rayleigh signal statistics (Jameson and Kostinski 1996). When the statistics are Rayleigh, it turns out that this ratio is simply *q*! (Jameson and Kostinski 1996, eq.2, p. 1848), but when the statistics are non-Rayleigh, in the manner described above, the ratio can become much larger. Using (2) we compute this ratio for *q*=2,3 and 4, and then normalize by *q*! as illustrated in Fig.1. This shows that by measuring these ratios it should be possible to estimate the clustering intensity, $\aleph$.

Calculations (Jameson and Kostinski 1996) indicate that for *q*=2,3,4 about 10, 50 and 200 independent *I* (pulses) are needed, respectively, in order to accurately estimate $\langle I^q \rangle / \langle I \rangle^q$ and, therefore, to characterize $\aleph$ over the domain

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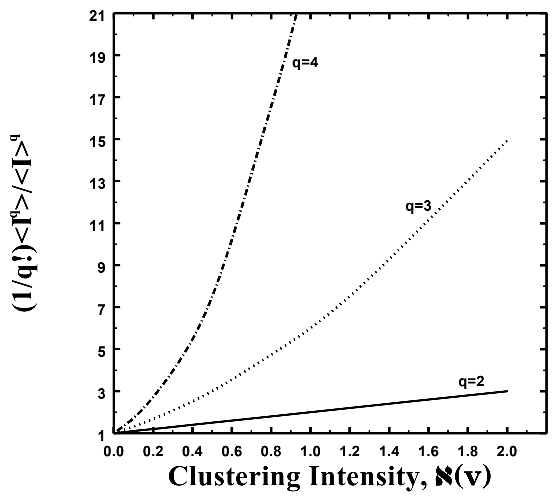

FIG.1: Intensity ratios, $\langle I^q\rangle)/\langle I\rangle^q$ normalized by the values expected from Rayleigh signal statistics (q!) for q=2,3 and 4 plotted as functions of clustering intensities, ℵ, that would be observed for the intensity distributions (2). Note that ℵ is a function of the size of the sample volume.

of observations. Since in our case each observation is statistically independent, there should be more than a sufficient number of independent samples. More details on how this might be done will be presented and may be found in Jameson (2003). Specifically, a high frequency (220 GHZ) radar with a 0.1Ebeam could easily be mounted and operated on an aircraft attempting to avoid locations of enhanced icing.

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