4B.2 THE 'AREA INTEGRATED Z/ZDR' TECHNIQUE FOR IMPROVED RAINFALL RATE ESTIMATES WITH OPERATIONAL POLARISATION RADAR

Robert J Thomspson *and Anthony Illingworth, University of Reading, UK

1 INTRODUCTION

Observations of differential reflectivity, Z_{dr} provide information on the mean raindrop shape and hence raindrop size. When combined with the conventional reflectivity it should be possible to estimate both the size and the concentration of the raindrops and so infer a more accurate rainfall rate than is possible from Z alone. However, it appears that in an operational environment the accuracy with which Z_{dr} can be estimated at each gate is not sufficient to realise the potential of the method. In this paper we suggest an alternative approach. The values of Z and Z_{dr} are examined at each gate over a region where the raindrop spectra are supposed to have the same characteristics, and these values are then used to derive an appropriate $Z = aR^b$ relationship for use over that region.

2 Z-R RELATIONSHIPS AND NORMALIZED GAMMA RAINDROP SPECTRA

The natural variability of rain drop spectra is well captured by the normalized gamma function:

$$N(D) = N_w f(\mu)\left(\frac{D}{D_0}\right)^{\mu} \exp\left(-\frac{(3.67+\mu)D}{D_0}\right) \quad (1)$$

where

$$f(\mu) = \frac{6}{(3.67)^4}\frac{(3.67+\mu)^{\mu+4}}{\Gamma(\mu+4)} \quad (2)$$

with three independent parameters, N_w, the normalised concentration, D_o, the median volumetric drop diameter, and μ a shape factor for the width of the spectrum. The normalization factor $f(\mu)$ is chosen so that for a given N_w the value of the liquid water is indepenent of μ. The numerical factor ensures that when $\mu = 0$ we have the conventional exponential form $N(D) = N_w exp(-3.67(D/D_0))$.

Assuming a Marshall-Palmer type raindrop spectrum, then as the rain becomes heavier D_0 increases but N_w and μ remain constant. Integration over suitably weighted values of Eqn(1) predicts Z varying as $N_w D_O^7$ and R as $N_w D_0^{4.67}$. Eliminating D_0 gives $Z = aR^b$ with $b = 1.5$ and a varying as $1/\sqrt{N_w}$ (Bringi and Chandrasekar, 2001). The oft quoted "factor of two" error in the value of R would then arise in natural rain if N_W varies by up to a factor of ten and consequently 'a' would change by up to a factor of three.

A constant value of N_w with varying rainfall rate is widely assumed, as in the 'ZPHI' technique of Testud et al (2000), but it is quite possible that N_w is a function of D_0. This will give rise to different values of b. For example, if N_w falls as $1/D_0$ then integration gives that Z varies as D_0^6 and R as $D_0^{3.67}$ and elimination gives us $Z = aR^b$ with $b = 1.63$. The index of 1.6 is widely used in Europe as a default Z-R relation. Alternatively, if N_w rises as D_0^2 then we would have Z and R varying as D_0^9 amd $D_0^{6.67}$, respectively, leading to a value of b=1.34, close to the default Nexrad Z-R relation. The extreme example would be D_0 remaining constant, in this case, as N_w rises, Z and R would scale together and we would have Z=aR, as has been suggested by List (1988).

Fig. 1 shows the full computed solutions for the various N_w versus D_0 combinations for μ = 5 using the correct terminal velocity relationship, rather than assuming V(D) varies as $D^{0.67}$, and demonstrates they are very close to the predicted power laws. The value of N_w for a rainfal rate of $1 mm\, hr^{-1}$ is fixed in all three cases to be $8000 m^{-3} mm^{-1}$ which for $\mu = 5$ implies a D_o of 0.867mm and a Z of 21.72dB, so the value of a in all three equations is 148, but b varies.

In the next section we show how analysis of observed values of Z and differential reflectivity Z_{dr} provides information on raindrop size and concentration and reveals how N_w and D_0 are linked and can be used to predict the best values of a and b to be used in a Z-R relation.

