

Marco A. Perez and Isztar Zawadzki*

*Department of Atmospheric and Oceanic Sciences, McGill University, Montreal, Canada***1. INTRODUCTION**

Authors as early as Austin (1947) have proposed the use of dual wavelength to better quantify precipitation estimates. Particularly, Atlas and Ulbrich (1974) and Atlas and Ulbrich (1977) describe the theoretical reasons for the almost linear relationship between attenuation at centimetric wavelengths and rain rate. These authors conclude that rain rate can be estimated with an accuracy around 20% from X-band attenuation. This is a significant improvement over the accuracy that can be obtained from reflectivity (~40%). Eccles (1979) provides encouraging evidence that X-band attenuation measured with a dual-wavelength radar can be used to obtain more accurate estimates of precipitation rates than using a conventional radar.

More recently, the interest in attenuation methods for estimating precipitation has been renewed with the launch of TRMM and plans for global precipitation measurements [Kumerow et al., 2001]. We will reexamine here the basic assumptions of dual wavelength precipitation estimates.

2. ATTENUATION OF MICROWAVES

The attenuated reflectivity measured at range r_0 is:

$$Z_A(r_0) = Z(r_0) - \int_0^{r_0} K dr \quad [dBZ] \quad (1)$$

where K is the two-way attenuation coefficient:

$$K = 2 \times 10 \log e \times 10^3 k \quad [dB/km] \quad (2)$$

$$k = -\frac{1}{dV} \left(\sum_{i=1}^N \sigma_{ti} \right) = \int_0^{\infty} N(D) \sigma_t(D) dD \quad [m^{-1}] \quad (3)$$

The total attenuation cross-section $\sigma_t(D)$ is the sum of the scattering cross-section and the absorption cross-section for a particle of diameter D .

The absorption cross-section is proportional to the mass of the particle. However, the dependence of the scattering cross-section on particle size is more complicated. In the optical limit ($D \gg \lambda$) it is proportional to the transversal area of the particle, while for small particles ($D \ll \lambda$) it is proportional to the sixth power of the diameter. In between these two limits the Mie-equations must be used to calculate the amount of scattered energy.

However, it is usually assumed that a power-law relation is a good approximation to $\sigma_t(D)$, i.e. $\sigma_t \approx aD^b$.

3. MEASUREMENT OF ATTENUATION

The attenuation coefficient can be calculated from dual-wavelength (DW) radar data by subtracting the measured attenuated reflectivity from the non-attenuated reflectivity from the longer wavelength and dividing this total attenuation by the path length.

Since reflectivity is measured with a limited precision we can only expect to measure total path attenuations higher than the accuracy of the reflectivity measurement. This error depends on the number of averaged independent samples and is usually of the order of 1 dB. In addition, both reflectivity signals must be cross calibrated at the beginning of the segment, therefore, the error in the estimation of reflectivity appears four times in the expression for attenuation. As a result, with current operational radar setups, we cannot measure the full range of attenuation coefficients needed for precipitation estimation without greatly sacrificing resolution: e.g. for an 8 km path length a reasonable minimum detectable attenuation would be

Corresponding author address: I. Zawadzki, AOS, McGill University, 805 Sherbrooke W., Montreal, QC H3A 2K6, Canada. Email: isztar@radar.mcgill.ca

0.25 dB/km, a value that on average requires reflectivities higher than 37 dBZ.

4. IMPROVING PRECIPITATION ESTIMATES

Among the many factors that affect the accuracy of radar estimates of rainfall, the uncertainty in the relationship between radar-measured reflectivity and rain rate is a very important issue.

This uncertainty stems from the fact that most of the contribution to Z comes from bigger drops due to the dependence of the back-scattering cross-section on the sixth power of the diameter while the main contribution to R is due to middle-size drops, since rain rate is roughly proportional to the 3.67th power of the diameter.

This is why the natural variability in the shape of the drop size distribution (DSD) affects the relationship between Z and R . The problem of estimating R from Z is equivalent to the problem of estimating the concentration of middle-size drops from the concentration of bigger drops.

The extra information provided by attenuation could be used to find local relationships between Z and K for a particular time period or spatial area. Then, by combining this local $Z-K$ relationship with a climatological $K-R$ relation, locally tuned $Z-R$ relationships could be obtained. Ryzhkov et al. (1996) propose a similar approach for polarimetric radars using K_{DP} .

5. DISDROMETER DATA

Long records of drop size distributions (DSD), coupled with a scattering model, can be used to derive climatological relationships between Z , R and K .

Our data come from a POSS disdrometer [Sheppard, 1990] located at McGill University in downtown Montreal and an OSP disdrometer [Hauser et al., 1984] which operated in central Costa Rica. Both instruments measure the average DSD for every minute. The data series are long enough to encompass a good deal of the natural variability of drop size distributions.

