6.1 PUGET SOUND TIDAL DATUMS BY SPATIAL INTERPOLATION

Kurt W. Hess* and Stephen K. Gill
National Ocean Service, NOAA

1. INTRODUCTION

The National Ocean Service (NOS) of the National Oceanic and Atmospheric Administration (NOAA) requires tidal datum information such as mean high water (MHW) and mean lower low water (MLLW) to support nautical charting, navigational safety, shoreline photogrammetry, and marine boundary determination. In addition, tidal datum information is needed for referencing NOS' bathymetric data (which is referenced to MLLW) to any one of the other vertical elevation reference systems. A software tool under development at NOS called VDatum (Milbert, 2002) is designed to transform among approximately 30 vertical reference datums. To be applicable over coastal waters, VDatum requires tidal datum fields, where the field describes the two dimensional, horizontal variability of the datum elevation. Tidal datum fields for VDatum have been produced by NOS for Tampa Bay and coastal southern Louisiana, the New York Bight, central coastal California, and Delaware Bay (Hess, 2001). Once VDatum has been established for a region, data can be incorporated into integrated bathymetric-topographic Digital Elevation Models for use in coastal GIS applications (Parker et al., 2001; Gesch and Wilson, 2002). VDatum will also be needed for carrying out the kinematic-GPS hydrographic surveying that NOS is planning to implement.

NOS routinely collects water level observations at shore-based stations along U.S. coasts and analyzes them to produce tidal datums. As described above, there is an important need to obtain two-dimensional tidal datum fields that cover the coastal waters between the water level stations. This paper discusses a method for obtaining tidal datum fields in Puget Sound, Washington, by the method of spatial interpolation of tidal data.

*Corresponding author address: Kurt W. Hess, Coast Survey Development Laboratory, Rm. 7826, 1315 East-West Highway, Silver Spring, MD 20910; e-mail: kurt.hess@noaa.gov.

2. TIDES AND DATUMS

Tidal datums at water level stations are elevation values that are determined from a time series of observations. For stations located along the coasts of the U.S. (except for the Great Lakes), the analysis starts with the identification of all the tidal extrema (highs and lows) in the record, and continues with the selection (within a 25-hour time period) of the higher of the two highs and the lower of the two lows. If only one high water is present in the time period, it is categorized as a higher high. Thus, for high water (for example), each day has either a high and a higher high, or a single higher high. The average of all the highs and the higher highs is called the Mean High Water (MHW), and the average of just the higher highs is called the Mean Higher High Water (MHHW). The process for producing Mean Low Water (MLW) and MLLW from the low waters is similar. The average of the MHW and the MLW is called the Mean Tide Level (MTL) and the average of the MHHW and the MLLW is called the Diurnal Tidal Level (DTL). Mean Sea Level (MSL) is the average of the hourly water levels. Where MSL is not computed, the MTL or DTL can be used as approximations. For further information on tidal datums, see Gill and Schultz (2001).

Observations made in a limited time period are adjusted to represent equivalent values for a 19-year National Tidal Datum Epoch (NTDE). The present NTDE of 1983-2001 was just implemented in April 2003 and replaced the previous 1960-1978 NTDE period. This recent epoch will give more accurate datums for locations where apparent sea levels are changing rapidly due to local land subsidence caused by mineral and ground water extraction, isostatic rebound following the last ice age, or tectonic motion.

Tidal datum values at NOS water level stations are routinely computed and are available to the public in the form of the station benchmark sheets. Within the Puget Sound, Washington, study area (between latitudes 47° 3' N and 48° 11' N, and longitudes 123° 11' W and 122° 10' W) there are 69 stations with historical tidal datum values. Those used in this study are shown in Figure 1. In this area, the value of the elevation of MHHW above MSL varies from 0.8 m at the northern end to 2.0 m at the southern end, and the MLLW varies from -1.6 m at the northern end to -2.5 m at the southern end. These changes are generally correlated with changes in the range of tide.

