9B6 SINGLE-ANTENNA INTERFEROMETRY TO MEASURE WIND

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1. INTRODUCTION

Components of the wind can be measured by the Doppler method, an interferometric technique, or by Tracking Reflectivity Echoes by Correlation (TREC) [Doviak and Zrnic, 1993; Briggs et al., 1950; Reinhart, 1979]. The Doppler method measures radial velocity, and the interferometric technique measures the wind component transverse to the beam axis. Both the Doppler and the interferometric techniques directly measure wind components (i.e., the velocity of randomly distributed scatterers advected by wind). The vector wind can be determined using the Doppler method (e.g., through a VAD type analysis) if the wind is uniform over a large area, But the interferometric technique can measure the vector wind within the resolution volume of the radar, thus providing finer resolution. Both of these techniques can estimate wind even if the reflectivity field is statistically homogenous. On the other hand, the TREC method tracks individual features in the reflectivity field to determine feature motions [Reinhart, 1979; Tuttle and Foote, 1990]. Thus, if reflectivity is not conserved, reflectivity motion is not same as the wind velocity; furthermore the TREC method does not work if the reflectivity field is homogenous.

The Doppler method uses the phase of auto-correlation function of received signals to estimate radial wind [Doviak and Zrnic, 1993]. Because the phase can only be measured within an interval of 2π, the Doppler method is prone to aliasing which causes the estimated wind velocity to alias with a period $\lambda/2T_s$ where T_s is the pulse repetition time. We develop an alternative method that is independent of the Nyquist limits, one that could supplement Doppler wind measurements.

The interferometric technique uses the magnitude of cross-correlation function, and thus the problem of phase aliasing is not involved. Interferometry for wind measurement can be based on spaced antenna (SA) radars in which scattered signals, received at separate antennas, are cross-correlated to estimate the transverse wind.

In this paper, we introduce single-antenna interferometry to measure the wind vector, and we study its feasibility for practical applications. We propose Angular Interferometry (AI) to determine transverse wind and a Range Interferometry (RI) to measure radial wind. The radial wind component is estimated from the cross-correlation function along range, whereas the transverse wind is obtained from the angular correlation function along the azimuth and elevation directions. It is noted that neither AI nor RI technique is a TREC method because both AI and RI techniques apply even if the reflectivity field is homogeneous.

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2. CONCEPTUAL DESCRIPTION

The accepted explanation for radar interferometric measurements of crossbeam wind is that the diffraction pattern of echoes advects across an array of receiving antennas at twice the speed that scatterers are advected across the beam (Briggs, 1984). Doviak et al. (1996), however, consider pairs of scatterers and receivers to prove that diffraction patterns do not advect at twice the speed of wind. Considering a pair of receivers, symmetrically placed about a transmitting antenna to form a pair of side-by-side bistatic scattering volumes, they showed that, by applying the reciprocity theorem, echoes in the two receivers are perfectly correlated when signals from one receiver is lagged a time difference equal to the time it takes the scatterers to advect from one scattering volume to the next (i.e., assuming the advection is along the baseline of the receiving antennas).

This later interpretation (i.e., spaced resolution volumes, rather than spaced antennas), leads us to an alternative explanation of interferometry applied to the measurement of wind advecting either discrete scatterers (e.g., rain drops) or Bragg scatterers [Doviak and Zrnic, 1993]. Ignoring the effects of turbulence, wind simply advects scatterers without changing their relative displacements. If the radar's resolution volume V_6 (Doviak and Zrnic, 1993) can be strictly translated the same vector distance the scatterers have moved, the phase path (i.e., from the transmitter to each of the scatterers and back to the receivers) differences would be the same for all the scatterers in each of the V_6s. For example, if scatterers are horizontally advected, V_6 must also be strictly translated horizontally (i.e., not simply by translating horizontally the center of V_6 as could be the case for a mono-static radar configuration). For V_6 to be strictly translated, both the transmitter and receiver must be translated horizontally the same vector distance or, if the transmitter is fixed, the receiver must be symmetrically and horizontally translated across the transmitter antenna, twice the distance that the scatterers have moved (i.e., a bistatic radar configuration is required). This explanation also applies if the scatterers and sample volumes advect radially. Thus, if the resolution volume strictly follows the motion of the scatterers such that the phases to all scatterers are fixed (i.e., ignoring turbulence), the magnitude of correlation function is maximal. Otherwise, the correlation of the received signal decreases.