Fig. 1 shows the results obtained for $Z > 10$ dBZ. This is a good range of reflectivities for conventional radar measurements. However, as previously discussed, DW radars can only expect to measure the attenuation produced by reflectivities higher than about 40 dBZ. Therefore, in order to simulate what relationships we can anticipate for them, we need to filter out lower reflectivities. These results are presented in Fig. 2.

Beside the evident deviation from a single power-law relationship, a striking fact emerges from the figures: the exponent in the relationships for $Z > 40$ dBZ becomes very close to one, particularly for the McGill dataset. This suggests that for higher values of Z , attenuation is almost proportional to reflectivity instead of the desired rain rate.

6. DISCUSSION

The local slope of the relationship between D and $\log \sigma_t$ is affected by a small bulge in $\sigma_t(D)$ produced by raindrops of diameters around 3 mm. This change in slope can be clearly appreciated in Fig. 3, where we plot $\sigma_t(D)$ together with two asymptotes for $\sigma_t \propto D^3$ and $\sigma_t \propto D^6$.

The slope variation implies that while for small drops the attenuation factor ($K \propto \sum \sigma_t$) is proportional to the third power of the diameters, for larger drops it is more proportional to the sixth power of the diameters (i.e. close to reflectivity).

7. IMPLICATIONS

For higher rain intensities the X-band specific attenuation is almost proportional to reflectivity. This statement contradicts a widespread belief in the radar community [Atlas and Ulbrich, 1977; Eccles, 1979; Delrieu et al., 1991] of X-band attenuation being a good estimator of rain rate.

The proportionality between attenuation and reflectivity has important implications for every method based on the use of X-band attenuation as a way of improving estimates of precipitation. For smaller rain intensities (where small drops dominate),

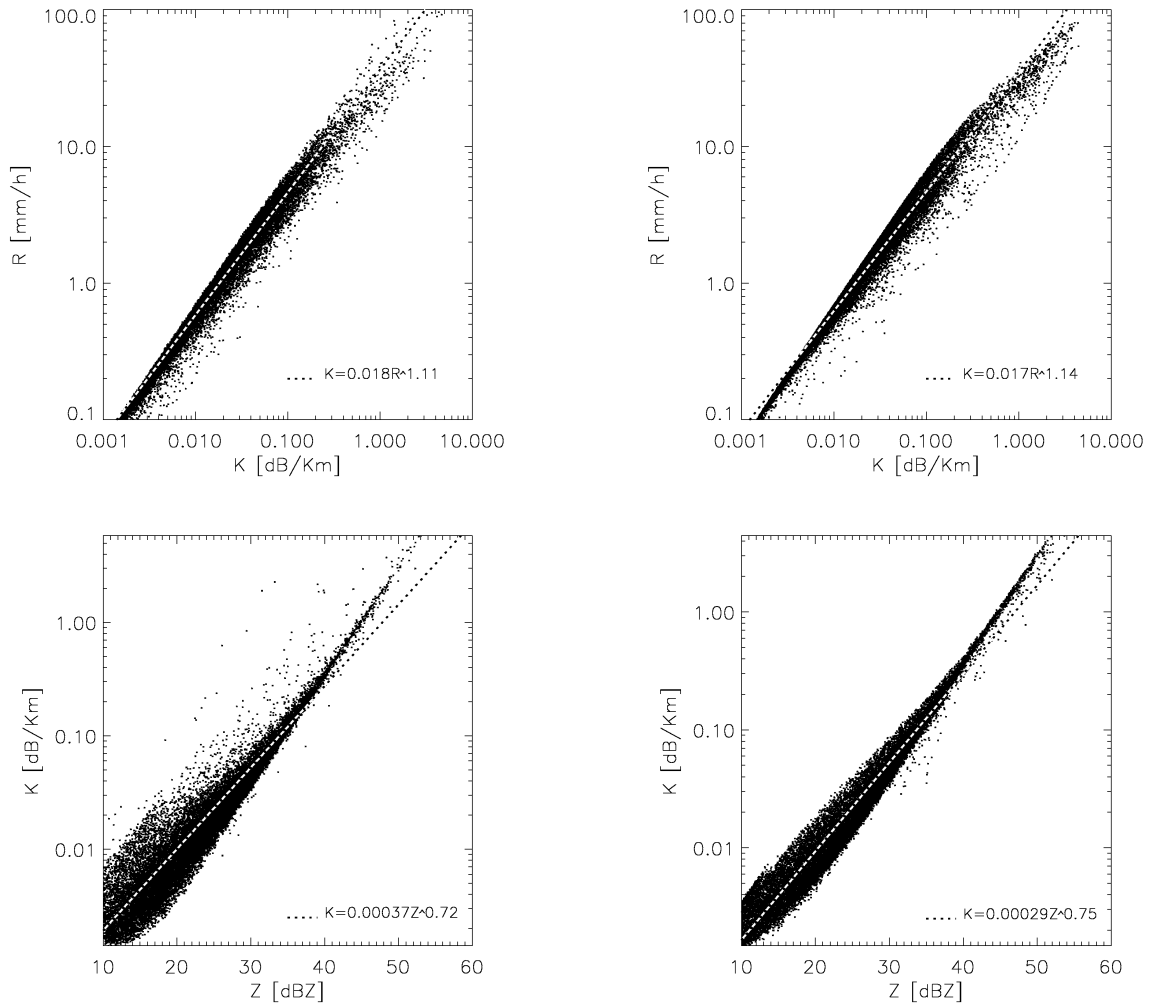


Fig. 1: Mean $K - R$ and $Z - K$ relationships from McGill POSS (left) and Costa Rica OSP (right), for $Z > 10$ dBZ .