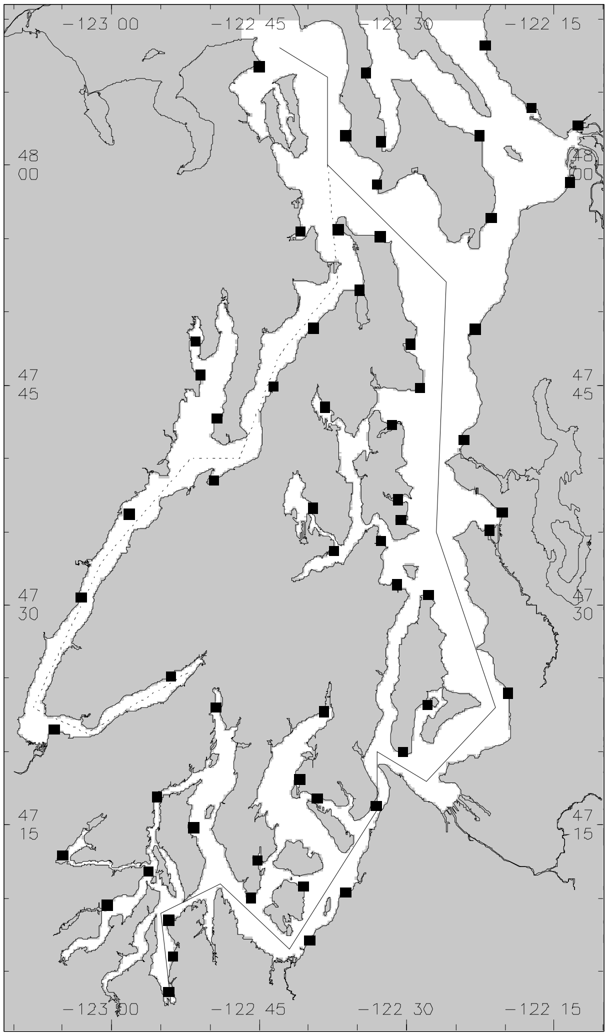

Figure 1. The Puget Sound study area with historical tide stations (squares), water cells in the computational grid (white area), the main axis of the Sound (solid line, with several straight segments, in white area), and the main axis of Hood Canal (dashed line).

3. SPATIAL INTERPOLATION

Spatial interpolation is used to generate the tidal datum fields. The interpolation method requires the datum field, *f*, to satisfy Laplace's Equation (LE),

$$\nabla^2 f = 0 \qquad (1)$$

and the solution is found numerically on a grid. No water depth data are used. The solution field matches the input data at the water level stations and takes into account land forms by the use of a specialized land-water boundary condition:

$$\frac{\partial f}{\partial n} = a\overline{\frac{\partial f}{\partial n}} \qquad (2)$$

where *n* is the normal direction, α is a constant, and the overbar signifies a local average. The interpolation method was shown to give useful digital representations of amplitude and phase distributions as produced by numerical models of tidally dominated bays, as well as datum planes, such as the ellipsoidally-referenced MSL or the MSL-to-MLLW difference (Hess, 2003; Hess, 2002).

The first step in the application of the interpolation method to Puget Sound, Washington, was to create a regular grid of square cells. A cell size of 231 m (0.125 nautical miles) was chosen to resolve many of the narrow passages in the Sound. This yields an undifferentiated grid with 329 cells in the eastward direction and 560 cells in the northward direction. A digitized coastline, obtained from the web site of NOAA's National Geophysical Data Center, was used to define the land-water boundary. A small number of natural passages were too narrow to be resolved automatically, and so were added by manual editing of 70 cells. The resulting grid (Fig. 1) has 37,280 water cells.

The next step was to apply the LE interpolation to the required datum levels. Tidal datums for stations were used for boundary values if the station was adjacent to water; of those available (69), 60 were used (Fig.1). The solution method for the LE, successive over-relaxation, required approximately 1,700 iterations to reach convergence for each of the datum fields (MHHW, MHW, MLW, and MLLW). Convergence was defined as when the maximum change between iterations in the numerical solution at any cell was less than 2.5×10^{-5} times the difference between the maximum and minimum input datum values. The spatially-interpolated tidal datums for MHHW, MHW, MLW, and MLLW for Puget Sound are shown in Figures 2 to 5, respectively.