Based on the above interpretations, we propose a single-antenna/receiver interferometric technique to measure wind. As shown in Fig. 1a, the AI technique tracks the motion of the scatterers in the angular direction to measure transverse wind. The angular correlation function of the signals from different directions is constructed to estimate transverse wind. Similarly, RI technique tracks the motion of the scatterers along range (Fig. 1b) and estimates the radial wind through the range correlation function.

The radar that uses the AI technique can have a phased array antenna or single dish reflector provided the signals in two adjacent directions are acquired rapidly to maintain signal correlation, and repeatedly to have enough independent samples. The phased array weather radar being assembled by National Severe Storms Laboratory (NSSL) has beam agility and is a good candidate to test the AI technique for wind measurement. For a single dish antenna, the angular correlation functions at positive and negative angles about the transmitting beam can be obtained with a monopulse radar. If the beam shifts in the direction that follows the displacement of the scatterers, the correlation of the signals is high. Beam shifts in the opposite direction leads to a lower correlation. Hence, the ratio of the correlation functions of signals from V_6s displaced in the direction of the wind and against it should give transverse wind information.

3. FORMULATION

Consider a mono-static radar located at the origin of a spherical coordinate (0,0,0). As illustrated by Doviak and Zrnic (1993, section 11.5), the received complex signal $V(\vec{r},t)$ can be represented by an integration of refractive index perturbations $\Delta n(\vec{r}',t)$ in the radar's resolution volume centered at $\vec{r}:(r,\hat{s})$ with r as range and $\hat{s}$ as angular position,

$$V(\vec{r},t) = A\int \frac{\Delta n(\vec{r}',t)}{r'^2} \exp\left(-\frac{|\hat{s}'-\hat{s}|^2}{4\sigma_\theta^2} - \frac{(r'-r)^2}{4\sigma_r^2} - 2jk(r'-r) \right) d\vec{r}' \quad (1)$$

where A the constant that depends on radar parameters. The first term in the exponent is the angular weighting function, the second term is the range weighting function, and $2jk(r'-r)$ is the phase term. The range resolution is σ_r, the standard deviation of the echo power weighting function, and $\sigma_\theta = \gamma\lambda / D$ is the width of an angular pattern that is related to the commonly used one-way 3dB beam width as $\theta_1 = 2.36\sigma_\theta$. Parameters λ, *D*, and γ are wavelength, antenna diameter, and antenna efficiency factor respectively.

Assuming that the transverse correlation length, l_T, of Δ*n* is much smaller than *D* and resolution volume width (i.e.,$\beta_T = \sqrt{l_T^2+\sigma_t^2\tau^2}$ << D, $r\sigma_\theta$), that the radial correlation length l_z is sufficiently small (i.e., $\beta_z = \sqrt{l_z^2+\sigma_t^2\tau^2}$ <<σ_r, l_z <<$\sigma_t\tau$), defining the cross-correlation function $C(\vec{r}_d,\tau) = \langle V(\vec{r}_1,t_1)V^*(\vec{r}_2,t_2)\rangle$, performing the integrations, and because the magnitude of the correlation function is used to estimate wind using interferometric techniques, we have

$$|C(\vec{r}_d,\tau)| \approx S\exp\left(-\frac{|r\vec{s}_d - \vec{v}_T\tau|^2}{8r^2\sigma_\theta^2} - \frac{(r_d - v_r\tau)^2}{8\sigma_r^2} - 2k^2\sigma_\theta^2 v_T^2\tau^2 - 2k^2\sigma_t^2\tau^2 \right) \quad (2)$$

where the angular and range separations of the two resolution volumes are $\vec{s}_d = \hat{s}_1 - \hat{s}_2 \equiv \theta_d\hat{s}_d$, and $r_d = r_1 - r_2$.

Equation (2) shows that the de-correlation in signals from separated resolution volumes depends on mean wind and turbulence as well as radar characteristics. The first term is due to angular separation of the resolution volumes with respect to the transverse displacement of the scatterers. The second term is related to the range separation and the radial displacement of the scatterers. The de-correlation caused by the resolution volume separation is compensated by scatterers' displacement matching the resolution volume displacement. The third term is the Fresnel term accounting for the phase change due to scatterers' motion in a rectilinear direction rather than the *θ* direction. The last term is due to turbulence. Turbulence and the Fresnel term usually cause faster de-correlation of received signals than the displacement of resolution volumes. To effectively use the single antenna interferometric technique to measure wind, either turbulence has to be negligibly small, or beam-widths sufficiently narrow.

