9A.1 A GRAPHICAL INTERPRETATION OF CLOUD MICROPHYSICAL PROCESSES AND ITS RELEVANCE FOR RADAR RAINFALL MEASUREMENT

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1. INTRODUCTION

The microphysical aspects of the relationship between radar reflectivity factor Z and rainfall rate R are examined. Three special modes that a $Z-R$ relationship may attain are revealed, depending on whether the variability of the raindrop size distribution is governed by variations of the drop number density, drop size, or a coordinated combination thereof with constant ratio of mean drop size and number density. The number-controlled case results in linear $Z-R$ relations that have been observed for steady and statistically homogeneous or equilibrium rainfall conditions. Most rainfall situations, however, exhibit a variability of drop spectra that is facilitated by a mix of variation of drop size and number density, which results in the well-known power-law $Z-R$ relationships.

Only few rainfall conditions result in physically meaningful relationships between Z and R. Graphical ways to identify those situations are presented. The mathematical basis is laid out in section 2, whereas a graphical interpretation thereof is given in section 3.

2. MATHEMATICAL FRAMEWORK

The raindrop size distribution (m^{-3} mm^{-1}) can be described by a gamma function of the form

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \tag{1}$$

where D (mm) is the drop diameter, N_0 (m^{-3} mm$^{-(1+\mu)}$) is a concentration scaling parameter, Λ (mm^{-1}) the slope coefficient, and μ the distribution shape factor (e.g., Ulbrich 1983). An exponential raindrop size distribution (e.g., Marshall and Palmer 1948) is obtained for $\mu = 0$, in which case N_0 (m^{-3} mm^{-1}) becomes the intercept parameter. In the limiting case of $\mu \to \infty$, the gamma distribution approaches a Dirac δ-function — i.e., a monodisperse raindrop spectrum. Thus, (1) represents a wide range of analytical forms for the raindrop size distribution.

The radar reflectivity factor Z (mm^6 m^{-3}) and rain rate R (mm h^{-1}) are defined as

$$Z = \int_0^{\infty} N(D) D^6 dD \tag{2}$$

and

$$R = \frac{6\pi}{10^4} \int_0^{\infty} N(D) D^3 v(D) dD \tag{3}$$

respectively. The relation between drop diameter and its fall velocity (m s^{-1}) shall be approximated by a power-law expression of the form

$$v(D) = v_0 D^p \tag{4}$$

with the coefficients $v_0 = 3.778$ (m s^{-1} mm^{-p}) and $p = 0.67$, as proposed by Atlas and Ulbrich (1977). The power law (4) is the only functional form consistent with power-law relationships between rainfall integral parameters such as Z and R (e.g., Uijlenhoet 2001).

To ease interpretation of subsequent results in microphysical terms, the coefficients N_0 and Λ are replaced by the raindrop number density N_{tot} (m^{-3}), that is the total number of drops per unit volume of air

$$N_{tot} = \int_0^{\infty} N(D) dD = \frac{N_0}{\Lambda^{(1+\mu)}} \Gamma(1+\mu) \tag{5}$$

and the mass-weighted mean drop diameter D_m (mm)

$$D_m = \frac{\int_0^{\infty} N(D) D^4 dD}{\int_0^{\infty} N(D) D^3 dD} = \frac{4+\mu}{\Lambda} \tag{6}$$

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respectively, where use was made of (1) and the gamma function

$$\int_0^\infty e^{-x}x^n dx = \Gamma(n+1) \quad (7)$$

Expressions of the radar reflectivity factor Z and the rain rate R in terms of N_{tot} and D_m are obtained by inserting (1) into (2) and (1) and (4) into (3), respectively, plus replacing N_0 and Λ by using (5) and (6), to yield

$$Z = N_{tot}\frac{\Gamma(7+\mu)}{\Gamma(1+\mu)}\left(\frac{D_m}{4+\mu}\right)^6 \quad (8)$$

and

$$R = N_{tot}\frac{6\pi v_0}{10^4}\frac{\Gamma(4+p+\mu)}{\Gamma(1+\mu)}\left(\frac{D_m}{4+\mu}\right)^{(3+p)} \quad (9)$$