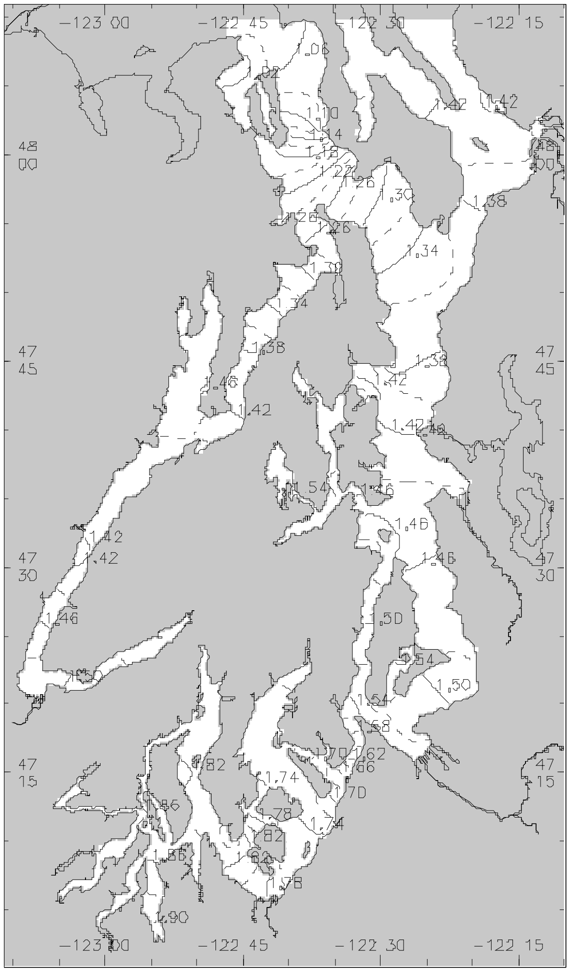

Figure 2. Contours of the interpolated MHHW tidal datum field (m).

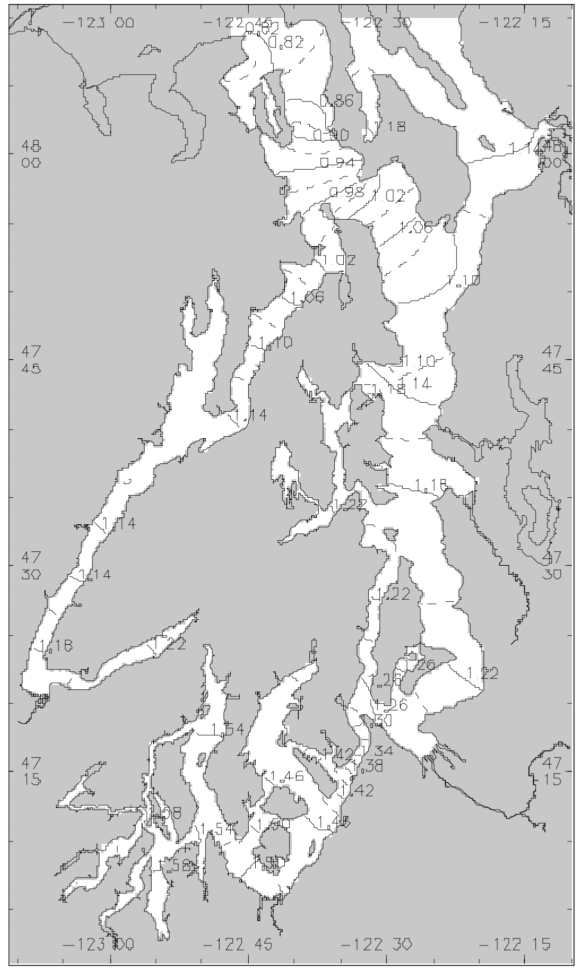

Figure 3. Contours of the interpolated MHW tidal datum field (m).

4. DISCUSSION

The accuracy of the computations is, in general, difficult to assess since there are few published datum fields based solely on observations. However, the accuracy was estimated in the following manner. For the MHHW datum, 60 additional approximations of the datum field were generated. To generate each new field, one of the 60 stations was removed as input, a different station for each new field. Then the value from each new field at the cell where the datum input was removed was compared to the value at the same cell in the original field, where the datum input was used. The root mean square (RMS), average, and maximum differences (original value minus new value) are 3.4 cm, -0.3 cm, and 16.4 cm, respectively. The maximum difference, at Bush Point on the west side of Whidbey Island (at 48° 2.0' N and 122° 36.2' W), occurs in a region with a relatively large horizontal datum gradient (appx. 3 cm per km).

Another useful, but non-rigorous, assessment of accuracy was made by plotting the interpolated tidal datums along the axes of the main channels (shown in Figure 1) and the datums at the nearby tide stations (Figure 6). The results show that the interpolated solution varies little from the input data values.

A third approach to assessing accuracy is to compare interpolated fields with the datums from tidal hydrodynamic models. For example, a tidal model for Puget Sound has been applied to the task of computing tidal datum fields (Mofjeld et al., 2002). In a comparison of the spatially interpolated solution and the modeled datum at 471 locations, the RMS difference for all datums was 5.2 cm. Statistics on the differences between the two models are shown in Table 1.