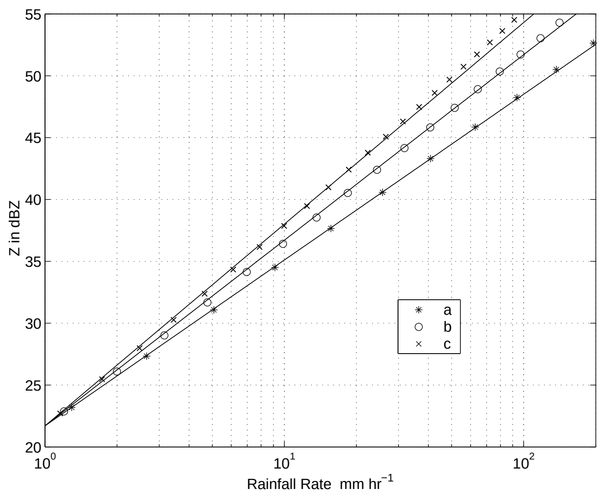

Figure 1: The value of Z and R computed for $\mu = 5$ and $N_w = 8000 m^{-3} mm^{-1}$ for $D_o = 0.867 mm$ and R=1$mm\, hr^{-1}$. a) $N_w \propto D_o^2$ b) N_w const, c) $N_w \propto D_o^{-1}$. The straight lines all have a = 148, but b is 1.34, 1.5 and 1.63, respectively.

* *Corresponding author address:* Anthony J Illingworth, Dept. of Meteorology, University of Reading, Earley Gate, PO Box 243, Reading RG6 6BB, UK; e-mail: a.j.illingworth@reading.ac.uk.

3 THE LINK BETWEEN DIFFERENTIAL REFLECTIVITY, DROP CONCENTRATION AND DROP SIZE

The differential reflectivity, Z_{dr}, essentially measures the mean raindrop shape and hence once the drop shapes are known can be related to the equivolumetric diameter, D_0, in the normalized gamma function. Because Z_{dr} is a ratio it is independent of N_w, so for a given Z_{dr} (constant D_0) both Z and R will scale with N_w. For each value of Z_{dr} we can calculate the value of Z for a rainfall rate of $1 mm\,hr^{-1}$: Z($1 mm\,hr^{-1}$) = f(Z_{dr}). If we were to observe Z_{obs} is a factor 'x' above this value of Z, then we can say that the rain rate is x $mm\,hr^{-1}$. In Fig 2 are plotted the values of Z and Z_{dr} expected for a rainfall rate of 1 $mm\,hr^{-1}$ for various values of μ using the drop shapes of Goddard et al (1995) based on comparing observed Z_{dr} values above a ground based disdrometer. Essentially the same curves are produced if the later laboratory shapes of Andsager et al (1999) are used. For comparison with Fig 1 the solid line in Fig 2 describes the values of Z and Z_{dr} for N_w = $8000 m^{-3} mm^{-1}$. We see that R=$1 mm\,hr^{-1}$ has a Z of 21.7dBZ and a Z_{dr} of just 0.14dB. For this value of N_w a Z_{dr} of 0.5dB would be associated with a rainfall rate of just under $10 mm\,hr^{-1}$.

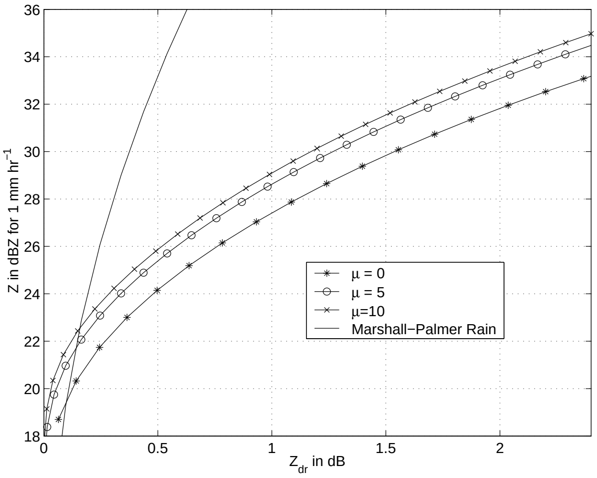

Figure 2: The value of Z for a rainfall rate of 1mm/hr as a function of the observed Z_{dr} for different values of μ. The solid line are the values of Z and Z_{dr} for const $N_w = 8000 m^{-3} mm^{-1}$.

The slopes of the lines in Figure 2 provide us with an estimate of the required accuracy of the Z_{dr} estimate. The steep slope for low rainfall rates means that for a rainfall rate of $3 mm\,hr^{-1}$ to be estimated to within 25% then Z_{dr} must be known to better than 0.1dB. For $10 mm\,hr^{-1}$ this can be relaxed to 0.2dB. Many flood producing situations in Europe result from prolonged rainfall of less $10 mm\,hr^{-1}$; unfortunately achieving an accuracy of Z_{dr} at each gate of 0.1dB is impossible for an operational radar (see Illingworth, 2003 for a detailed discussion). The statistical noise in the Z_{dr} estimator means that even if the correlation between the H and V returns is 0.98, then 60 independent samples are needed to achieve 0.2dB accuracy. Unrealistically long dwell times would be needed for 0.1dB. In addition, for operational radars with imperfect antennas, effects such as reflectivity gradients in the presence of mismatched sidelobes, mismatched beam patterns in H and V leading to a lowering of the correlation, and triple scattering echoes will all contribute noise into the individual Z_{dr} estimates.