By letting $r_d = 0$ in (2), we obtain the magnitude of angular correlation function of signals from sample volumes at the same range,

$$|C_A(\tau)| \approx S\exp\left(-\frac{|r\vec{s}_d - \vec{v}_T\tau|^2}{8r^2\sigma_\theta^2} - \frac{v_r^2\tau^2}{8\sigma_r^2} - 2k^2\sigma_\theta^2 v_T^2\tau^2 - 2k^2\sigma_t^2\tau^2 \right) \quad (3)$$

It can be seen that if the displacement of the resolution volume center matches the transverse displacement of the scatterers in a time τ, the cross-correlation reaches maximum.

Similarly, letting $\vec{s}_d = 0$ in (2), we obtain range cross-correlation function magnitude as

$$| C_R(\tau) | \approx S \exp\left(-\frac{(r_d - v_r \tau)^2}{8\sigma_r^2} - \frac{v_T^2 \tau^2}{8r^2\sigma_\theta^2} - 2k^2\sigma_\theta^2 v_T^2 \tau^2 - 2k^2\sigma_t^2\tau^2 \right) \quad (4)$$

The maximal correlation occurs at the time lag that scatterers' motion matches range separation of sample volume.

4.WIND ESTIMATION AND ERROR ANALYSIS

4.1 Wind estimation from correlation ratios

To measure the wind vector, we estimate the wind components in transverse and radial directions through the angular and range correlation functions. We use the cross-correlation ratio (CCR) method developed to estimate wind from SA measurements [Zhang et al., 2003]. From the logarithm of a ratio between the angular correlation function (3) at positive and negative lags, we obtain the transverse wind component

$$v_T = \frac{2r\sigma_\theta^2 L_A(\tau)}{\theta_d \tau} \quad (5)$$

where the logarithm of angular CCR is $L_A(\tau) = \ln[| C_A(\tau) | / | C_A(-\tau) |]$.

Similarly, we obtain radial velocity

$$v_r = \frac{2\sigma_r^2 L_R(\tau)}{r_d \tau} \quad (6)$$

from the range CCR, $L_R(\tau) = \ln[| C_R(\tau) | / | C_R(-\tau) |]$.

4.2 Standard error of wind estimates

In theory, the wind components can always be determined using (5) and (6). In practice, however, the angular and range correlation functions are not known, and only their estimates are obtained from measurements.

Because the wind components are linearly proportional to the correlation ratios as in (5) and (6), the standard deviation, *SD,* of the wind estimates ($\hat{v}_T$ and $\hat{v}_r$) are simply related to the *SD* of the correlation ratios ($\hat{L}_A(\tau)$ and $\hat{L}_R(\tau)$), respectively, as

$$SD(\hat{v}_T) = \frac{2r\sigma_\theta^2}{\theta_d \tau} SD\left(\hat{L}_A(\tau)\right) \quad (7)$$

and

$$SD(\hat{v}_r) = \frac{2\sigma_r^2}{r_d \tau} SD\left(\hat{L}_R(\tau)\right) \quad (8)$$

For a cross-correlation function magnitude written in the form

$$| C(\tau) | = S \exp\left(-\frac{(\tau - \tau_p)^2}{2\tau_c^2} - \eta \right) \quad (9)$$

where the Gaussian parameters τ_c, τ_p and η are coherence time, time-lag to cross-correlation peak, and a de-correlation parameter, respectively, the *SD* of the correlation ratio estimates $\hat{L}(\tau)$ has been derived in Zhang et al. (2003) as

$$SD(\hat{L}) = \frac{\tau}{\tau_c \sqrt{M_I}} \frac{1}{\rho_0} \left(1 + \frac{2\tau_p^2}{\tau_c^2} - \rho_0^2 \right)^{1/2} \quad (10)$$

where M_I is the number of independent samples and ρ_0 is the cross-correlation coefficient at zero lag.

Because the angular correlation function (3) and range correlation function (4) can be written in the form of (9), the standard errors of the estimates for the transverse wind and radial wind components can be derived and evaluated.

The standard deviation of the transverse wind estimates is plotted as a function of turbulence at various ranges in Fig. 2a. It shows the *SD* increases as the turbulence becomes stronger and range increases. The increase in *SD* is related to the fact that stronger turbulence and larger beam size lead to shorter correlation times of the received signals. The fixed level of *SD* at weak turbulence is due to the de-correlation by the Fresnel term. It is possible to obtain a valid wind estimate with the AI technique at near ranges if turbulence is weak.