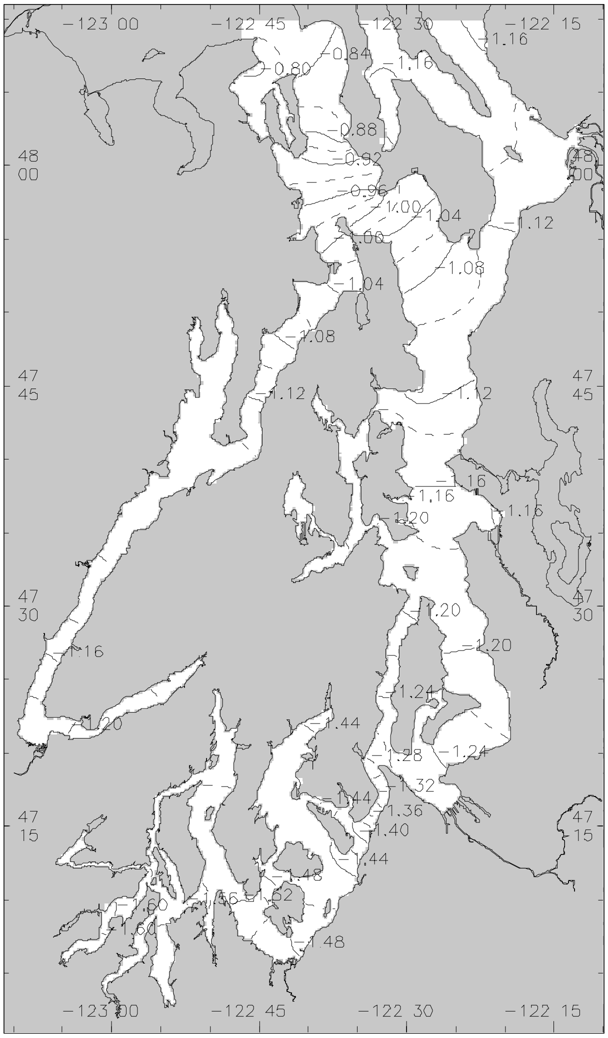

Figure 4. Contours of the interpolated MLW tidal datum field (m).

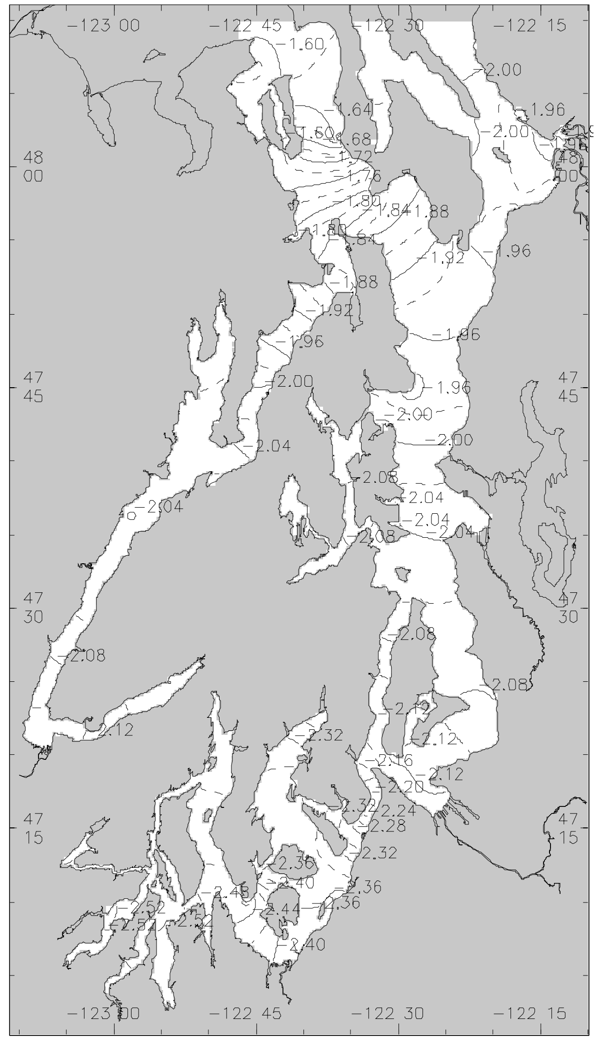

Figure 5. Contours of the interpolated MLLW tidal datum field (m).

TABLE 1

Differences in tidal datum fields (interpolated minus hydrodynamic model-based values).

Tidal Datum	Avg. Diff. (cm)	RMS Diff. (cm)	Max. Abs. Diff. (cm)
MHHW	-5.8	6.4	11.8
MHW	-2.6	3.3	7.5
MLW	-1.3	2.9	10.9
MLLW	-5.9	7.0	18.4

Since hydrodynamic models incorporate the physics of water flow, they can produce datum fields that more accurately represent bathymetric influences. However, these models typically require long periods of time (months to years) to calibrate and, because of the complexity of the flow field, usually do not exactly match the data at the tide stations. By contrast, spatial interpolation matches the data at the tide stations but involves no tidal physics. NOS is exploring the approach of taking the results from hydrodynamic models, where they exist, and then spatially-interpolating the errors to produce final, corrected datum fields.