In the next section we outline a different approach which relies on the characteristics of Z and Z_{dr} to provide information on drop sizes and concentrations.

4 THE AREA INTEGRATED Z/ZDR TECHNIQUE

Observations of Z and Z_{dr} taken in convective rain over a 4km square region of a low level PPI with the narrow beam (0.28deg) Chilbolton S-band radar are displayed in Fig.3. Values of Z are ranging from 30dBZ to 55dBZ. Even with this high quality research radar the data points show considerable scatter; the spread in Z for a given Z_{dr} is over 10dB indicating that inferred values of N_w over this small region are scattered over an order of magnitude. Similarly, for a given Z there is a big spread of observed Z_{dr}, indicating that for a particular value of Z the rainfall rates are changing by over a factor of two. The dash-dot line is a polynomial fit of Z against Z_{dr} but of course since the rainfall rates are distributed around this in a logarithmic fashion, if the scatter reflects reality, then the mean rainrates will be above this line. We suggest that if the scatter is due to random noise arising from instrumental imperfections, using individual data points will lead to a biased retrieval in the rainfall rate.

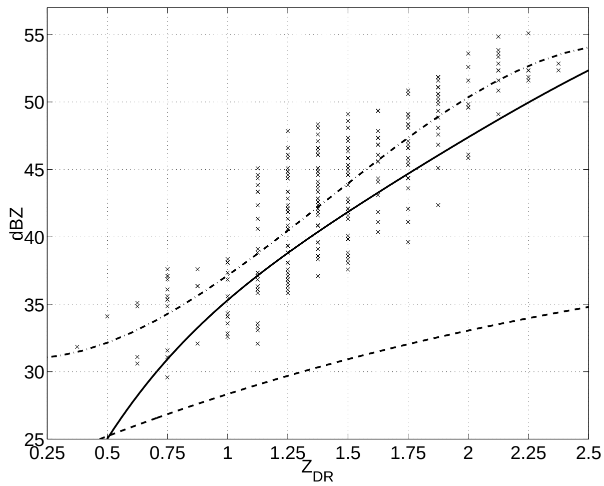

Figure 3: Observed values of Z and Z_{dr} for a 4km square region of convective rain. Solid line - Marshall-Palmer; Dashed line: R=1 $mm\,hr^{-1}$; Dash-dot line - polynomial fit.

A polynomial fit of Z as a function of Z_{dr} is plotted as the dash-dot line in Fig 3 and is about 2-3dB above the Marshall-Palmer line, indicating that the values of N_w are approximately constant but rather higher than $8000 m^{-3} mm^{-1}$, and that a should be lower than the 148 in Fig 1. The difficulty with this approach is that a polynomial fit of Z_{dr} against Z would give a very different relationship.

The alternative approach is shown in Fig 4, where the indivdual observations of Z and Z_{dr} in Fig 3 have been converted into Z and LogR and plotted, together with the b=1.5 Marshall-Palmer line from Fig 1. Because the data are so scattered, a regression of Z on Z_{dr} will produce a different result from the regression of Z_{dr} on Z. In this case the most appropriate fit is the so-called 'standard deviation' or 'SD' line. If we compute the average value of the data points, $\overline{logR}$ and $\overline{logZ}$ and their standard deviations σ_{logR} and σ_{logZ}, then the SD line passes through the point $\overline{logR}$ and $\overline{logZ}$ with the gradient $\sigma_{logZ}/\ \sigma_{logR}$. In this convective case the SD line suggests we have a=433.5 and b=1.33.

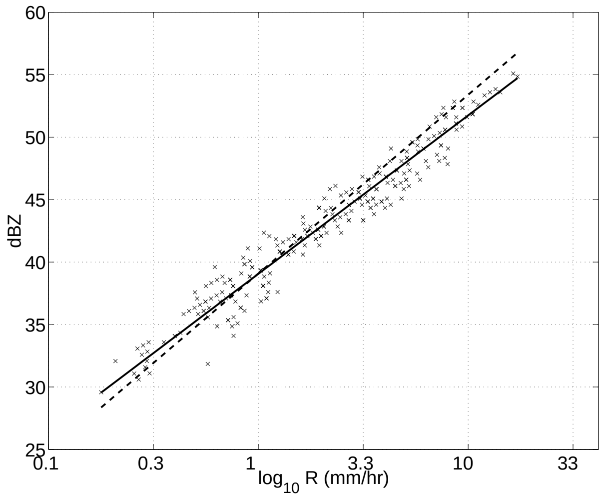

Figure 4: The data in Fig 3 plotted in logZ-logR space. Solid line: S-D line equivalent to $Z = 433R^{1.33}$. Dashed line: Marshall-Palmer with b=1.5.