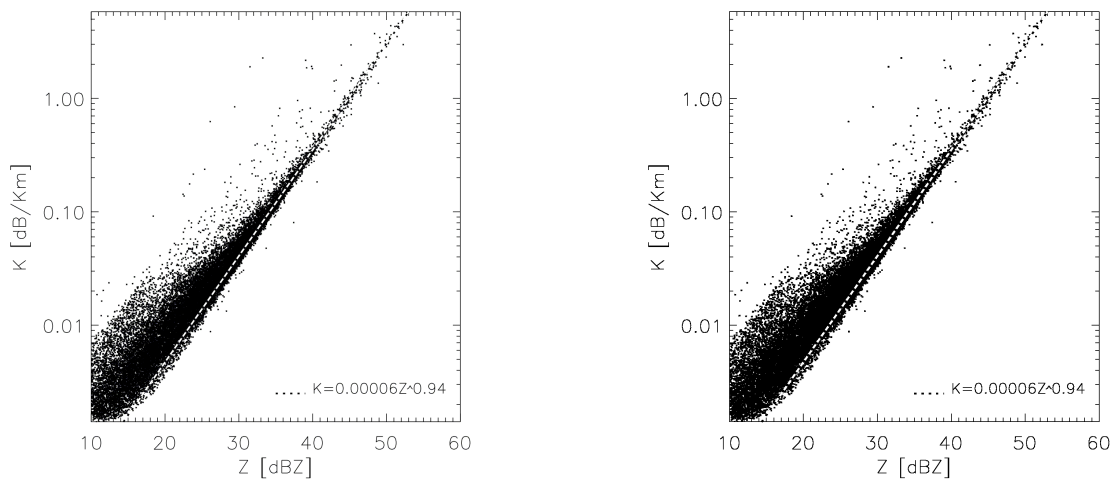


Fig. 2: Mean $Z - K$ relationships from McGill POSS (left) and Costa Rica OSP (right), for $Z > 40$ dBZ .

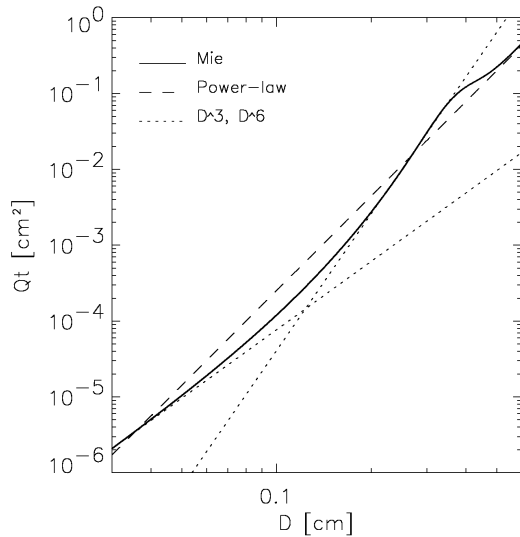


Fig. 3: $\sigma_t(D)$ for X-band, $T = 10^\circ\text{C}$.

attenuation is closely related to precipitation rate, however, for higher rain intensities (increasing concentration of bigger drops) it becomes almost proportional to reflectivity.

A troublesome fact for DW radars is that it is precisely in the higher range of reflectivities that they can hope to measure K with reasonable accuracy. In particular, the method we mentioned before for locally tuning the $Z-R$ relationship from attenuation measurements relies on K being almost linearly related to R . Obviously, this is not the case for the range of reflectivities where K can be measured. Therefore, we have to conclude that the tuning algorithm would not work for S- and X-band dual-wavelength radars. Neither will work any other method that depends on measurements of X-band attenuation at high rain rates to improve upon reflectivity-only estimates. That is, for higher intensities the X-band attenuation does not add much information to that provided by reflectivity alone.

8. CALIBRATION

DW measurements of attenuation are not affected by radar calibration errors. Attenuation is calculated as a difference of reflectivities, therefore, calibration errors are removed and only relative changes are important.

This characteristic could be exploited for externally calibrating DW radars. If we were to measure enough $Z-K$ pairs for a climatologic relationship to appear, then the deviation of the measured relation from the climatologic one would give the calibration error of the radar.

A preliminary study using a limited DW dataset from the McGill radar compared to an external disdrometer has produced encouraging results.

9. REFERENCES

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