The relationship between the radar reflectivity factor and the rainfall rate is often expressed as a power law,

$$Z = \alpha R^\beta \quad (8)$$

(e.g., Marshall 1969; Battan 1973). There are different ways to obtain such a relationship. For simplicity, we only consider the case of exponential raindrop size distributions ($\mu = 0$). Detailed analyses based on gamma, exponential, and monodisperse drop size spectra are discussed in Steiner et al. (2003). Solving (9) for N_{tot} and substituting the obtained expression into (8), plus setting $\mu = 0$ and using the numerical values of v_0 and p, results in

$$Z = 270.55(D_m)^{2.33}R \quad (11)$$

On the other hand, solving (9) for D_m instead and using that expression in (8) yields

$$Z = 28561\left(\frac{1}{N_{tot}}\right)^{0.63}R^{1.63} \quad (12)$$

Alternatively, Eq. (3) combined with (1) could be solved for either N_0 or Λ, and the respective result substituted into the combination of (1) with (2). The former approach, and using (6) to replace Λ by D_m, results in (11) as well. The latter approach, however, combined with Eqs. (5) and (6) to replace N_0 by the raindrop number density N_{tot} and mean drop size D_m, produces

$$Z = 10531\left(\frac{D_m}{N_{tot}}\right)^{0.50}R^{1.50} \quad (13)$$

Equations (11)-(13) characterize three micro-physically distinct $Z-R$ relationships. For a linear $Z-R$ relation like (11) to be physically meaningful, the mean drop size D_m has to remain constant and the variability of the raindrop size distribution be entirely facilitated by variations in drop number density (*number controlled case*). Power-law $Z-R$ relations (12) with exponent $\beta = 1.63$ are the consequence of a constant drop number density N_{tot}, while the variability of the drop spectrum may be accommodated through variations in mean drop size (*size controlled case*). Finally, power-law $Z-R$ relations (13) with exponent $\beta = 1.5$ are valid for a constant ratio D_m/N_{tot} — i.e., the mean drop size and drop number concentration may vary, albeit only in a coordinated fashion. This latter case represents situations with constant N_0 (*constant intercept case*), as may be seen from (5) and (6).

The infinitesimal possibilities of obtaining a $Z-R$ relation for combinations of mean drop size and number concentration easily explain the multitude of such relationships reported in the literature (e.g., Stout and Mueller 1968; Battan 1973).

3. GRAPHICAL INTERPRETATION

It is noteworthy that independent of the form of the raindrop size distribution there appear to be three microphysical modes that result in physically meaningful (as opposed to purely statistical) $Z-R$ relationships. This result is consistent with the one obtained by Uijlenhoet et al. (2003), which is based on a scaling-law formalism that doesn't make any assumptions about the raindrop size distribution. Smith and Krajewski (1993) arrive at a similar result for lognormal raindrop size distributions and a statistical rainfall model (Smith 1993) based on the arrival rate and characteristic properties of raindrops. Steiner et al. (2003) demonstrate that the variability of the raindrop size distribution is bound by either size-controlled (constant N_{tot}) or number-controlled (constant D_m) conditions, with conditions of a mixed control (constant N_0) embedded in between those extremes.

Figure 1 shows the relationship between the drop number density N_{tot} and the mass-weighted mean drop size D_m as a function of the intercept coefficient N_0 and rain rate R as determined by Eqs. (1) and (3)-(6) for $\mu = 0$. This key figure enables distinction of the three microphysical modes that the $Z-R$ relationship may attain.

Linear $Z-R$ relations ($\beta = 1$), characterized by Eq. (11), have been found previously for so-called equilibrium conditions in rainfall, where all variability of the raindrop size distribution is controlled by variations in raindrop concentration (e.g., List 1988). Jameson and Kostinski (2001) call this condition statistically homogeneous rain. Equilibrium or statistically homogeneous rainfall conditions reflect a balance between drop collisions, coalescence, and breakup (e.g., Hu and Srivastava 1995). If such conditions truly occur in nature, one might find them within the efficient warm-rain process dominated growth phase of intense tropical rainfall (e.g., hurricanes), severe and long-lasting mid-latitude storm systems (e.g., supercells), or maybe persistent heavy orographic rainfall. Displaying raindrop size spectra information from such conditions in Fig. 1, one expects all fluctuations to take place along the vertical axis, while D_m remains approximately constant.