The observations in stratiform are rather different and are displayed for a 4km square in Fig 5. In this case the range of the data is less with Z values between 28dBZ and 40dBZ and the highest values of Z_{dr} only 1.8dB. Although there is again a large scatter of the individual data points, there is a definite suggestion that when compared to the const N_w Marshall-Palmer line, N_w is falling as the drops get larger and D_0 increases.

The stratiform data is replotted in logZ-logR space in Fig 6. Because of the smaller dynamic range the relative scatter of the data points is much greater than in Fig 4, and so polynomial fits or regression lines will give inconsistent values. The S-D line gives $Z = 321R^{1.62}$.

5 CONCLUSIONS

We suggest that the different $Z = aR^b$ relations can be explained in terms of the variability of N_w and D_0 in the normalized gamma function used to represent the range of naturally occurring drop size spectra. As rainfall becomes heavier D_0 generally increaseas and if N_w remains constant then $b = 1.5$. If N_w falls with increasing D_0 then b rises above 1.5, but if N_w falls then b is lower than 1.5. In theory, observations of Z_{dr} combined with Z should enable values of N_w and D_0 to be derived at each gate, but in pracice the values at individual gates are too noisy for this to be possible.

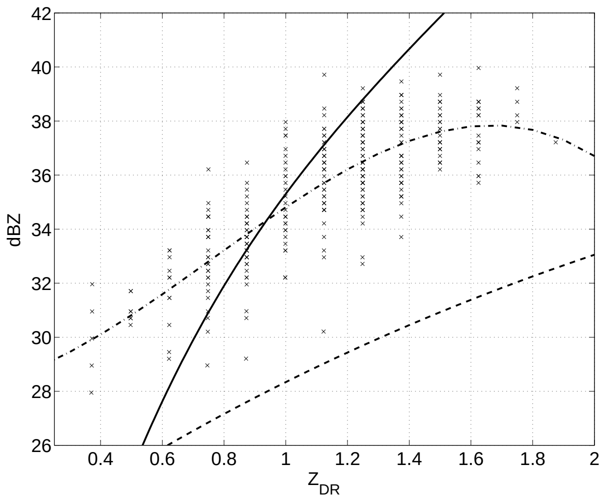

Figure 5: Observed values of Z amd Z_{dr} as for Fig 3 but for stratiform rain.

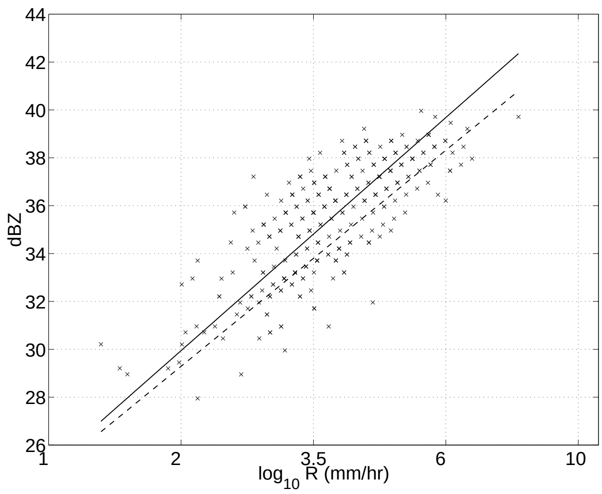

Figure 6: AS for Fig 4 but for 4km square region of stratiform rain. Solid line is S-D line equivalent to $Z = 321R^{1.62}$.

We suggest a new approach, whereby the values of logZ and logR derived from observed Z and Z_{dr} over a small region are examined, and an 'SD' fit through the data provide a better estimate of a and b in the Z-R relation. This method assumes that the instrumental effects inroduce random noise into both of the observables, Z and Z_{dr}, so that an averaged value will approach reality. However, becasue the inferred rain rates are logarithimically distributed in Z/Z_{dr} space, averaging of the noisy values at individual gates could produce a biased value of rainfall.

Initial tests on a 4km saqure region of convective rain and a similar sized region of stratifrom rain, yield the relationships, $Z = 433R^{1.33}$ and $Z = 321R^{1.62}$. This seems to confirm the many emprical findings recently published, that the convective rain has a higher concentration of raindrops than stratiform, and it also suggests that higher values of b, such as the 1.6 used as a default in Europe' really are more appropriate for stratiform conditions. Stratiform rainfall originates from melting snow, and it may be that in heavier precipitation the increased aggregation of snowflakes is responsible for the fall in N_w.

It seems that this method could be applied operationally and, when consideration is given to the broader beamwidth of such radars, appropriate values of a and b derived over each square region of side 10 or 20km. We plan to investigate this further.

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