Finally, since datum values at locations other than water level stations are difficult to obtain, especially offshore, NOS has plans to develop rapidly-deployable buoys with GPS positioning capability, but these would not be ready for a few years.

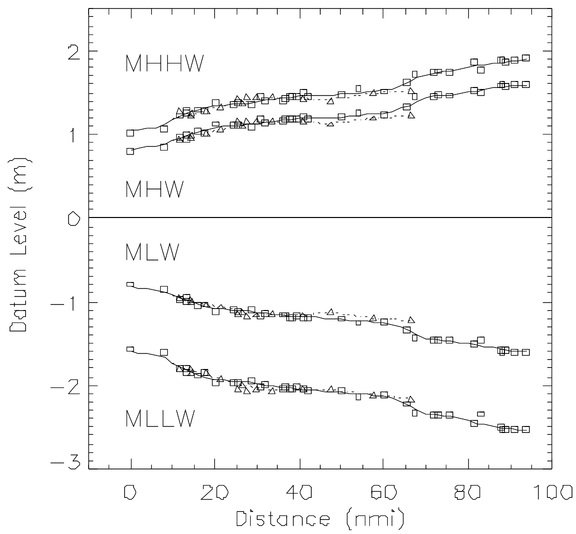

Figure 6. For four tidal datums, the interpolated elevations are shown along the main channel (solid line) and data from nearby tide stations (squares), and along Hood Canal (dashed line) and nearby stations (triangles). The main axis and Hood Canal axis are shown in Fig.1.

5. SUMMARY AND CONCLUSIONS

The spatial interpolation of tidal datums is a fast and relatively accurate method of producing tidal datum fields. For Puget Sound, the MHHW field ranges from 0.8 m at the northern end to 2.0 m at the southern end, and the MLLW ranges from -1.6 m at the northern end to -2.5 m at the southern end. The relative accuracy of the MHHW field was estimated to be 3.4 cm. In coastal regions where a tidal hydrodynamic model has been calibrated, the modeled datum field can be corrected by interpolating the errors at the tide stations to produce a more accurate field. These datum fields will facilitate the development of the VDatum tool for the Puget Sound region.

REFERENCES

Gesch, D., and R. Wilson, 2002: Development of a seamless multisource topographic/ bathymetric elevation model for Tampa Bay. *Marine Technology Society Journal*, 35(4), 58–64.

Gill, S. K., and J. R. Schultz, 2001: Tidal Datums and Their Applications. Silver Spring, Maryland. NOAA Special Publication NOS CO-OPS 1, 111p. + appendix. [Available from Center for Operational Oceanographic Products and Services, Rm. 6633, 1325 East-West Highway, Silver Spring, Maryland 20910]

Hess, K.W., 2001: Generation of Tidal Datum Fields for Tampa Bay and the New York Bight. NOAA Technical Report NOS CS 11, 43p. [Available from Coast Survey Development Laboratory, Rm. 7806, 1315 East-West Highway, Silver Spring, MD 20910]

_____, 2002: Spatial interpolation of tidal data in irregularly-shaped coastal regions by numerical solution of Laplace's equation. *Estuarine, Coastal and Shelf Science*, 54(2), 175-192.

_____, 2003: Water level simulation in bays by spatial interpolation of tidal constituents, residual water levels, and datums. *Continental Shelf Research*, 23(5), 395-414.

Milbert, D.G., 2002: Documentation for VDatum (and VDatum Tutorial); Vertical Datum Transformation Software. Ver. 1.06 (http:// nauticalcharts.noaa.gov/bathytopo/vdatum.htm).

Mofjeld, H. O., A. J. Venturato, V. V. Titov, F. I. González, and J. C. Newman, 2002. Tidal Datum Distributions in Puget Sound, Washington, Based on a Tidal Model. NOAA Technical Memorandum OAR PMEL-122, 35pp. [Available from the Pacific Marine Environmental Laboratory, 7600 Sand Point Way, Seattle, WA 98115]

Parker, B.B., D. Milbert, R. Wilson, J. Bailey, and D. Gesch, 2001: Blending bathymetry and topography: the Tampa Bay demonstration project. *Proceedings, U.S. Hydrographic Conference 2001,* Norfolk, VA*,* The Hydrographic Society of America, 11p.