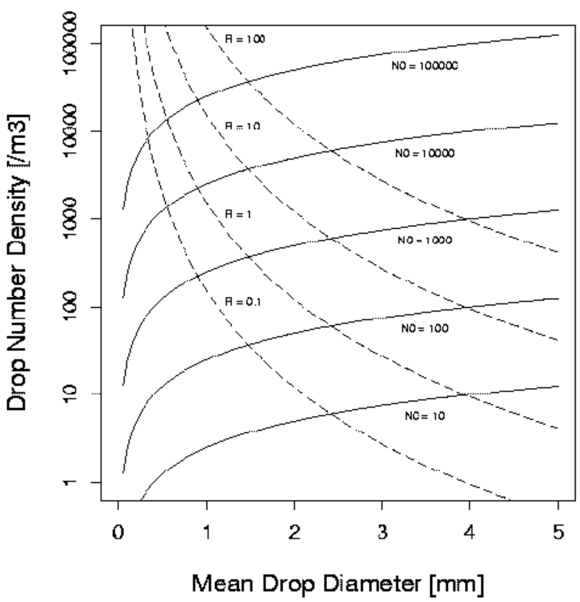

Figure 1. Relationship between N_{tot}, D_m, N_0, and R based on exponential raindrop spectra.

The other limiting case, where all variability of the raindrop spectrum is controlled by variations in characteristic drop size, is described by (12), with a multiplicative factor of the $Z-R$ relation that depends only on the raindrop number density N_{tot} and an exponent $\beta = 1.63$. Such conditions can be recognized in Fig. 1 by fluctuations along the horizontal axis, while N_{tot} remains approximately constant. Evidence suggests that such situations may occur in dissipating convective cells (e.g., Carbone and Nelson 1978) or size sorting due to wind shear and/or turbulence (e.g., Gunn and Marshall 1955), although it seems that these conditions may not last very long and thus be rather rare.

Equation (13) highlights the case of a power-law relationship between Z and R with exponent $\beta = 1.5$ and a multiplicative factor α that depends both on the raindrop concentration N_{tot} and the mean drop size D_m. This type of rainfall is characterized by variations in raindrop spectra such that the ratio D_m / N_{tot} (proportional to N_0) remains constant, which can be recognized in Fig. 1 by rainfall variations following lines of constant N_0. This is essentially the kind of rainfall reported by Marshall and Palmer (1948) and many others since.

Provided that there are three special (physical) modes for the $Z-R$ relationship, one wonders about the microphysical and dynamic conditions that may produce these modes, and how the transition from one to another mode might be facilitated. Such transitions are likely not smooth and may be marked by discontinuities in the raindrop size distributions (N_0-jumps), reflecting significant changes in the underlying microphysical growth processes. An observed characteristic of N_0-jumps, as originally pointed out by Waldvogel (1974), is that they occur for approximately constant rain rate. Order of magnitude changes in N_0, therefore, take place along the dashed lines in Fig. 1, highlighting that such changes are the result of modifications of both mean drop size and number concentration. Microphysical processes associated with jumps to larger N_0 (accompanied by a decreasing D_m and increasing N_{tot}) are increased breakup of raindrops due to enhanced number of collisions or onset of riming, which tends to suppress aggregation and thus inhibit the formation of larger snowflakes that would melt into bigger raindrops. Significant decreases in N_0, on the other hand, are related to an increase in mean drop size and decrease in number concentration, which may be the result of increased coalescence and thus a rapid growth of raindrops and/or increased aggregation of snowflakes. Not every drastic change in N_0, however, qualifies as a N_0-jump in the "classical" sense of Waldvogel (1974), where the rain rate remains approximately constant.

4. CONCLUSIONS

Analytically it can be shown that there are three special modes that a $Z-R$ relationship may attain. These three distinct modes are associated with conditions where the variability of the raindrop size distribution is controlled by either variations in drop number concentration N_{tot}, mean drop size D_m, or a combination thereof in which the ratio D_m/N_{tot} is constant. The variability of the raindrop size distribution is bound by either size-controlled (constant N_{tot}) or number-controlled conditions (constant D_m), with conditions of a mixed control (constant N_0) embedded in between those extremes. Moreover, these three special modes represent physically meaningful $Z-R$ relationships, in contrast to many empirically obtained $Z-R$ relations that are of a statistical nature.

The three special modes of the $Z-R$ relation can be graphically identified within a parameter space spanned by the drop number density N_{tot} and the mean drop size D_m (Fig. 1). Moreover, those special microphysical conditions leave distinct footprints in this parameter space that are clearly different from the signatures associated with marked transitions (N_0-jumps) from one microphysical growth mode to another.

Identification of the appropriate type of $Z-R$ relationship (e.g., linear versus power law) can have a significant effect on the radar-based rainfall estimation.

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