The *SD* of the radial wind estimates is plotted in Fig. 2b. The *SD* increases as turbulence and range resolution increase.

5. SUMMARY AND CONCLUSIONS

We present a single-antenna interferometry to measure three-dimensional wind. We provide an alternative explanation of the interferometric technique for wind measurement using multiple antennas. We extend the interferometric technique of SA radars having multiple receivers to single-antenna radars. We propose Angular Interferometry (AI) to determine the transverse wind, and Range Interferometry (RI) to determine the radial wind from time series data collected at different angles and range gates. Angular and range cross-correlation functions for single-antenna radar measurements are derived based on wave scattering from random fluctuations of refraction index, but it is also applicable to discrete scatterers. The wind components are estimated from the ratio of the correlation function at positive and negative lags. The feasibility of the method is studied through error analysis. The standard errors of the estimated wind velocities are derived and their sensitivity to resolution, sample time, and turbulence are analyzed.

It has been shown that the AI technique needs small beam size while the RI technique requires fine range resolution, as well as weak turbulence to perform well. The advantage of single-antenna interferometry for wind measurement is that it does not require multiple receivers, and the problem of Doppler aliases does not exist. The data collection and processing are simple and straightforward. Single-antenna Interferometry can be applied, without much additional hardware, to wind profilers, weather radars, as well as middle and upper atmospheric radars. The limitation is the requirement of high resolution and weak turbulence.

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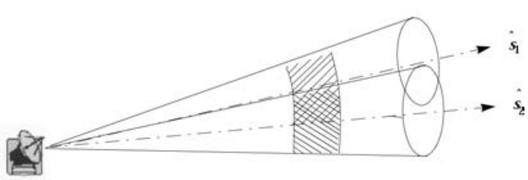
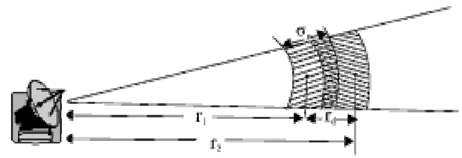


(a)



(b)

Figure 1. Configuration for single-antenna interferometry. (a) Angular Interferometry (AI) of signals from different directions $\hat{s}_1$ and $\hat{s}_2$, and (b) Range Interferometry (RI) of signals from different ranges r_1 and r_2. Range separation is r_d. Range resolution is σ_r.

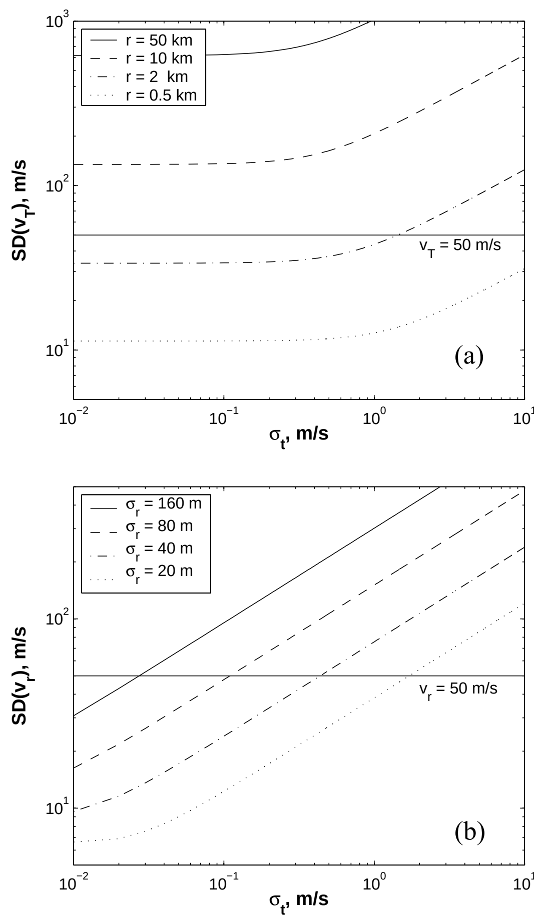


Figure 2. Standard deviation of wind estimates as a function of turbulence for (a) transverse wind at various ranges; (b) radial wind estimates for various range resolutions, v_t = 0 m/s. Other parameters are $\sigma_\theta = 0.42$ degree, T_d = 60 s, λ = 0.1 m.